The effect of interventions on realignment probabilities

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Abstract

The expected future change of the exchange rate within its currency band and the expected realignment rate are estimated using the Regime–Switching Model. There exists an unobserved variable $s_t$, which characterises the equilibrium state of the expected future change of the exchange rate within its currency band at any time $t$ with certain probabilities. Different values of $s_t$ correspond to states with high and low risk of realignment, respectively. The probabilities of switching between one regime and another depend on central bank intervention in the foreign exchange market. Daily data on intervention by Norges Bank are used. The data contain relatively few actual realignments, and the sample distribution of realignments may not be representative enough to capture the discrete changes in the exchange rate caused by a non-zero subjective probability of realignment (even when no realignment has in fact taken place). This causes the very well known peso problem in the estimation. © 2000 Elsevier Science B.V. All rights reserved.

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1. Introduction

As Rogoff (1984) states, sterilised interventions can influence the exchange rate either by altering public perceptions of future monetary policy or by affecting the exchange rate risk premium. The latter effect is often called the ‘portfolio balance’
effect of interventions (cf. Loopesko, 1984; Rogoff, 1984; Dominguez, 1989; Dominguez and Frankel, 1993), while the former is called the ‘signalling’ effect (cf. Mussa, 1981; Lewis, 1995; Fatum and Hutchison, 1999; Mundaca, 1999a,b). The ‘portfolio balance’ effect is one in which sterilised interventions change the currency denomination of relative asset supplies and thereby the exchange risk premium, if assets are imperfect substitutes. The ‘signalling’ effect is one in which sterilised interventions provide new information about policy objectives.

In this paper we study the ‘signalling effect’ of central bank intervention on future exchange rate development, by focusing on the effect of sterilised interventions on the exchange market participants’ subjective probabilities of future realignments. We obtain estimates of the realignment expectations, which enable us to draw conclusions about the credibility of the exchange rate policy. Here, exchange market participants are assumed realise that the exchange rate may change from an initially stable state to a state of currency crisis as a result of changes in monetary policy. Reasonably, information about central bank intervention policies should in principle signal central bank preferences about the management of the exchange rate.

Under a currency band regime, the central bank may choose between defending a narrower implicit band than the official one with frequent interventions, and allowing the exchange rate to move within the whole band with intervention only at the edges. If the latter is chosen, it is a sign of a desire for some monetary independence. In this context, monetary independence can be achieved when the exchange rate moves freely within its band while the monetary authorities can adjust interest rates to local conditions, at least for a short time. For example, by lowering the domestic interest rate in a recession and increasing it during a boom. On the other hand, frequent interventions to defend a narrower band will signal that the objective is to avoid large variations in the exchange rate. This policy may lead to a reduction of not only the expected depreciation of the exchange rate within the band, but also the subjective probability of changing to a state of currency crisis. In the current study, one objective is to analyse whether frequent interventions that aim at holding the exchange rate close to its central parity, reduce the subjective probability that the exchange rate will switch to a state of currency crisis in the next period. This will be compared with the case where interventions occur only at the edges and the exchange rate is then let to move within the whole band. If market participants believe that it becomes harder to defend the band once the exchange rate reaches the band edges (especially the weak edge), interventions may increase instead of reduce the subjective probability that the exchange rate will switch to a state of currency crisis at some future time. Maintaining the credibility of the currency band will then be harder when the exchange rate is allowed to move within its whole band.

The so called ‘drift-adjustment’ method suggested by Bertola and Svensson (1993) has been used quite widely to estimate realignment expectations within currency bands. However, it can be shown that the estimates of the expected realignment rates (à la drift adjustment) are not consistent\(^2\), because this method

\(^2\) An estimator, \( \hat{\Theta} \), is consistent if by increasing the sample size it can be made to lie arbitrarily close to the true value, \( \Theta \), with a probability arbitrarily close to one. This is often denoted by \( \text{plim}_{n \to \infty} \hat{\Theta} = \Theta \) where \( \text{plim} \) stands for probability limit.
does not resolve the so called ‘peso problem’\(^3\). It is here proposed the well-known Markov–Switching Model for taking care of the ‘peso problem’ and providing consistent estimates of the expected rate of realignment.

This paper has three objectives. The first is to estimate the expected future change of the exchange rate within its currency band, and the expected realignment rates. It is assumed that exchange market participants have certain probabilities that the depreciation rates within the currency band may change from one equilibrium state (regime) to another, but do not know when the changes will occur. The probabilities associated with future regime switches are then specified and estimated. In the model structure, the exchange rate can switch between two equilibrium states, a high risk and a low risk of realignment state.

To achieve this objective, the transition probabilities are first assumed to be constant over time but also to be depend on central bank interventions. Previous studies, using other econometric models, take the probabilities of realignment to depend on some macroeconomic fundamentals, but not on central bank interventions. See for example, Blanco and Garber (1986), Grilli (1990), Edin and Vredin (1993), Goldberg (1994). A theoretical model with exogenous realignment probabilities has been developed by Bertola and Caballero (1992). Filardo (1991), Weinbach (1998), Filardo and Gordon (1998) have also considered a Switching–Markov Model with endogenous transition probabilities but using different data set.

The second objective of this study is to analyse whether the transition probabilities and the expected depreciation rates depend on the type of central bank intervention policy.

The last objective is to analyse how different types of information on interventions may affect the expectations of the exchange market participants. This will be analysed when the transition probabilities depend on central bank interventions. We distinguish between information on daily interventions and on accumulated interventions over a number of days (5 and 25 days). In particular, information on 1-day intervention may not be sufficient to convince the market that the central bank is committed to defend the currency band.

The model is tested using daily data on sterilised spot interventions and the exchange rate, for two different exchange rate policy regimes pursued by Norges Bank between October 1986 and February 1990, with the policy change, as announced by the authorities, occurring in mid-June of 1988\(^4\).\(^5\). In the first period, the exchange rate fluctuated between the upper and lower edges of the official currency band with interventions occurring mainly close to and at the edges. In the

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\(^3\) See Mundaca (1999c) for further discussion.

\(^4\) The Norwegian target zone until 19 October 1990 had a band equal to ±2.25% around a par exchange rate specified in terms of a trade-weighted basket of foreign currencies. This exchange rate is called the Norwegian currency basket index. From 22 October 1990 until 11 December 1992, the Norwegian krone was pegged to the ECU with a band also equal to ±2.25%.

\(^5\) The period of study begins in October 1986, because the data on interventions are only available from that date.
second period, the exchange rate was only allowed to move within an unofficial implicit narrower band, within which there have been more frequent interventions by the central bank.

The high degree of contemporaneous (same day) interaction between the exchange rate and intervention leads to an econometric problem that needs to be faced. Interventions normally have an immediate impact on the exchange rate, while exchange rate movements also have an immediate effect on the decisions to intervene (the reaction functions of the central bank). If most of the interaction between exchange rates and intervention occurs within a single day, and we do not have information on the timing of the intervention, it is not easy to determine empirically the causal direction of contemporaneous interaction. To deal with this problem, interventions enter as explanatory variables in the subjective probabilities, with a 6-day lag. The same holds for accumulated interventions over 5 and 25 days. Note that this procedure does not make intervention exogenous; it is only a way to deal with the problem of contemporaneous interaction. Mundaca (1999a,b) studies the effect of interventions on the exchange rate, while treating them as simultaneously determined.

Section 2 contains an account of the theoretical background for the present study. Section 3 presents the econometric model, while Section 4 presents the data and methodological considerations. Section 5 consists of the empirical results and Section 6 is the conclusion.

2. Theoretical background

Let us define:

\[ x_t \equiv e_t - c_t, \]  

where \( c_t \) denotes the log of the central parity of the exchange rate band, and \( x_t \) is the deviation of the (log of the) exchange rate, \( e_t \), from \( c_t \). A realignment is typically defined as a change in \( c_t \). Assuming uncovered interest rate parity (UIP) we write Eq. (1) in terms of rates of change and conditional expectations on information available at time \( t (I_t) \):

\[ \frac{E_t \Delta c_{t+\tau}}{\tau} \equiv \delta(\tau) - \frac{E_t \Delta x_{t+\tau}}{\tau}, \]  

\( ^6 \) The data on interventions are spot interventions. Note that, in general, a central bank intervention at day \( t \) implies a deal on the amount of intervention and the exchange rate level, this is called a spot intervention. Nevertheless, the settlement takes place on day \( t + 2 \). Norges Bank makes public the spot intervention with a lag of an average of 6 days from the day this intervention was made.

\( ^7 \) UIP implies that if the domestic currency is expected to depreciate, interest rates on assets denominated in terms of this currency will exceed those abroad by the expected rate of depreciation, i.e. \( \delta(\tau) = \frac{E_t \Delta e_{t+\tau}}{\tau} dt \).
where \( \delta(\tau) \) is the differential between the domestic and foreign interest rates with maturity \( \tau \). \( \tau \) and \( dt \) represent the fraction of the year that corresponds to the maturity.

Whereas data on \( \delta(\tau) \) are usually available, data on \( E_t \Delta x_{t+\tau} / \tau \; dt \), the expected depreciation rate, are not. The estimation of the expected rate of depreciation within the band is usually hindered by a 'Peso Problem'. As Svensson (1993) has pointed out, this problem arises when the market perceives even a small probability that a realignment may take place (even when, in fact, no realignment has actually taken place), inducing a likely jump in \( x_{t+\tau} \). The data typically show relatively few actual realignments, and the sample distribution of realignments may not be representative enough to capture the discrete changes in \( x_{t+\tau} \).

The drift adjustment method defines the expected rate of devaluation as:

\[
E_t \Delta x_{t+\tau} + p_t \tau \left( E_t \left( \frac{\Delta x_{t+\tau}}{\tau \; dt[I_t, R]} \right) - E_t \left( \frac{\Delta x_{t+\tau}}{\tau \; dt[I_t, NR]} \right) \right) = \delta_t(\tau) - E_t \left( \frac{\Delta x_{t+\tau}}{\tau \; dt[I_t, NR]} \right),
\]

where \( p_t(\tau) \) is the probability at time \( t \) that a realignment will take place in the period between \( t \) and \( t+\tau \), and \( R \) and \( NR \) stand for realignment and no-realignment, respectively. The method suggests that one can obtain \( E_t(\Delta x_{t+\tau} / \tau \; dt[I_t, NR]) \) by regressing the realised depreciation rates within the band, \( x_{t+\tau} \), exclusive of the observations around the time an actual realignment took place\(^{10}\) on the information set \( I_t \).

In our opinion, such a regression yields the estimated rate of depreciation within the band, conditional upon ex-post actual no-realignment, not conditional upon ex-ante expectations of no-realignment. The 'Peso Problem' still needs to be taken care of. The frequency of the actual realignments observed in the data need not coincide with the frequency of the non-zero (unobservable) subjective realignment probabilities, even if they are small.

An alternative procedure is needed to estimate consistently \( E(\Delta x_{t+\tau} / \tau \; dt[NR, I_t]) \) or, better still, \( E(\Delta x_{t+\tau} / \tau \; dt[I_t]) \), while taking care of the peso problem\(^{11}\).

\( ^8 \) Eq. (3) is obtained from considering, \( E_t \Delta x_{t+\tau} = \delta_t(\tau) - p_t(\tau)E_t(\Delta x_{t+\tau} / \tau \; dt[I_t, R]) - (1 - p_t(\tau))E_t(\Delta x_{t+\tau} / \tau \; dt[I_t, NR]). \)

\( ^9 \) This estimate is then subtracted from \( \delta_t(\tau) \) to obtain an estimate of the expected rate of devaluation.

\( ^{10} \) In practice, the observation at the time of realignment is sometimes excluded, but observations, previous to the time of realignment may also be excluded. This is done in order to net out the jump of the exchange rate within the band that usually occurs at the point of realignment (Svensson, 1993). If no realignment takes place during the studied period, no observations are excluded.

\( ^{11} \) It is misleading to argue that the method is only concerned with what happens with the depreciation of the exchange rate when there has not been an actual realignment. After all, the initial objective was apparently (and should be) to estimate devaluations (or realignment) expectations to make an assessment of exchange rate regime credibility.
3. A model for expected realignment and central bank interventions

Our aim is to obtain consistent estimates of the expected realignment rates. We then need first to estimate the mean of the expected depreciation of the exchange rate within its band, and take care of the ‘Peso Problem’.

Using the models of Lindgren (1978) and Hamilton (1989), we assume that there exists an unobserved random variable, \( s_t \), that takes on the value one or two with certain probabilities that correspond to the states of high and low risk of realignment, respectively\(^{12}\). That is, \( s_t + \tau = 1, 2 \) characterises the state of the change of the exchange rate within the band at an arbitrary time \( t + \tau \).

At an arbitrary time \( t + \tau \), the market then knows that the current change in the exchange rate within the band is drawn from a mixture of two normal distributions. When the depreciation within the band is in state 1, it is drawn from the distribution \( \mathcal{N}(\mu_1, \sigma_1^2) \); and when it is in state 2, it is drawn from the distribution \( \mathcal{N}(\mu_2, \sigma_2^2) \). At time \( t + \tau \), the market then knows that\(^{13}\):

\[
\frac{\Delta x_{t+\tau}}{\tau} = \frac{(x_{t+\tau})}{\tau} = \mu_i + \epsilon_{t+\tau} \quad \text{if} \quad s_{t+\tau} = i; \quad i = 1, 2; \tag{4}
\]

where \( \mu_i \) and \( \sigma_i \) denote the mean and variance, respectively, of the depreciation of the exchange rate within the band in state \( s_t + \tau = i \), while \( \epsilon_{t+\tau} \) is drawn from a distribution \( \mathcal{N}(0, \sigma^2) \). Thus, the density function of \( \Delta x_t/\tau \) conditional upon \( s_{t+\tau} \) is:

\[
f\left(\frac{\Delta x_{t+\tau}}{\tau} \mid s_{t+\tau} = i; \theta_i \right) = \frac{1}{\sqrt{2\pi\sigma_i}} \exp\left(-\frac{\left((\Delta x_{t+\tau}/\tau) - \mu_i\right)^2}{2\sigma^2_i}\right) \tag{5}
\]

where \( \theta = (\mu_i, \sigma_i) \) and \( i = 1, 2 \).

Two informational possibilities are possible, (i) the market observes the current state (either state 1 or 2) and knows exactly when the equilibrium state switch will take place; or (ii) the market does not know with certainty when the state switch will take place, but it does know the current regime \( s_t + \tau \).

If (i) is the case, using Eq. (4), we obtain white noise, zero mean forecast errors:

\[
\frac{\Delta x_{t+\tau}}{\tau} - \frac{E_t \Delta x_{t+\tau}}{\tau} = \epsilon_{t+\tau}, \tag{6}
\]

\(^{12}\) Note first that a realignment implies either \( \Delta c > 0 \) or \( \Delta c < 0 \). The former implies a devaluation and latter, a revaluation. We think that there could be a high risk of realignment not only when the exchange rate approaches the edges of its band (the weak or the strong edge) but also when the exchange rate is well inside the band. The currency crisis of the 90s in Europe is empirical evidence that the latter is possible.

\(^{13}\) The maturity \( \tau \) in this study is 3 months, therefore \( \tau \) dr is approximately 1/4 year, because \( \tau \) is 63 days and \( \tau \) dr is 1/253. \( \Delta x_{t+\tau}/\tau \) represents the rates of depreciation within the band between \( t \) and \( t + \tau \), which is a quarter. The 3-month horizon (as chosen by Svensson (1993) and others) is useful for the analysis of realignment expectations to be presented later, in which the 3-month interest rate differential plays an important role. For this reason, Eq. (3) will be studied further.
Eq. (6) implies that the market (and the economist) knows exactly when there will be a switch from the low to the high risk of realignment state and vice versa. In this case though, there is no ‘Peso Problem’ to be concerned about. However, it is more realistic to assume case (ii)\textsuperscript{14}. For this purpose, we assume, as Engel and Hamilton (1990), the following Markov chain for the evolution of the unobserved state variable, $s_{t+\tau}$:

$$p(s_{t+\tau} = 1 | s_t = 1) = p_{11}$$
$$p(s_{t+\tau} = 2 | s_t = 1) = p_{12} = 1 - p_{11}$$
$$p(s_{t+\tau} = 2 | s_t = 2) = p_{22}$$
$$p(s_{t+\tau} = 1 | s_t = 2) = p_{21} = 1 - p_{22};$$

where $p_{11}$ is the transition probability of remaining in the high risk of realignment state (state 1) from date $t$ up to and including $t + \tau$, given that the process is in state 1 at time $t$, and $p_{22}$ is the transition probability of remaining in the low risk of realignment state (state 2) from date $t$ up to and including $t + \tau$, given that the process is in state 2 at time $t$.

When central bank interventions affect the equilibrium state, the functional forms of these transition probabilities, as also suggested by Diebold et al. (1994), are the following:

$$p_{11}(t) = p_s(s_{t+\tau} = 1 | s_t = 1) = \frac{\exp(\alpha_10 + \alpha_11NetI_{t-\ell})}{1 + \exp(\alpha_10 + \alpha_11NetI_{t-\ell})}$$
$$p_{22}(t) = p_s(s_{t+\tau} = 2 | s_t = 2) = \frac{\exp(\alpha_20 + \alpha_21NetI_{t-\ell})}{1 + \exp(\alpha_20 + \alpha_21NetI_{t-\ell})}$$

If $\alpha_{11}$ and $\alpha_{21}$ are zero, we will have the original Markov Switching Model with exogenous probabilities. Eqs. (8a) and (8b) are logistic functions of the $\alpha$ parameters and $NetI_{t-\ell}$, where $\ell$ denotes a lag of six days. $NetI$ denotes net interventions, the sum of international reserves sold and bought by the central bank, e.g. a negative $NetI$ means a net selling of reserves to defend the currency.

We define $NetI$ in three different ways, (i) as the net intervention that took place 6 days ago and it is denoted by $INT(1D)$; (ii) as the accumulated net intervention over 5 days until 6 days ago, denoted by $SUM INT(5D)$; (iii) as the accumulated net intervention over 25 days until 6 days ago, denoted by $SUM INT(25D)$.

\textsuperscript{14} Kaminsky (1993) model, which is an extension of Hamilton’s model, may also be used. Kaminsky assumes that the market participants do not observe the current regime directly, so that the probability of being in a certain regime at a certain time needs to be estimated as well. See also Engel (1985), Lewis (1989). This consideration should not affect our argument.
As explained above, these definitions of interventions are made to deal with the econometric problem caused by contemporaneous (same day) interaction between the exchange rate and interventions. Note also that the different definitions of Net I will serve to test whether information on, for example, a one-day intervention is sufficient to enable the market participants to form expectations about central bank intervention policy and commitment to defending the currency band.

The specification of the conditional density of \( \Delta x_{t+\tau}/\tau \) (Eq. (5)) and the transition probabilities defined by Eq. (7) or Eqs. (8a) and (8b) are all that is needed to describe the stochastic structure of the switching process.

The EM algorithm is used to obtain maximum likelihood estimates of the parameters of the model. Such an algorithm with constant transition probabilities is documented in Hamilton (1990), while the algorithm with endogenous transition probabilities is described in Diebold et al., (1994).¹⁵

The conditional expectation of the future change in the exchange rate within the band for the two states is defined as follows:

\[
\text{If } s_t = 1, \quad E_t (x_{t+\tau} - x_t) = \frac{p_{11} \mu_1 + (1 - p_{11}) \mu_2}{\tau}
\]

\[
\text{If } s_t = 2, \quad E_t (x_{t+\tau} - x_t) = \frac{p_{22} \mu_2 + (1 - p_{22}) \mu_1}{\tau}
\]

where \( p_{11} \) and \( p_{22} \) can be either exogenous or endogenous.¹⁶

4. The Norwegian exchange rate regimes and methodological consideration

We consider daily data on interventions and the Norwegian exchange rate for the period from October 1986 to February 1990, while the authorities declared them-

¹⁵ The EM algorithm maximises the incomplete-data log likelihood (of \( \Delta x_{t+\tau}/\tau \)) via iterative maximisation of the expected complete-data log likelihood conditional upon the observable data. Filardo (1991) and Boldin (1992) develop an approach based on the numerical maximisation of the incomplete data log likelihood. See the Appendix A to this paper for a very brief description of the expected complete-data log likelihood function and the EM algorithm for the maximisation of this function with respect to all the parameters of the model.

¹⁶ The forecast errors are in this case:

\[
\frac{\Delta x_{t+\tau}}{\tau} = \frac{E_t \Delta x_{t+\tau}}{\tau} = \begin{cases} 
(\mu_2 - \mu_1) (1 - p_{22}(t)) + \epsilon_{t+\tau} & \text{if } s_{t+\tau} = 2, s_t = 2 \\
(\mu_2 - \mu_1) p_{11}(t) + \epsilon_{t+\tau} & \text{if } s_{t+\tau} = 2, s_t = 1 \\
(\mu_1 - \mu_2) p_{22}(t) + \epsilon_{t+\tau} & \text{if } s_{t+\tau} = 1, s_t = 2 \\
(\mu_1 - \mu_2) (1 - p_{11}(t)) + \epsilon_{t+\tau} & \text{if } s_{t+\tau} = 1, s_t = 1
\end{cases}
\]

Note that the error term depends on the agents' subjective assessment of the probability of future realignment.
selves that a change in their intervention policy occurred in mid-June of 1988. In the period October 1986–July 1988, the stochastic process of the exchange rate depends on interventions occurring mostly close to/at the edges of the currency band and the exchange rate is allowed to move within the whole official band. In the second period, July 1988–February 1990, the exchange rate depends on interventions occurring mostly within the band (and the exchange rate moves within a narrower band than the official one).

From Figs. 1 and 2, we see that from October 1986 until mid-June 1988, Norges Bank sold and bought foreign currency (in millions of USD) basically only when the Norwegian currency basket index approached the upper and lower limits of its bands (114.5 and 109.5, respectively), i.e. when the Norwegian currency was at its weakest and strongest\textsuperscript{17}. In contrast, Figs. 3 and 4 indicate that from mid-June of 1988 until February of 1990 Norges Bank intervened more frequently, seemingly to reduce movements of the currency basket index within the band. Notice that in November 1988 the bank also intervened at the edge of the band, obviously to prevent the currency basket index from moving outside the band.

\textsuperscript{17} A couple of times during November 1987, Norges Bank also intervened when the Norwegian currency was well inside the band. This was apparently to prevent it from moving too fast to the weaker edge of the band. This was not systematic behaviour as it was in our second period.
Fig. 2. The Norwegian currency basket index first period.

Fig. 3. Norges bank’s interventions second period.
As mentioned above, there is generally a high degree of contemporaneous (same day) interaction between the exchange rate and interventions which leads to econometric problems. We deal with this problem by entering our intervention...
variable (NET $I_{t-1}$) with a lag of 6 days. This approach is reasonable because in practice, Norges Bank makes available the amount of net intervention twice a week, with a lag of an average of 6 days from the day the deal on intervention (spot intervention) took place\textsuperscript{18}.

We do not provide estimates for the determinants or reaction functions of interventions for Norges Bank; these are presented in detail in Mundaca (1999a,b). It was found that in the period from June 1988 to February 1990, deviations of the exchange rate from its parity were the variable that most affected intervention decisions and that the interest rate differential was quite low (see Fig. 6). Over the period October 1986–June 1988, large movements of the exchange rate within its band predominated and the Norwegian krone was always vigorously defended whenever it reached the weaker edge of its band. During this period, the interest rate differential was relatively high (see Fig. 5).

One could consider both periods as one sample. As will be shown, the two periods are quite distinct and characterised by completely different realignment expectations\textsuperscript{19}. It is also interesting to see how the different types of intervention policy on the part of Norges Bank have affected the market participants’ subjective probabilities of realignment. Certain types of intervention policy may be more effective in reducing realignment expectations than others. In terms of our model,

\textsuperscript{18} This 6-day lag includes the 2-day delay in making the transaction (transfer of foreign exchange) effective.

\textsuperscript{19} The results of the estimation of the whole period are always available upon request.
Table 1
Parameter estimates with constant transition probability (estimates of means and variances in percentages)\(^a\)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>NOK (first period)</th>
<th>NOK (second period)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\mu_1)</td>
<td>6.952 (0.515)</td>
<td>6.295 (0.360)</td>
</tr>
<tr>
<td>(\mu_2)</td>
<td>-6.766 (0.141)</td>
<td>-0.586 (0.136)</td>
</tr>
<tr>
<td>(\sigma_{12})</td>
<td>36.719 (4.339)</td>
<td>7.062 (1.332)</td>
</tr>
<tr>
<td>(\sigma_{22})</td>
<td>4.043 (0.410)</td>
<td>5.513 (0.449)</td>
</tr>
<tr>
<td>(p_{11})</td>
<td>0.9903 (0.0075)</td>
<td>0.995 (0.008)</td>
</tr>
<tr>
<td>(p_{22})</td>
<td>0.993 (0.0054)</td>
<td>0.998 (0.002)</td>
</tr>
<tr>
<td>Log (L)</td>
<td>-617.97</td>
<td>-499.43</td>
</tr>
</tbody>
</table>

\(^a\) Standard deviations in parentheses.

we test, for example, whether an intervention policy that aims to keep the exchange rate close to its parity, reduces the probabilities of switching to the regime of high risk of realignment once the exchange rate is in the regime of low risk of realignment.

![Fig. 7. 3-M exchange rate change within band and smoothed probability \(s_t = 1\).](image-url)
It was decided not to use the regime-switching model here to determine whether agents anticipated the switch in policy regime. A model for accommodating uncertainty about the intervention policy will be developed in future research.

5. Estimation results

Section 5.1 presents the results of the switching-regime model when the transition probabilities are assumed to be exogenous, and Section 5.2 the results when the transition probabilities are assumed to depend on interventions.

5.1. Constant transition probabilities

The results are presented in Table 1. Figs. 7 and 9 show both the quarterly percentage change of the Norwegian currency and the smoothed probability that the process was in state 1, the high risk of realignment state, for the first and second periods, respectively. These two are plotted as functions of time. Smoothed probabilities are obtained using the full sample of observations and the maximum likelihood estimates of the parameters of the model with constant transition probabilities.
The maximum likelihood estimates associate state 1, high risk of realignment, with a depreciation of 6.952% in our first period of study, and 6.295% in our second period of study. In state 2, the Norwegian currency appreciates by 6.766% in our first period in comparison with only 0.586% in the second period. The high risk of realignment state is mostly characterised by a highly depreciated Norwegian krone and high volatility, while the low risk of realignment state is characterised by an appreciated currency and low volatility. See Table 1.

The estimates also indicate that the Norwegian currency has long fluctuations characterised by plateaus. The point estimates of $p_{11}$ and $p_{22}$ are 0.99 for both first and second periods. This is evidence that once the Norwegian currency enters either the high or the low risk of realignment state, it is likely to stay in that state for a relatively long time.

The relationship between the interest rate differential and the expected depreciation of the Norwegian krone within its band ($E[\Delta x_t | .]$, equation (10)) yields the expected realignment rates, as defined in Eq. (2). This can be observed in Figs. 8 and 10 for the first and second periods, respectively, together with the smooth probability that the krone was in state 1. Note also that when we assume constant

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$20$ The interest differential is between the Norwegian interest rate and a basket of foreign interest rates (from the same countries whose currencies form the Norwegian exchange rate basket).
transition probabilities, we consequently obtain constant expected changes in the exchange rate within its band. This is not the case with time-varying transition probabilities.

Fig. 8 indicates that in our first period, the interest rate differential was always higher than the expected change of the exchange rate within its band in both states\(^{21}\). Thus, the expected realignment rates were positive even when the probability that the Norwegian krone was in the high risk of realignment state was very small. For example, when this probability was close to zero (there was actually an expected appreciation rate of 6.544%), the interest rate differential was well over 5\% (see Fig. 8). This contrasts the results for the second period, which are shown in Fig. 10. There, the interest rate differential was lower than the estimated expected depreciation within the band in the high risk of realignment state (6.132\%), and only slightly above zero when an appreciation of 0.561\% was expected during the low risk of realignment state.

\(^{21}\) The expected depreciation of the Norwegian krone within its band \(E_t (x_{t+1} - x_t)/\tau dt = [p_{11} \mu_1 + (1 - p_{11}) \mu_2] \) for our two periods of study at each state is,

<table>
<thead>
<tr>
<th>State</th>
<th>First period</th>
<th>Second period</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s_t = 1)</td>
<td>(E[\Delta x_{t+1}] = 6.744%)</td>
<td>(E[\Delta x_{t+1}] = 6.132%)</td>
</tr>
<tr>
<td>(s_t = 2)</td>
<td>(E[\Delta x_{t+1}] = -6.544%)</td>
<td>(E[\Delta x_{t+1}] = -0.561%)</td>
</tr>
</tbody>
</table>
In the first period, even very small probabilities of being in the high risk of realignment state\textsuperscript{22} have then coexisted with relatively high interest rate differentials, while in the second period, one could conclude that a negative change was expected in the central parity ($E\Delta c_{j,t} < 0$). We will next study whether these results change when time-varying transition probabilities are considered.

5.2. Time-varying transition probabilities. The effect of central bank interventions

We will now take up the second issue of this paper. Did intervention by selling foreign currency reduce the likelihood of the krone remaining in the high risk of realignment state once it was in that state? Did buying of foreign currency increase likelihood of the krone of remaining in the low risk of realignment state once, it was in that state? If the answers are yes, one would expect $z_{11}$ and $z_{21}$ (equation Eqs. (8a) and (8b) respectively) to be greater than zero.

The results for our first period are presented in Table 2. For all types of information about interventions, note that the means and the variances of the depreciation of the Norwegian currency in both states are the same as the ones obtained with constant transition probabilities (see Table 1). Those estimates are significant at the 5\% level or less.

Table 2
Parameter estimates with time-varying transition probabilities (estimates of means and variances in percentages)\textsuperscript{a}

<table>
<thead>
<tr>
<th>Param</th>
<th>First period</th>
<th>Second period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>INT (1D)</td>
<td>SUM INT (5D)</td>
</tr>
<tr>
<td>$\mu_1$</td>
<td>7.019 (0.146)</td>
<td>6.9437</td>
</tr>
<tr>
<td>$\mu_2$</td>
<td>(0.122)</td>
<td>(0.065)</td>
</tr>
<tr>
<td>$\sigma_{12}$</td>
<td>36.383</td>
<td>36.796</td>
</tr>
<tr>
<td>$\sigma_{22}$</td>
<td>(1.033)</td>
<td>(0.810)</td>
</tr>
<tr>
<td>$z_{10}$</td>
<td>4.09 (0.0452)</td>
<td>4.005</td>
</tr>
<tr>
<td>$z_{11}$</td>
<td>(0.0401)</td>
<td>(0.0439)</td>
</tr>
<tr>
<td>$z_{20}$</td>
<td>4.249</td>
<td>4.235</td>
</tr>
<tr>
<td>$z_{21}$</td>
<td>(0.0305)</td>
<td>(0.3784)</td>
</tr>
<tr>
<td>$\log (L)$</td>
<td>796.63</td>
<td>956.41</td>
</tr>
</tbody>
</table>

\textsuperscript{a}Standard errors are in parentheses.

\textsuperscript{22}The probability of leaving state 2, $1 - p_{22}$, was very small, 0.007.
Regarding the estimates of the time-varying transition probabilities, all the intercept estimates are found to be significant at the 10% level or less. We also find that, with a 10% significance level, past information (with a 6-day lag), about one-day interventions involving buying of foreign currency, reduces the likelihood of the krone remaining in the low-realignment state once it is in this state. Note that in this state, the krone appreciate and the central bank mostly bought foreign currency.

The other parameters are statistically insignificant at any reasonable level. These results indicate that the Norwegian exchange rate band had low credibility and interventions were not very effective. Seemingly, only the very high interest rates prevented a realignment from occurring. In future work, one should consider the effect of the interest rate differential on such realignment expectations. The purpose of this paper was however specifically to analyse the role of interventions on realignment expectations.

The results for our second period are also presented in Table 2. Tables 1 and 2 here seem to indicate that in this second period, the means and variances differ accordingly to whether or not the transition probabilities depend on interventions by Norges Bank. Estimates of the means and variances are now significant at the 5% level or less. Note also that if there were two equilibrium states during this period, they were characterised by great differences in the variances between states and almost none in the means. Our results indicate a depreciation of around 0.4% in both states. The krone was instead quite volatile at the beginning of this second period.
periods, as the data also indicate, but not quite so volatile at the end of period. (See Figs. 4 and 9).

Recall that in this second period, there quite frequent interventions, both buying and selling. All the parameters of the transition probability equations except two, are significant at the level of 5% or lower. We still find that past information (with a 6-day lag), about 1-day interventions involving buying (selling) foreign currencies, reduces (increases) the likelihood of the krone remaining in the state of low risk of realignment, once it was there. However, this 1-day buying (or selling) of foreign currency does not seem to affect the likelihood of the krone remaining in or leaving the high risk of realignment state, once it was in this state.

Conversely, information on intervention selling foreign currency for 5 or 25 consecutive days reduced the likelihood of the krone remaining in the high risk of realignment state. Moreover, selling foreign currency in the last 25 days increases the likelihood of the krone remaining in the low risk of realignment state, given that the Norwegian krone was in this state initially. Figs. 11–13 show the expected

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23 The same figures for the first period are available upon request. They are not presented here, since the transition probabilities do not appear to have been affected by interventions. Note that the estimated expected change of the exchange rate within its band and the expected realignment rates are functions of such probabilities.
realignment rates for the case where the Norwegian krone was in the high risk of realignment state (state 1), as well as the smoothed probability that the currency was in this state in the second period when the transition probabilities were modelled as time-varying.

These figures correspond to information on 1-day interventions (1D), accumulated interventions for 5 days, and accumulated interventions for 25 days respectively. A comparison with Fig. 10 reveals two main differences. The first is that in Figs. 11–13, after December 1988 (observation 60), the krone appears to have returned, with high probability, to the high risk of realignment state, even though it was kept close to its central parity. It remained in this state until March of 1989 (observation 145). Between December 1988 and March 1989, Norges Bank was very active in buying foreign currency. After March 1989, when there was also more selling, did the krone leave this state of high risk of realignment. Intervening by buying foreign currency then decreased the probability of the krone of remaining in the low risk of realignment state.

The second difference is in the size of the expected realignment rates. They here appear to be always positive when the Norwegian krone was in the high risk of realignment state. The interest rate differential up to the beginning of 1989 was higher than the expected depreciation within the band. This is in contrast to what was observed when the transition probabilities were exogenous.
Contrary to the results we obtain when considering constant transition probabilities for the second period, Figs. 11–13 indicate that the expected realignment rates are positive although they decrease indicating that the Norwegian exchange rate band also gain credibility over time.

6. Conclusions

In this present paper, we have tried to estimate the realignment expectations for the Norwegian currency band during the period 1986 to 1990, under two states, a high and a low risk of realignment state. One major objective was to abandon the restrictive assumption of the Switching-Regime Models regarding the constancy of the transition probabilities. We have assumed that these probabilities are time-varying and depend on central bank interventions. We also determine the role played by these interventions in the formation of realignment expectations and the switching between the high and low risk of realignment states. The other major objective was to present an alternative methodology to the widely used drift adjustment method in order to deal with the peso problem and obtain consistent estimates of these expected realignment rates.

Our general conclusion is that in a currency band regime, when the exchange rate is allowed to move within the whole band and central bank interventions occur only at the edges, as in Krugman (1991) target zone model with marginal, rather than infinitesimal interventions, central bank interventions can be relatively ineffective in eliminating expectations of realignment. However, when the exchange rate is not allowed to move to the edges of its band and interventions are intramarginal, interventions appear to be much more effective in changing expectations about a realignment in the desired direction, i.e. reducing the probability of the krone remaining in the high risk of realignment state and increasing the probability of it remaining in the low risk of realignment state. In other words, such an intervention policy signals to the market that the central parity of the currency band will be maintained.

Another result is that the more persistent the central bank is about keeping the exchange rate away from the edges of its own band using frequent interventions, the stronger the (desired) effect of interventions on market expectations.

Finally, in both periods, it was difficult for Norges Bank to intervene by buying foreign currency without making the Norwegian krone more variable and more depreciated. Such interventions reduced the probability of the krone remaining in the low risk of realignment state.

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24 Note that the probability of leaving this state \((1 - p_{11})\) was as small as 0.005 and \(\mu_2\) was also small (in absolute terms).

25 We write \(\Delta x_t\) instead of \(\Delta x_t,_{-} = x_t'_{-}/t\) purely to simplify notation.
Acknowledgements

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Appendix A

There are two set of parameters governing the density function of \( \Delta x_t/\tau dt \), \( \theta = (\mu, \sigma)' \) and \( z = (z_{10}, z_{11}, z_{20}, z_{21})' \). It turns out that in the process of forming the conditional likelihood function of \( \Delta x_t/\tau dt \), one also needs to make assumptions about the probability law governing the initial unobserved states. It is computationally simplest to define \( \rho = p(s_1 = 1) \); that is, \( \rho \) is defined as the unconditional probability of being in state 1 at time 1. Again, this is use to construct the first term of the likelihood function.

Hamilton (1990), Diebold et al. (1994) show that by differentiating Eqs. (8a) and (8b) with respect to all the parameters to be estimated, one obtains the following order conditions:

\[
\sum_{i=1}^{T} p(s_i = i|\Psi; \Theta^{(j-1)}) (\Delta x_{t} - \mu_{i}^{(j)}) = 0 \quad (A.1)
\]

\[
\sum_{i=1}^{T} p(s_i = i|\Psi; \Theta^{(j-1)}) \left( \frac{1}{\rho} \right) - 1 = 0 \quad (A.2)
\]

\[
\sum_{i=1}^{T} p(s_i = i|\Psi; \Theta^{(j-1)}) \left( \frac{(\Delta x_{t} - \mu_{i}^{(j)})^2}{(\sigma_{i}^{2})^{(j)}} - 1 \right) = 0 \quad (A.3)
\]

Here, \( \Psi \) denotes the vector of observations, \( \Psi = (\Delta x_{T}, \Delta x_{T-1}, ..., \Delta x_{1}; \text{NETI}_{T}, \text{NETI}_{T-1}, ..., \text{NETI}_{1}) \), \( \Theta \) is the parameter vector \( \Theta = (\mu_{1}, \mu_{2}, \sigma_{1}, \sigma_{2}, z_{10}, z_{11}, z_{20}, z_{21}, \rho) \), while \( j \) indicates the iteration number and \( i = 1, 2 \) are the regimes.

From Eqs. (A.1), (A.2) and (A.3), we obtain the EM equations for the Maximum Likelihood Estimators:

\[
\mu_{i} = \frac{\sum_{t=1}^{T} \Delta x_{t} p(s_i = i|\Psi; \Theta^{(j-1)})}{\sum_{i=1}^{T} p(s_i = i|\Psi; \Theta^{(j-1)})} \quad (A.4)
\]

\[
\rho^{(j)} = \frac{p(s_i = i|\Psi; \Theta^{(j-1)})}{\sum_{i=1}^{T} p(s_i = i|\Psi; \Theta^{(j-1)})} \quad (A.5)
\]

\[
(\sigma_{i}^{2})^{(j)} = \frac{\sum_{t=1}^{T} (\Delta x_{t} - \mu_{i}^{(j)})^2 p(s_i = i|\Psi; \Theta^{(j-1)})}{\sum_{i=1}^{T} p(s_i = i|\Psi; \Theta^{(j-1)})} \quad (A.6)
\]
It is worth noting that the estimator of \( \rho \) is an approximate maximum likelihood estimator. It is the smoothed probability that the process was in state 1 at time 1 in the EM algorithm. Thus, the full sample of all observables, \( \Psi \), are conditional upon that assumption. See Weinbach (1995) for an exact maximum likelihood estimator for \( \rho \).

The remaining first order conditions are for the transition probability functions. It is only these first-order conditions that will be different from the ones that are obtained when the transition probabilities are assumed to be exogenous.

The EM equations for \( p_{11} \) and \( p_{22} \), when these are assumed to be exogenous (see Hamilton (1990) for more details) are:

\[
\begin{align*}
    p_{11}^{(j)} &= \frac{\sum_{t=2}^{T} p(s_t = 1, s_{t-1} = 1 | \Psi; \Theta^{(j-1)})}{\sum_{t=2}^{T} p(s_t = 1 | \Psi; \Theta^{(j-1)})} \\
    p_{22}^{(j)} &= \frac{\sum_{t=2}^{T} p(s_t = 2, s_{t-1} = 2 | \Psi; \Theta^{(j-1)})}{\sum_{t=2}^{T} p(s_t = 2 | \Psi; \Theta^{(j-1)})}
\end{align*}
\]

(A.7) \hspace{1cm} (A.8)

Assuming the logit transition probability functions Eqs. (8a) and (8b), the remaining first order conditions appear to be nonlinear in the \( \alpha \) parameters. The EM equations are then:

\[
\begin{align*}
    \sum_{t=2}^{T} \text{Net}_t \varphi(s_t = 1, s_{t-1} = 1 | \Psi; \Theta^{(j-1)}) - p_{11}(t)p(s_t = 1 | \Psi; \Theta^{(j-1)}) &= 0 \quad (A.9) \\
    \sum_{t=2}^{T} \text{Net}_t \varphi(s_t = 2, s_{t-1} = 2 | \Psi; \Theta^{(j-1)}) - p_{22}(t)p(s_t = 2 | \Psi; \Theta^{(j-1)}) &= 0 \quad (A.10)
\end{align*}
\]

For each iteration, an unconstrained nonlinear optimization is used. See Hamilton (1990), Weinbach (1998), Diebold et al. (1994) for more details. The EM procedure amounts to the following: Begin with an initial guess for \( \Theta \) (denoted \( \Theta_0 \)) in order to start the EM algorithm. For these parameter values and for \( \Psi \), the sample of observations for \( \Delta x_t \) (the changes of the exchange rate within its band) and \( \text{Net}_t \varphi \) (Central Bank Interventions), calculate the smoothed state probabilities. These smoothed probabilities are then used to

\[26\] To draw an inference about the historical state the process was in at some date \( t \), one could obtain the ‘smoothed’ and ‘filter’ inferences. The ‘smoothed’ inference about the state of the process at date \( t \) is calculated using the full sample of ex-post available information \( \Psi \). A ‘filter’ inference is one that uses ex-post information only up to date \( t \). As a first step in calculating the ‘smoothed’ inferences, one needs to calculate and store the ‘filter’ inferences together with the conditional likelihoods.
update parameter estimates, $\Theta$, say $\Theta_1$. These new estimates should now be used to obtain a new set of smoothed probabilities and then to obtain a $\Theta_2$. The convergence criterion adopted may be based upon various standard criteria, such as the change in the log likelihood from one iteration to the next, the value of the gradient vector, or $||\Theta^{(0)} - \Theta^{(1)}||$ is less than $10^{-8}$.

References