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A Faster Algorithm for Computing the Conditional Logit Likelihood

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Abstract

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1 Introduction

Following [Wooldridge(2002), 15.73], the *conditional logit* likelihood for an individual is

$$L_i(\beta) = \frac{\exp(\sum_{t\in Y_i} x_{it}\cdot\beta)}{\sum_{a\in R^{T_i}_{n_i}} \exp(\sum_{t\in a} x_{it}\cdot\beta)} \tag{1}$$

where $\beta = (\beta_1, \ldots, \beta_K)$ is a parameter vector, $x_{it} = ((x_{it})_1, \ldots, (x_{it})_K)$ is a vector of covariates for period t. T_i is the number of observations for this individual, Y_i is the set of positive observations, $n_i = \operatorname{card} Y_i$ is the number of positive observations, and $R^m_j = \{a \subset \{1, \ldots, m\} : \operatorname{card} a = j\}$ is the collection of all subsets of $\{1, \ldots, m\}$ with j elements. Thus, we have $Y_i \in R^{T_i}_{n_i}$, hence $0 < L_i < 1$. (Strict inequality because trivial individuals (all or none positive observations) have been eliminated from the dataset.)

Since we are only considering a single individual here, we drop the i subscript and use the formula

$$L = \frac{\exp(\sum_{t\in Y} x_t\cdot\beta)}{\sum_{a\in R^T_n} \exp(\sum_{t\in a} x_t\cdot\beta)} = \frac{C}{D} \tag{2}$$

where we have named the numerator and denominator C and D.

This is a straightforward likelihood, the only obstacle being the size of the R^T_n in the denominator. There are $\binom{T}{n}$ terms in the outer sum, each term is an exp of an n-length sum. We thus have to do $\approx n\binom{T}{n}$ arithmetic operations.

The sum in the denominator makes the model intractable for anything but small T and n. We show that the denominator can be computed with less effort.

2 A recurrence relation

Let $h_t = \exp(x_t \cdot \beta)$. The denominator D in (2) can now be rewritten

$$D = \sum_{a \in R_n^T} \exp(\sum_{t \in a} x_t \cdot \beta) = \sum_{a \in R_n^T} \prod_{t \in a} \exp(x_t \cdot \beta) = \sum_{a \in R_n^T} \prod_{t \in a} h_t \tag{3}$$

For $1 < j < m$, remember the defining relation of Pascal's triangle ([Pascal(1654)]):

$$\binom{m}{j} = \binom{m-1}{j-1} + \binom{m-1}{j},$$

a formula which reflects the underlying disjoint union of sets:

$$R_j^m = \{\{m\} \cup a : a \in R_{j-1}^{m-1}\} \bigcup R_j^{m-1}.$$

We split the last sum in (3) accordingly, i.e. one part with $T \in a$, the other with $T \notin a$:

$$D = \sum_{a \in R_n^T} \prod_{t \in a} h_t = h_T \sum_{a \in R_{n-1}^{T-1}} \prod_{t \in a} h_t + \sum_{a \in R_n^{T-1}} \prod_{t \in a} h_t. \tag{4}$$

Now, if for arbitrary j, m with $1 \le j \le m \le T$ we write

$$z_j^m = \sum_{a \in R_j^m} \prod_{t \in a} h_t$$

we can write (4) as

$$D = z_n^T = h_T z_{n-1}^{T-1} + z_n^{T-1}.$$

Since n, T are arbitrary, we have in general, for $1 < j < m \le T$ the recurrence relation

$$z_j^m = h_m z_{j-1}^{m-1} + z_j^{m-1}. \tag{5}$$

The initial conditions are

$$z_1^m = \sum_{t=1}^m h_t \quad \text{for } m = 1..T,$$

and

$$z_m^m = \prod_{t=1}^m h_t \quad \text{for } m = 1..n.$$

We have

$$D = z_n^T, \tag{6}$$

requiring of the order nT arithmetic operations rather than $n\binom{T}{n}$. (Actually the order is roughly $T \min(n, T-n)$, the worst case nT is for $n = T/2$)

3 The gradient

Note that we want to compute the log-likelihood, i.e. $\log L$ where L is given by (2). For maximization, we also need the gradient. We have

$$\log L = \log C - \log D = \sum_{t \in Y} x_t \cdot \beta - \log D. \tag{7}$$

For the i'th partial derivative, we thus get

$$\frac{\partial \log L}{\partial \beta_i} = \sum_{t \in S} (x_t)_i - \frac{D'}{D}. \tag{8}$$

D' is equally demanding to compute as D, but we may in fact differentiate our recurrence relation (5) to get a recurrence relation for each partial derivative:

$$(z_j^m)' = h_m' z_{j-1}^{m-1} + h_m (z_{j-1}^{m-1})' + (z_j^{m-1})'. \tag{9}$$

We have

$$h_m' = \frac{\partial h_m}{\partial \beta_i} = h_m (x_m)_i. \tag{10}$$

Considering the i'th partial derivative

$$g_j^m = (z_j^m)' = \frac{\partial z_j^m}{\partial \beta_i}$$

and using (10) in (9), we get the following recurrence relation:

$$g_j^m = h_m g_{j-1}^{m-1} + g_j^{m-1} + (x_m)_i h_m z_{j-1}^{m-1}.$$

The only difference between this recurrence and (5), is that this one has a heterogeneity term $(x_m)_i h_m z_{j-1}^{m-1}$, and the initial conditions are the derivatives of the z_j^m conditions, i.e.

$$g_1^m = \sum_{t=1}^{m} (x_t)_i h_t \quad \text{for } m = 1..T,$$

and for g_m^m with $m = 1..n$ we can use the recurrence relation

$$g_m^m = (z_m^m)' = (h_m z_{m-1}^{m-1})' = h_m' z_{m-1}^{m-1} + h_m g_{m-1}^{m-1}. \tag{11}$$

To sum up, we have, for the derivative of the denominator in (2):

$$D' = g_n^T. \tag{12}$$

As with D this requires far less operations than computing the sum over R_n^T.

4 Summary

We have the following results:

Theorem 4.1. *Let* $D = \sum_{a \in R_n^T} \exp(\sum_{t \in a} x_t \cdot \beta)$ *be as in Section 1. For* $1 \leq t \leq T$*, denote* $h_t = \exp(x_t \cdot \beta)$*. For* $1 < j < m \leq T$*, let*

$$z_j^m = h_m z_{j-1}^{m-1} + z_j^{m-1}.$$

Assume $z_1^m = \sum_{t=1}^m h_t$ *for* $m = 1..T$*, and* $z_m^m = \prod_{t=1}^m h_t$ *for* $m = 1..n$*. Then*

$$D = z_n^T.$$

Corollary 4.2. *Computation of the log-likelihood defined in (2) can be done in the order of* nT *steps.*

Here's a short fortran snippet, utilising Theorem 4.1 for computing D, given n, T and h. Note, by the way, that the program may be made simpler, though probably not faster, by simplifying the initial conditions by starting one step further back and use as initial condition: $z_0^m = 1$ for $m = 0..n$ and $z_m^{m-1} = 0$ for $m = 1..n$.

```
integer :: m,j,n,T
double precision :: z(1:T,1:n), h(1:T), D
z(1,1) = h(1)
do m=2,T-n+1
   z(m,1) = z(m-1,1) + h(m)
end do
do m=2,n
   z(m,m) = z(m-1,m-1)*h(m)
end do
do j=2,n
   do m=j+1,T-n+j
      z(m,j) = h(m)*z(m-1,j-1) + z(m-1,j)
   end do
end do
D = z(T,n)
```

Also note that the program is not recursive. A purely recursive implementation of Theorem 4.1 is aesthetically pleasing, but will have problems reusing results. E.g. consider computing $z_3^5 = h_5 z_2^4 + z_3^4$. The z_2^4 expands as $h_4 z_1^3 + z_2^3$ whereas $z_3^4 = h_3 z_2^3 + z_3^3$. The quantity z_2^3 occurs in both of these expressions and should only be computed once. A purely recursive implementation (which is not caching results), will compute z_2^4 and z_3^4 independently and not take advantage of this reuse. This will roughly double the work for each recursion level, i.e. it will introduce a slow-down factor of something in the vicinity of 2^n.

We strongly suggest that software providers implement this algorithm rather than the direct summation or the recursive variant. (Stata's manual may suggest that Stata is using a recursive algorithm, but the details are unknown to us.)

By differentiation of the above recurrence, we get

Theorem 4.3. *Let D and h_t be as above. For $1 < j < m \leq T$, and $1 < i \leq K$, let*

$$g_j^m = h_m g_{j-1}^{m-1} + g_j^{m-1} + (x_m)_i h_m z_{j-1}^{m-1}$$

where z_j^m is as in theorem 4.1. Assume $g_1^m = \sum_{t=1}^m (x_t)_i h_t$ for $m = 1..T$ and that g_m^m for $m = 1..n$ are given by (11). Then

$$\frac{\partial D}{\partial \beta_i} = g_n^T$$

Clearly, it is equally easy to get a recurrence relation for the Hessian.

A Smalltalk & Example

So we have T different numbers, $h_1, h_2, \ldots, h_T$. We want to pick n of these numbers and multiply them together. In fact, we want to pick n different numbers in every possible way, and add all these n-length products. This sum we call z_n^T.

The main idea in this note is to place these products in two classes (sets). The first set (call it A) contains all the products with h_T as a factor, the second set (call it B) contains all the products which *don't* have h_T as a factor. If we remove h_T from each product in A, we are left with a set of products of $n-1$ numbers among $\{h_1, h_2, \ldots, h_{T-1}\}$. In fact, this set contains all such products, so its sum is what we call z_{n-1}^{T-1}. If we multiply by h_T we get all the numbers in A, thus the sum of all numbers in A is

$$h_T z_{n-1}^{T-1}. \tag{13}$$

The set B, by construction, consists of all n-length products of the numbers $\{h_1, h_2, \ldots, h_{T-1}\}$, thus its sum is what we have called

$$z_n^{T-1}. \tag{14}$$

Adding together (13) and (14) we get

$$z_n^T = h_T z_{n-1}^{T-1} + z_n^{T-1}. \tag{15}$$

For each term on the right hand side, we use the same argument, thus

$$z_{n-1}^{T-1} = h_{T-1} z_{n-2}^{T-2} + z_{n-1}^{T-2}$$

and

$$z_n^{T-1} = h_{T-1} z_{n-1}^{T-2} + z_n^{T-2}.$$

Note that the quantity z_{n-1}^{T-2} appears in both of the right hand sides. This process stops when either the subscript becomes 1, or the subscript becomes equal to the superscript (the right hand side is not meaningful then), this is exactly our initial conditions. I.e. z_1^m is the sum of all products with 1 factor, i.e. it's $\sum_{t=1}^m h_t$. The number z_m^m is the sum of all m-length products in $\{h_1, \ldots, h_m\}$. There is only one, i.e. $z_m^m = \prod_{t=1}^m h_t$.

As an example, assume $T = 4$ and $n = 2$. We have the numbers h_1, h_2, h_3, h_4. We have the collection R_n^T:

$$R_n^T = R_2^4 = \{\{1,2\},\{1,3\},\{1,4\},\{2,3\},\{2,4\},\{3,4\}\}$$

consisting of all sets with two elements from $\{1,2,3,4\}$. So we want to compute

$$z_2^4 = \sum_{a \in R_2^4} \prod_{t \in a} h_t = h_1h_2 + h_1h_3 + h_1h_4 + h_2h_3 + h_2h_4 + h_3h_4.$$

The idea is to split up this sum, one term with those containing h_4 (which we then may use as a common factor), and one without h_4:

$$h_4 \sum_{a \in R_1^3} \prod_{t \in a} h_t + \sum_{a \in R_2^3} \prod_{t \in a} h_t = h_4(h_1 + h_2 + h_3) + (h_1h_2 + h_1h_3 + h_2h_3). \quad (16)$$

The last term is handled the same way:

$$h_1h_2 + h_1h_3 + h_2h_3 = h_3(h_1 + h_2) + h_1h_2. \quad (17)$$

Now, in the original sum there were 6 terms, so 6 multiplications and 5 additions, close to $2\binom{4}{2} = 12$ operations. If we count the operations in our divide and conquer method, there are 2 multiplications and 2 additions in (17), and there are 3 additions and 1 multiplication in (16), a total of 8 operations. (Or 7 if we do $h_1 + h_2$ only once.)

The reader is invited to do the same with $T = 5, n = 3$ to understand the $n \times T$ matrix below. I.e. to check that z_3^5 in the lower right corner evaluates to the sum of all 3-length products from $\{h_1, h_2, \ldots, h_5\}$. Indeed, computing z_3^5 requires 12 arithmetic operations, compared to $3\binom{5}{3} - 1 = 29$ for the full sum. The matrix can be filled in left to right, top to bottom. Entries with $\cdot$ are undefined or not needed to reach the bottom right corner. The upper row and main diagonal are filled in according to the initial condition in Theorem 4.1. Note how the other entries are instances of (15).

$$\begin{bmatrix} z_1^1 = h_1 & z_1^2 = h_2 + z_1^1 & z_1^3 = h_3 + z_1^2 & z_1^4 = \cdot & z_1^5 = \cdot \\ z_2^1 = \cdot & z_2^2 = h_2 z_1^1 & z_2^3 = h_3 z_1^2 + z_2^2 & z_2^4 = h_4 z_1^3 + z_2^3 & z_2^5 = \cdot \\ z_3^1 = \cdot & z_3^2 = \cdot & z_3^3 = h_3 z_2^2 & z_3^4 = h_4 z_2^3 + z_3^3 & z_3^5 = h_5 z_2^4 + z_3^4 \end{bmatrix}$$

Note the similarity with Pascal's triangle. This matrix is the same z as we build up in the fortran snippet above. Also, note that at each step we only use the cell immediately to the left and the one above it, thus, if nT is very large, we may save memory by not storing the full matrix. The above matrix results in the following formula for z_3^5:

$$\begin{aligned} z_3^5 &= h_5(h_4(h_1+h_2+h_3)+h_3(h_1+h_2)+h_1h_2) \\ &\quad +h_4(h_3(h_1+h_2)+h_1h_2)+h_1h_2h_3. \end{aligned}$$

Computing (2) directly, typically requires $n\binom{T}{n}$ steps, so the speedup will be around

$$\frac{n\binom{T}{n}}{nT} = \frac{\binom{T}{n}}{T},$$

a quantity which easily becomes large. For our example with $T = 21, n = 10$ we have $\frac{\binom{T}{n}}{T} \approx 17000$. For the more ambitious example $T = 60, n = 12$, the factor is approx 23×10^9, or 1 second compared to 740 years.

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