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Epidemics and increasing returns to scale on social distancing

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It is shown that the standard Susceptible Infectious Recovered model of an epidemic implies that there for a large set of epidemic parameter values there will be increasing returns to scale if the objective is to limit the economic cost of infection. The explanation is that if an epidemic has a high basic reproduction number, a given amount of social distancing will not have much effect. The same amount may however be very effective if the reproduction number is lower (but still larger than one).

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Introduction.

There is a considerable literature on the economic management of epidemics. Historically this literature has to a considerable extent been mostly, but not exclusively, concerned with vaccination. As is to be expected, results will depend on the biological characteristics of the pathogen at hand. A very important distinction is between endemic diseases and transient epidemics. Endemic diseases, although subject to episodic spikes, require a different policy perspective than transient epidemics, see Goldman and Lightwood (2002), Barrett and Hoel (2006) for discussions of endemic diseases. Here the focus is on transient epidemics such as the common flu and (hopefully) Covid-19. Several articles analyze the management of transient epidemics by utilizing the well established Susceptible-Infectious-Recovered (SIR) model, developed by McKendrick and Kermack (1927), see e.g. Morton and Wickwire (1974), Francis (1997) (2004) and Nævdal (2012). Several papers have examined the effect of policy interventions. E.g has Brito et al. (1991) and Geoffard and Philipson (1996) examined vaccination policies. Gersovitz and Hammer (2004) examined the case of several instruments. Nævdal (2012) identified the possibility of increasing returns to scale on pre outbreak vaccination efforts. For some parameter values and stocks of unvaccinated individuals in the population it turned out that the marginal value of vaccination is an increasing function of the number of vaccinated, i.e. increasing returns to scale. Nævdal (2012) explained this with a “brush fire” effect where a vaccination in a very fast spreading epidemic has little effect unless followed up with more vaccination. Here I show that the same argument applies to social distancing as a policy measure.

The analysis is done with a very simple deterministic model in order to highlight how the epidemic dynamics may imply increasing returns to scale.

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The SIR - model

The model has 3 variables. x is the number of susceptibles, y is the number of sick and z is the number of individuals who are immune rate. It is assumed that the population is a constant n so that $x + y + z = n$. From hereon n is normalized to 1. The infection rate $\dot{y}$ is proportional to the product of the number of infected and the number of susceptibles. An individual can acquire immunity by recovering from the disease. The equations of motion are given by:

$$\dot{x} = -\beta xy \tag{1}$$

$$\dot{y} = \beta xy - \gamma y \tag{2}$$

$$\dot{z} = \gamma y \tag{3}$$

Here β and γ are positive constants. γ has the interpretation that γ^{-1} is the expected duration of the disease for an infected. Thus the duration of the epidemic for an infected individual is exponentially distributed with intensity γ. β is the contact rate and is a product of the transmissibility of the pathogen and the number of interactions an individual has per day. The basis reproductive number R_0 is given by

$$R_0 = \beta\gamma^{-1} \tag{4}$$

This is roughly the number that an infected person will infect in the beginning of the epidemic. In the basic SIR model, this number together with initial values of state variables will determine the path of the epidemic.

Since $\dot{x} + \dot{y} + \dot{z} = 0$ it holds that $x(t) + y(t) + z(t) = x(0) + y(0) + z(0) = 1$ for all t. Also, the system has an infinite number of steady states. Any triple $(x, y, z) = (x^*, 0, z^*)$ such that $x^* + z^* = 1$ is a steady state. There are no steady states with positive values of y.

The initial conditions are $x(0) = 1 - \varepsilon$ and $y(0) = \varepsilon$. From (2) it is immediately clear that the epidemic reaches it's apex, $\max_t y(t)$ when $x = \gamma/\beta = R_0^{-1}$. We can

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derive a single differential equation for y as a function of x. We denote this function $Y(x)$.

$$y = Y\left(x\right) = \frac{\dot{y}}{\dot{x}} = -\frac{\beta xy - \gamma y}{\beta xy} = -1 + \frac{R_0^{-1}}{x}, \; Y\left(1-\varepsilon\right) = \varepsilon \tag{5}$$

Solving (5) yields:

$$Y\left(x\right) = 1 - x + R_0^{-1}\left(\ln\left(x\right) - \ln\left(1-\varepsilon\right)\right) \tag{6}$$

The solution in (6) shows how x and y moves in tandem during an epidemic. Note that as long as ε is small, it has very little impact on the path. In the absence of any interventions the number of individuals who will be susceptible after an epidemic is given by the value x, denoted x_{min} such that:

$$Y\left(x_{\min}\right) = 1 - x_{\min} + R_0^{-1}\left(\ln\left(x_{\min}\right) - \ln\left(1-\varepsilon\right)\right) = 0 \tag{7}$$

We now modify (4) in order to account for social distancing measures. Over a time interval [0, T] where T we have that

$$R = R_0 - h \tag{8}$$

Here h is some, possibly constant, function of time over the interval [0, T] where $T < \infty$. For $t > T$ we have that $h = 0$. One way of interpreting h is simply as the reduction in the number of potentially infective social interactions per day, scaled to be in the same units as R_0. Thus the definition of social distancing used here is different than that employed by e.g Gollier (2020) where a fraction of the population is in lock down.

It is easy to show that for any $R_0 > 1$ there is an interval of steady states $S = \left[x_{\min}, R_0^{-1}\right]$ that is the set of feasible endpoints for the epidemic. x_{min} represents a worst case scenario where the maximum number of individuals, $1 - x_{min}$, have been infected. With interventions, the medically best possible outcome is that the disease ends with $1 - R_0^{-1}$ having been infected. If we restrict h to constant functions over we can identify x_h which is, roughly, the long run number of people never infected given a constant value of h until T. Whenever $x > R_0^{-1}$, the epidemic will always reappear when h is set equal to 0. x_h is determined by a modification of (7).

$$Y\left(x_h\right) = 1 - x_h + \left(R_0 - h\right)^{-1}\left(\ln\left(x_h\right) - \ln\left(1-\varepsilon\right)\right) = 0 \tag{9}$$

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It is not possible to find an explicit solution that solves (9) for x_h as a function of h.[1] However it is straight forward to plot x_h as a function of h for given values of R_0 and ε. This is done in Figure 1.

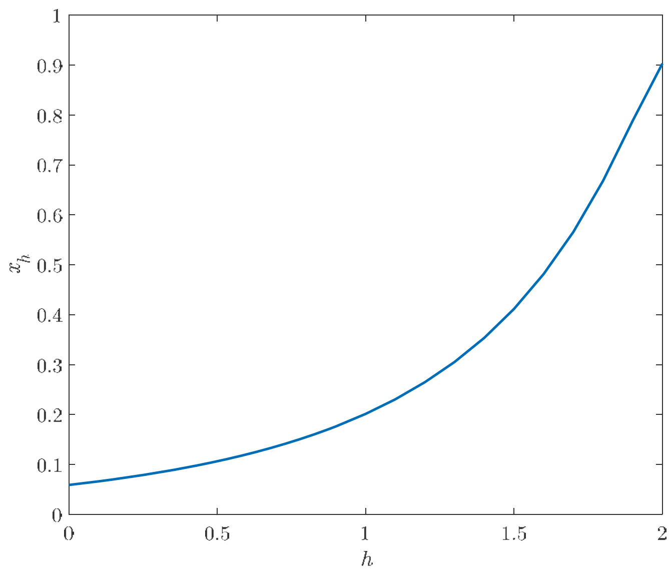

Figure 1. The long run stock of remaining susceptibles, x_h, as a function of social distancing

The important thing to note about the relationship between h and x_h is that x_h is a convex function of h, implying that the higher h, the more effective is a marginal increase in h at reducing the number of infected during the course of an epidemic. This is confirmed in Figure 2 where the time paths of x for an epidemic with $R_0 = 2.5$ is plotted. The effect of setting $h = 0.1$ has a negligible effect. x_0, that is x_h when $h = 0$, is 0.89. $x_{0.1} =$ is 0.88, a reduction of 1 percentage point. However if h is set to 1 and then further increased to 1.1, we have that $x_{1.5} = 0.58$ and $x_{1.1} = 0.51$, a reduction of 7 percentage points.

Herein lie the explanation for increasing returns to scale. When R_0 is large, marginal increases in social distancing simply has very little effect on the outcome of the epidemic. When R_0 is small or substantial social distancing is in

[1] An explicit solution can be found using the Lambert function.

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effect, marginal increases in social distancing has a much larger effect on the epidemic outcome.

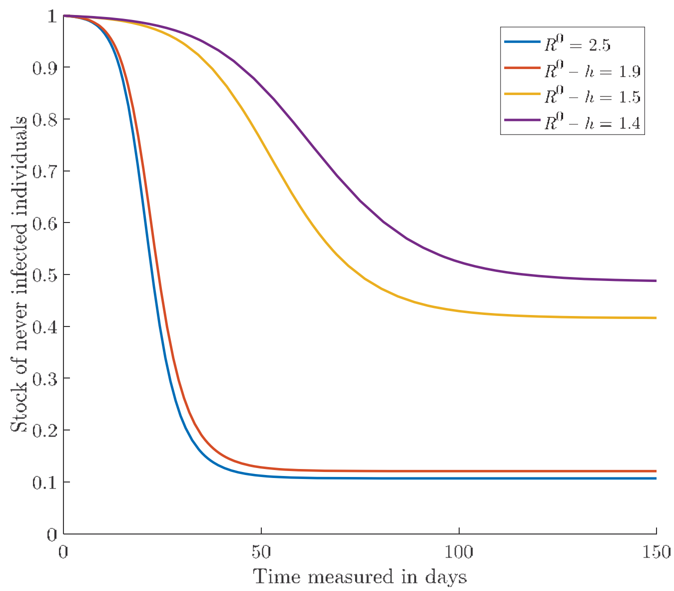

Figure 2. The stock of susceptible individuals over time at different levels of social distancing.

The economic benefits of social distancing

To see how social distancing affects the economic costs of an epidemic we can change the model so that R_0 becomes a function of distancing efforts:

$$\begin{gathered} R_0 = (\beta - h)\gamma^{-1} \\ \downarrow \\ \beta(h) = \gamma R_0 - h \end{gathered} \tag{10}$$

We write β as a function of distancing efforts. Thus the epidemic equations for x and y may be written:

$$\begin{gathered} \dot{x} = -(\gamma R_0 - h)xy,\ x(0) = 1 - \varepsilon \\ \dot{y} = (\gamma R_0 - h)xy - \gamma y. \quad y(0) = \varepsilon \end{gathered} \tag{11}$$

Here h is a measure of social distancing. Then a very simple model of an epidemic is:

$$V(h) = \int_0^\infty -wye^{-rt}dt \tag{12}$$

Here wy is the economic surplus lost if y individuals are ill for a unit of time. In the simulation below w is normalized to one. Let us examine the benefits of social distancing without examining the costs by doing a thought experiment. Assume that we fix β so that $h > 0$ for $x \geq x_{crit}$. When x goes below x_{crit} we set $h = 0$. The benefit from such an intervention is shown in Figure 3.

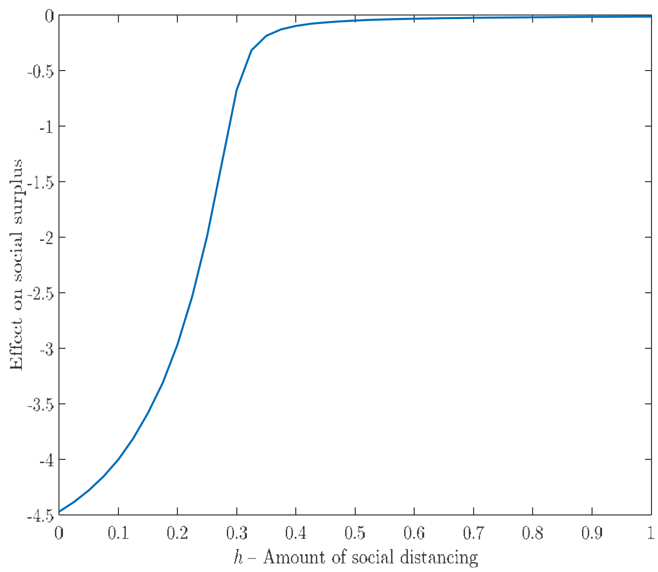

Figure 3. Relationship between the cost of , *h*. It is clear that the benefits exhibit increasing returns to scale for *h* in the interval 0 to 0.3. Thereafter there are diminishing returns that quickly go to zero. Here x_{crit} is set to 0.1.

The specification of the epidemic cost is very simple. In particular they are linear with respect to the stock of infected. This is unlikely to be the case when an epidemic is serious with respect to mortality, infectiveness and health outcomes. If the instantaneous marginal cost of y depends on the magnitude of y, this may, paradoxically, imply returns to scale become diminishing. This is in line with Nævdal (2012) who found that more serious epidemics exhibited diminishing returns to scale on vaccination efforts.

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Summary and policy implications

A transient epidemic is in many ways like a brush fire. A high R_0 has the same effect as a severe drought has on brush land. The drier the vegetation, the more vegetation is consumed, the quicker is a specific area consumed and the less is the effect of a bucket of water. This has some implications for economic management. A very dry area may be a lost cause. However, if it pays to throw one bucket of water on the fire it pays even more to throw a second bucket. The same goes for epidemics. There may be increasing returns to scale to social distancing and other efforts to control the epidemic. The results have some very clear policy implications.

1) With increasing returns to scale, a corner solution, i.e. no distancing, may be optimal.
2) *If* it pays to apply social distancing as a policy, then it is often the case that if it pays to do a little it pays even more to do a lot.
3) An issue not covered in the present paper is what happens when individuals respond behaviorally to an epidemic threat by choosing to socially distance themselves, Garibaldi *et al* (2020). Does this negate the need for public intervention? In general, the answer is no as the individual only receives part of the benefits from their own behavior. Additionally, the analysis here indicates that for some parameter values increasing returns to scale implies that individual self distancing may increase the value of public efforts.

It should also be noted that if alternative measures of social distancing are employed, this will affect results. The approach to modeling social distancing employed by Gollier (2020) implies that β becomes a quadratic function of the fraction of people in lockdown. This would likely strengthen the results in the present paper.

The results underscore the need for economic analysis to be founded on a solid understanding of the mathematical dynamics of an epidemic.

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