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Stochastic energy market equilibrium modeling with multiple agents[1]

Kjell Arne Brekke[2], Rolf Golombek[3], Michal Kaut[4], Sverre A.C. Kittelsen[5] and Stein W. Wallace[6]

Abstract

The energy markets are characterized by many agents simultaneously solving decision problems under uncertainty. It is argued that Monte Carlo simulations are not an adequate way to assess behavioral uncertainty; one should rather rely on stochastic modelling. Drawing on economics, decision theory and operations research, a simple guide on how to transform a deterministic energy market equilibrium model - where several agents simultaneously make decisions - into a stochastic equilibrium model is offered. With our approach, no programming of a stochastic solution algorithm is required.

JEL classification: C63; C68; D58; D81; Q28; Q40;Q54

Key words: uncertainty; stochastic equilibrium; Monte Carlo; energy modeling

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1 Introduction

Agents in the European energy market have experienced, and may continue to experience, considerable uncertainty: during the last financial crisis demand for energy dropped significantly, and in the future the Paris agreement may trigger radical changes in the energy markets. Such abundant uncertainties can have huge consequences on investment in the energy industry. At the same time, if there is reluctance to invest in some technologies, for example, due to expectations about high fossil fuel prices or high taxes on greenhouse gas emissions, the market may look more promising for other technologies, like renewables. Thus, to fully analyze the impact of uncertainty the *interdependences* of different technologies, energy goods and agents have to be taken into account – this calls for a multi-dimensional *equilibrium* model that captures the essential characteristics of the energy industry.

It is, however, not trivial to solve, or even formulate, a model where many heterogeneous decision makers face uncertainty. Thus, it is not surprising that most analyses assume full certainty, or, if uncertainty is analyzed, rely on Monte Carlo simulations instead of examining the behavior of agents optimizing under uncertainty. Some examples of Monte Carlo simulations, which are also termed what-if analysis, sensitivity analysis, or scenario analysis, are IPCC (2014) on how short- and long-term factors have impact on mitigation pathways, Egerer et al. (2015) on how alternative scenarios have impact on intermittent renewables in the German electricity system, and Fæhn and Isaksen (2016) on impacts, within a technology-rich CGE model, of regulatory climate policy uncertainty.

In order to properly handle uncertainty, the basic ideas of stochastic programming, see, for example, Kall and Wallace (1994), should be built upon. Here, a crucial distinction is made between decisions made before the uncertainty is revealed, and decisions made afterwards. A key insight is that it is not valid to solve the model scenario by scenario – as is the case with Monte Carlo simulations - and then try to extract an overall picture from these solutions.

Some examples of papers applying stochastic programming, that is, truly *stochastic* equilibrium models, are Bjørndal et al. (2010) on gas transportation networks, Ehrenmann and Smeers (2008) on capacity markets for power, Chin and Siddiqui (2014) on capacity expansion in electricity markets, and Cabero et al. (2010) on hydrothermal generation. In addition, there are numerical stochastic models for natural gas markets: world-wide gas market modelling (Egging 2010), long-term evolution of the European gas market (Abada 2012), investment uncertainties in the European gas market (Bornaee 2012), security of supply (Egging and Holz 2016), market power (Baltensperger and Egging 2017), and risk aversion (Egging et al. 2017). In addition, some papers have developed specialized solution techniques for these stochastic models: Gabriel and Fuller (2010) present a Benders-decomposition algorithm for stochastic complementarity problems. This type of algorithm is further developed in Egging (2013) and tested on a large-scale, stochastic, multi-period, mixed complementarity problem. Devine et al. (2016) offers an alternative approach where a stochastic model is solved sequentially, using a rolling-horizon approach.

To sum up, most of the literature relies on Monte Carlo simulations, which do not have a sound theoretical basis. In particular, scenarios are not assigned probabilities but are rather seen as some of the possible future states; see Higle and Wallace (2003) for an example showing what can go wrong within this type of approach. There is, however, also a literature drawing on stochastic programming. Here, each scenario is assigned a probability and the sum of probabilities equals one. One challenge with the latter literature is that special algorithms are needed. In particular, there is no easy way to transform a numerical deterministic model into a stochastic model; one has to start the stochastic modeling from scratch. The purpose of the present paper is to bridge the gap between these literatures: The contribution of our paper is to offer a general guide on how to transform a deterministic energy market equilibrium model, where several agents simultaneously make decisions, into a stochastic equilibrium model when the algorithm, and software, for the deterministic model can be used also for the stochastic model. In a companion paper, this guide is used to transform an existing, large-scale, *deterministic* energy market model – LIBEMOD, see Aune et al. (2008) – into a stochastic model, and this stochastic model is used to analyze the impact of economic uncertainty on the Western European energy markets, see last part of Brekke et al. (2013). This example illustrates that our approach works in practice.

Because one should make a clear distinction between decisions taken under uncertainty and decisions taken after the uncertainty has been revealed, the model of uncertainty presented in this paper has two periods. In period 1, some agents make decisions under uncertainty, typically to determine their future capacities through investments. In the beginning of period 2, the uncertainty is revealed and all agents learn the true state of the economy, that is, which scenario that has materialized. Then all agents make decisions; producers determine how much to produce (given the predetermined capacities), arbitrators determine how much to trade, and consumers determine how much to consume.

For each realization of the uncertainty, that is, for each scenario, the model determines supply of, and demand for, all goods from all agents and the corresponding vector of prices that clears all markets. In fact, the stochastic equilibrium model determines simultaneously all quantities (investment, production, trade and consumption) and all market clearing prices for *all* scenarios. The determination of quantities and prices are based on the assumption that all agents have rational expectations, that is, when investment decisions are taken in the first period, agents take into account the probability distribution over the scenarios and the equilibrium prices of all scenarios.

In spirit, our approach to modeling uncertainty is similar to the discussion of uncertainty in Debreu's (1959, chapter 7) classic 'Theory of Value', where uncertainty is represented by a discrete event tree. In our terminology, each branch of Debreu's event tree is called a scenario. Hence, in our model uncertainty is represented by a set of scenarios. Each scenario is one possible future realization of the uncertainty, and each scenario is assigned a probability.

Below, the simple equilibrium model with investment, production, trade and consumption is solved both when there is no uncertainty (Section 2) and when investment decisions are taken under uncertainty (Section 3). In particular, it is demonstrated how the idea in Wets (1989), which is more formally described in Rockafellar and Wets (1991), to find the solution of a stochastic problem for a *single* agent can be used to solve a stochastic *equilibrium* model where *several* agents make decisions simultaneously. This gives a guide on how to transform a deterministic equilibrium model into a stochastic one. For a given specification of scenarios, the stochastic equilibrium model finds the optimal solution under uncertainty.

Note that although the formulation of the stochastic model is in line with the basic ideas of stochastic programming, the model is not solved by stochastic programming. By building on an idea in Rockafellar and Wets (1991), a simple strategy to transform a pre-existing deterministic energy market model, where several agents simultaneously make decisions, into a stochastic equilibrium model is developed.

In Section 4 Monte Carlo simulations are discussed, which is a much used method to assess the impact of uncertainty. However, it is argued that there is a fundamental problem with this approach; the Monte Carlo method does not produce a consistent representation of the behavior of the actors. Hence, one should use a stochastic model to examine how the energy markets work under uncertainty. Finally, Section 5 concludes.

2 A deterministic energy market equilibrium model

This paper shows how the general idea in Rockafellar and Wets (1991) to find the solution of a stochastic problem for a *single* agent can be used to transform a *deterministic equilibrium* model with *multiple* agents into a *stochastic* equilibrium model with multiple agents. To this end, a simple deterministic equilibrium model for a two-region electricity market is first set up. In each region, there is production and consumption of electricity, and consumption in a region depends on one (utility) parameter.

The simplicity of the model makes the impact of uncertainty more transparent. It is shown that in the case of no uncertainty, there will be no investment in transmission between the two regions, and hence no trade in electricity. It is demonstrated that this result does *not* depend on the values of the demand parameters. In contrast, with stochastic demand parameters (Section 3) there will be investment in transmission in equilibrium. This capacity will be utilized if the realizations of the two stochastic demand parameters differ, which means that one region has higher demand for electricity than the other.

2.1 The model

Consider electricity production and consumption in two regions. In each region i, $i=1,2$, there is a representative producer i. Initially, a producer has no production capacity, but he can invest in capacity at a constant unit cost c. There is no cost of operating the capacity, so production will equal capacity (K_i). There is also a transmission company, which may invest in a transmission line between the two regions. Initially, there is no transmission capacity, but the transmission company can invest in capacity (K_T) at a constant unit cost (c_T).

The model has two periods (but discounting between the periods is neglected). In period 1, the agents may invest in capacity. In the beginning of period 2, the new capacities are available, and there is production and consumption like in any standard deterministic model. Assume that the electricity producer in market *i* can sell electricity in this market only, whereas the transmission company can buy electricity in one market and sell this electricity in the other market. It is further assumed that all agents are price takers and cannot exercise market power.

In period 1, the electricity producer knows that in the next period the price of electricity will be p_i. The electricity producer in region *i* will therefore maximize $(p_i - c)K_i$. The Kuhn-Tucker first-order complementarity condition of this problem is

$$p_i \leq c \perp K_i \geq 0 \tag{1}$$

where the complementarity operator $\perp$ indicates that one or both of the weak inequalities must hold as strict equalities. Hence, if it is optimal to invest in production capacity $(K_i > 0)$, then $p_i = c$, whereas if $p_i < c$ it is not optimal to invest in production capacity $(K_i = 0)$.

In period 1, the transmission company determines its investment in transmission capacity. Let z_1 be electricity bought in market 1 by the transmission company. This quantity is exported to market 2 and then sold in market 2 by the transmission company. Correspondingly, let z_2 be electricity bought in market 2 by the transmission company and then exported to market 1. Profits of the transmission company are then $(p_1 - p_2)z_2 + (p_2 - p_1)z_1 - c_T K_T$. Of course, exports cannot exceed the transmission capacity, and hence in period 2 the transmission company faces the following two restrictions:

$$\begin{aligned} z_1 &\le K_T \perp \gamma_1 \ge 0 \\ z_2 &\le K_T \perp \gamma_2 \ge 0 \end{aligned} \tag{2}$$

where γ_i is the shadow price associated with the constraint on the amount of exports from market i. Maximizing profits with respect to transmission capacity and export quantities, the first-order conditions are:

$$\gamma_1 + \gamma_2 \le c_T \perp K_T \ge 0 \tag{3}$$

$$\begin{aligned} p_2 - p_1 &\le \gamma_1 \perp z_1 \ge 0 \\ p_1 - p_2 &\le \gamma_2 \perp z_2 \ge 0. \end{aligned} \tag{4}$$

In period 2, the electricity producers will use their entire production capacity (as long as demand is positive) because there are no variable costs of production.

In each region, there is a representative consumer. His consumption of electricity (x_i) gives him gross utility $2\theta_i\sqrt{x_i}$, where θ_i is the utility parameter of the representative consumer in market *i*. This parameter may depend on a number of factors, for example, the temperature. Henceforth, a high parameter value is associated with cold weather, and therefore a high utility of electricity for heating. The consumer in region *i* maximizes his net utility $2\theta_i\sqrt{x_i} - p_i x_i$. The first-order condition for the consumer is:

$$x_i = \left(\frac{\theta_i}{p_i}\right)^2. \tag{5}$$

Finally, the market clearing conditions are:

$$\begin{aligned} K_1 + z_2 - z_1 &= x_1 \\ K_2 + z_1 - z_2 &= x_2. \end{aligned} \tag{6}$$

Hence, in each region domestic production of electricity (K_i) plus net imports of electricity $(z_j - z_i, j \neq i)$ is equal to consumption of electricity in this region (x_i).

2.2 The deterministic market equilibrium

The market equilibrium in this case is obvious. For prices approaching zero, demand is infinite. Hence, there will be production of electricity, which requires investment in production capacity in period 1; $K_i > 0$. With an interior solution for production capacity, $p_i = c$. Therefore, prices are equal between the two markets, and it will not be profitable to invest in transmission capacity to export electricity between the two regions $(K_T = 0)$. Hence, there will be no trade. Technically, $p_1 = p_2$, γ_i is indeterminate (as long as each γ_i is nonnegative, see (2), and the sum is less than c_T, see (3)), and $z_1 = z_2 = 0$. Thus the equilibrium is characterized as follows:

$$\begin{aligned} K_i &= x_i = \left(\frac{\theta_i}{c}\right)^2 \\ p_i &= c \\ K_T &= 0. \end{aligned}$$

Note that no matter the value of (θ_1, θ_2), the optimal solution is always $K_T = 0$. While this may seem like a very robust result, as demonstrated below $K_T = 0$ is not the equilibrium in the stochastic model.[7]

3 A stochastic energy market equilibrium model

3.1 Modelling uncertainty

The deterministic model is now transformed to a stochastic model by letting the demand parameters be random. Suppose that there are two possible values of $\theta_i \in \{\theta_L, \theta_H\}$ for each market. This makes four possible combinations: $(\theta_1, \theta_2) \in \{(\theta_L, \theta_L), (\theta_L, \theta_H), (\theta_H, \theta_L), (\theta_H, \theta_H)\}$. Denote each of the four outcomes as a scenario *s*, $s \in \{1,2,3,4\} = S.$ The probability that scenario *s* materializes is q_s where $\sum_{s=1}^{4} q_s = 1.$

With uncertainty the information available to the decision maker at the time of making the decision has to be specified. It will be assumed that investment decisions are taken under uncertainty in period 1. In the beginning of period 2, agents learn the true scenario, and production, trade and consumption decisions are taken. Hence, these decisions are determined after the uncertainty has been resolved.

Let us now consider the maximization problem of electricity producer *i*. The straightforward formulation would be to maximize

$$\sum_{s=1}^{4} q_s (p_{is} - c) K_i = \left(Ep_i - c\right) K_i$$

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[7] For a detailed discussion, taken from stochastic programing, on single-agent optimization where the deterministic solution differs qualitatively from the stochastic one, see Wallace (2000).

where Ep_i is the expected value of the price. This would give the first-order condition

$$Ep_i \le c \perp K_i \ge 0.$$

While this would of course work, the present paper seeks to find a strategy that makes the changes as small as possible when a (pre-existing) deterministic model is transformed into a stochastic model. This strategy should be able to handle a large set of models, for example, multi-period models with learning, see Section 5. To this end, a model formulation from Rockafellar and Wets (1991) is employed.

To explain the Rockafellar and Wets approach, suppose the deterministic model is solved simultaneously for each of the four scenarios. This would simply amount to specifying the first-order conditions four times, once for each scenario. This could be done by adding an index *s* for the scenarios to each variable. Thus, the first-order condition for the electricity producers would be

$$p_{is} \le c \perp K_{is} \ge 0.$$

This condition has to be satisfied for each electricity producer *i* and each scenario *s*. Such a simultaneous solution would only require a scenario index on the variables. However, this will *not* be the solution to the stochastic problem: with no link between the scenarios, the production capacity would be $K_{is} = \left(\theta_{is} / c\right)^2$, and hence a producer would have a different capital stock for each scenario. But this does *not* make sense: because capital has to be chosen *before* the scenario is revealed, the capital stock must be the *same* in all scenarios. Therefore, $K_{i1} \ne K_{i2}$ cannot be a solution when capital is chosen *before* the firm knows the scenario. The condition that $K_{is} = K_{is'}$

for all $s, s' \in S$ has to be imposed. Below, this restriction is specified as $K_{is} = K_i$ for $s = 1, 2, 3, 4$ and it is referred to as the *implementability constraint*.

The discussion above implies that under uncertainty, the investor cannot maximize profit for each individual scenario separately. With uncertainty, the aim of the electricity producer is to find the production capacity in each scenario (K_{is}) that solves the following problem:

$$\begin{aligned} &\max \sum_{s=1}^{4} q_s (p_{is} - c) K_{is} \quad \text{subject to} \\ &K_{is} = K_i \text{ for all } s. \end{aligned}$$

The first-order conditions are

$$\begin{aligned} q_s p_{is} &= q_s c + \omega_{is}', \;\; s = 1, 2, 3, 4 \\ \sum \omega_{is}' &= 0, \;\; s = 1, 2, 3, 4 \end{aligned}$$

where ω' is the shadow price of the implementability constraint. Now, define $\omega_{is} = \frac{\omega_{is}'}{q_s}$ – the probability adjusted shadow prices. The first-order conditions can then be rewritten as

$$\begin{aligned} p_{is} &= c + \omega_{is}, \;\; s = 1, 2, 3, 4 \\ E\omega_i &= \sum_{s=1}^{4} q_s \omega_{is} = 0. \end{aligned} \tag{7}$$

Compared to the first-order condition in the deterministic case ($p_i = c$), see (1), the (probability adjusted) shadow price of the implementability constraint (ω_{is}) has been added and all variables have been indexed by s. In addition, there is a condition for the (probability adjusted) shadow price; its expected value should be zero. The number of equations has increased from 2 in the deterministic case (one for each region) to $2s+2$ in the stochastic case.

The first-order condition for investment in transmission capacity in the deterministic case is changed in the same way as the condition for investment in electricity production capacity; the first-order condition in the deterministic model, see (3), is extended by an additive term ε_s, which is the (probability adjusted) shadow price of the implementability constraint $K_{Ts} = K_T$, $s = 1,2,3,4$, and all variables are indexed by s. In addition, the expected value of the shadow price ε is zero:

$$\begin{aligned} &\gamma_{1s} + \gamma_{2s} \le c_T + \varepsilon_s \perp K_{Ts} \ge 0 \\ &E\varepsilon = 0. \end{aligned} \tag{8}$$

Actual transmission (trade) and consumption is decided in period 2, that is, *after* the scenario is known. Thus, to characterize these decisions no implementability constraint is needed; the conditions are therefore similar to the ones for the deterministic case, except that all variables are indexed by s. The first-order conditions for trade are thus

$$\begin{aligned} &p_{2s} - p_{1s} \le \gamma_{1s} \perp z_{1s} \ge 0 \\ &p_{1s} - p_{2s} \le \gamma_{2s} \perp z_{2s} \ge 0, \end{aligned} \tag{9}$$

whereas (4) is the corresponding first-order conditions under certainty.

Further, the first-order condition for the consumers is

$$x_{is} = \left(\frac{\theta_{is}}{p_{is}}\right)^2. \tag{10}$$

Relative to the deterministic case, see (5), the number of equations has increased from 2 (one for each region) to $2s$ in the stochastic case.

Finally, market clearing requires

$$\begin{aligned} K_{1s} + z_{2s} - z_{1s} &= x_{1s} \\ K_{2s} + z_{1s} - z_{2s} &= x_{2s}. \end{aligned} \tag{11}$$

Again, the only difference to the deterministic case, see (6), is that all variables have been indexed by s.

The stochastic model consists of equations (7)-(11). Note the difference to the deterministic model: First, each of the original first-order conditions has to apply in every scenario. Thus, the number of equations increases with a factor equal to the number of scenarios. Second, a shadow price for each equation that determines investment has to be added. Finally, there are conditions requiring each expected shadow price to be equal to 0.

While the stochastic model is much larger than its deterministic counterpart, the benefit of our approach is partly that the changes required to transform the deterministic model to a stochastic one are modest. Moreover, the stochastic model can be solved by using a standard mixed-complementarity problem solver, such as GAMS/PATH (Ferris et al., 2004), that is, the same software used to solve the deterministic model. Thus, there is no need to program special algorithms for stochastic programming. Note that all changes in the equations are linear. Hence, if the original deterministic problem is a well-defined, convex one, this remains true for the stochastic problem.

As discussed above, the downside of our approach is that the number of equations increases sharply. Thus, the computation time will increase, in particular for large models.[8] In a companion paper, see Brekke et al. (2013), the transformation strategy has been applied to a large-scale model of the European energy market. Of course, the stochastic model took more time to solve than the deterministic one; with 10 scenarios the solution time increased from 1 minute (the deterministic model) to 1 hour, and further to 25 hours with 30 scenarios.

3.2 The stochastic solution

As noted above, the deterministic solution implies $p_i = c$ for any value of (θ_1, θ_2). For the stochastic solution, this is no longer the case as $p_{is} = c + \omega_{is}$ and the shadow price of the implementability constraint (ω_{is}) will generally be non-zero. Thus, prices may deviate from unit cost. The intuitive reason is that an electricity producer has to decide on a capacity *before* demand is known. Thus the same amount of electricity is supplied when demand is low as when demand is high, and consequently the price will be low when demand is low and high when demand is high.

With uncertainty, the realizations of θ_1 and θ_2 may differ between regions and therefore also prices may differ between regions. This provides a market for transmission. It is easy to choose parameters such that the optimal stochastic solution is characterized by positive investment in transmission capacity. Thus, what seemed like a robust result in the deterministic model is not true in the stochastic model. Note that the stochastic equilibrium determines production, trade flows,

[8] In addition, the standard challenge to select good initial values is particularly tough for large models.

consumption, and prices in *all* scenarios simultaneously (in addition to investments – these do not differ between scenarios).

4 Monte Carlo simulations

Monte Carlo simulation is a method for numerical integration. If one wants to compute an expectation $Ef(\theta)$, where θ is a random variable, then Monte Carlo simulation may come handy. One simply draws a number of realizations of the stochastic variable $\theta,$ compute $f(\theta)$ and take the average. This crude method yields a valid estimate of $Ef(\theta)$ and it can be much improved upon, see, for example, Judd (1998, chapter 8).

For a numerical equilibrium model, Monte Carlo may seem as a viable option to assess the impact of uncertainty. One could draw a number of realizations of the stochastic variables and use the average as the expected outcome under uncertainty. Tempting as it is, there is a fundamental problem with this approach.

A key characteristic of equilibrium models is maximizing agents. Let $\pi(x,\theta)$ be the objective function of an agent where *x* is the vector of decision variables of this agent. Because the parameter θ is uncertain, the agent will maximize under uncertainty, that is, his choice follows from $\text{argmax}_x E\pi(x,\theta).$ Assume instead that one relies on Monte Carlo simulations. Then the following two-stage procedure may be used: First, for each realization of the stochastic variable $\theta,$ find the choice of the agent from $f(\theta) = \text{argmax}_x \pi(x,\theta).$ Next, compute the average of the choices and use this as the prediction of the choice made by the agent. Because $Ef(\theta) = E[\arg\max_x \pi(x,\theta)] \neq \arg\max_x E[\pi(x,\theta)],$ the Monte Carlo method does not produce a valid estimate of the behavior of the agent.

Below follows an illustration of this general discussion. The following system of equations corresponds to a Monte Carlo simulation of the original model:

$$p_{is} = c$$
$$K_{is} = x_{is} = \left(\frac{\theta_{is}}{p_{is}}\right)^2$$
$$\mathrm{K}_T = 0.$$

Here one obtains one value (solution) for each endogenous variable p_{is}, x_{is}, K_{is} for each scenario *s*, that is, for each realization of θ_{is}. In particular, the electricity capacities K_{is} will differ between the scenarios. Monte Carlo simulations thus simply ignore the fact that producers in the economy do not know which scenario that will materialize when they decide on investment. Put differently: Under Monte Carlo simulations the solution is found under the false assumption that producers consider the future as certain – which scenario that for sure will materialize differs between the simulations.

Comparing the Monte Carlo approach with the assumption that agents take the uncertainty into account when making decisions, note some major differences. In the Monte Carlo simulations, $p_{is} = c$ in all scenarios. Thus there is no variation in the price, but electricity production and electricity capacity will be different in each scenario. Moreover, there is no investment in transmission capacity. In contrast, with optimizing agents under uncertainty, electricity capacities, which are determined *before* the producer knows which scenario that will materialize, and production, which is equal to capacity, do not differ between the scenarios. Moreover, under uncertainty there is investment in transmission capacity, and the prices - $p_{is} = \theta_{is} / \sqrt{x_{is}}$ - differ between the scenarios because the parameters θ_{is} differ between the scenarios. To sum up: variables that differ between scenarios under Monte Carlo simulations do not differ between scenarios under uncertainty, and vice versa.

More fundamentally, according to economic theory, uncertainty changes the *behavior* of agents (compared with the case of no uncertainty). This is captured by the stochastic model but not by Monte Carlo simulations. For each Monte Carlo simulation, a realization of the stochastic variables, that is, one set of parameter values, is drawn from a probability distribution and then one finds the equilibrium in the resulting *deterministic* model. By simulating *n* times one finds *n* equilibria, all obtained from the same deterministic model with different parameter values. Needless to say, the realizations (parameter values) will in general differ between each of the *n* runs, but agents neglect uncertainty simply because the model is deterministic.

5 Discussion

Above, a guide to transform one-period numerical equilibrium models into stochastic models was derived. Our approach can, however, be extended to dynamic multi-period models with learning. The information available in different periods would then be represented by partitions of the set of scenarios; the decision makers in a given period do not yet know the exact scenario that will materialize in the future, only which subset the true scenario will belong to. Typically, decisions made in the first period will have to be the same in all scenarios, while decisions in a later period will be the same within a subset of scenarios, but different across subsets. In the last period the exact scenario will be known. Learning is represented by the gradually finer partition of the set of scenarios.

To take one simple example: Suppose the set of all scenarios is partitioned into two disjoint sets $S = S_1 \cup S_2$ where $S_1 \cap S_2 = \varnothing$, and assume that investors know, at the time of investment, which of these two subsets that will materialize, even if this was not known at some earlier stage. Hence, there is learning over time. The implementability constraints are now

$$K_s = K^j \text{ for all } s \in S_j,\ j = 1,2$$
$$\sum_{s \in S_j} q_s \omega_s = 0 \text{ for } j = 1,2.$$

Thus for a given information structure, one can handle implementability without altering the rest of the model. Also, irreversible investments can be handled by imposing the condition $K_{ts} \geq (1-\delta)K_{t-1,s}$, where t is time period and s is scenario.

Our transformation guide assumes that actors are risk neutral, that is, they maximize expected profits/utility. The approach presented above can, however, be extended to account for risk aversion, either by assuming that investments are decided by the firms' owners who are diversified in the financial market, or that investments are decided by risk-averse managers. In the first case, probabilities are replaced by weights derived from the prices of Arrow securities, that is, contingent claims that pay 1 \$ if a particular scenario materializes, see Arrow (1964). After replacing probabilities with normalized prices of contingent claims, all agents will behave as if they were risk neutral. Risk aversion will be reflected in the prices for contingent claims. This approach is similar to the use of equivalent martingale measures in finance; see, for example, Duffie (1996). With risk averse managers, a similar approach can be used, but in this case the scenario-weights will be firm specific, and thus some modest changes in the first-order conditions would be required.[9]

6 Conclusion

The energy markets are characterized by many agents simultaneously solving decision problems. A deterministic equilibrium model captures this complex structure by specifying the solution of the (deterministic) decision problems, along with relations taking interrelationships between actors and markets into account. How should this approach be extended to handle uncertainty?

The standard approach to analyze uncertainty is Monte Carlo simulations, that is, to run a deterministic equilibrium model for different parameter values. As argued in the present article, this is not an adequate way to assess behavioral uncertainty; one should rather rely on some type of stochastic modelling. It is, however, not necessary to start the stochastic modelling from scratch. Rather, one could transform the existing deterministic model into a stochastic model by following our guide:

- For each decision variable determined *under uncertainty*, one should introduce one (probability adjusted) shadow price in the first-order condition, and a corresponding equation stating that the expected value of this shadow price is zero. All decision variables, as well as (probability adjusted) shadow prices, should be indexed by the scenario.

[9] More details on how to account for risk aversion and to extend the method to cover dynamic models can be obtained from the authors upon request.

- For each variable that is determined *after* the uncertainty has been revealed, the first-order conditions from the deterministic case are not changed but all variables are indexed by the scenario because in general their values depend on the scenario.

Our guide implies that if there are l relations in the original deterministic model, the number of relations in the stochastic model is $ls+k$ when there are s scenarios and k variables are determined under uncertainty. If the number of scenarios is doubled, the number of relations increases by ls. Thus, whereas it is easy to transform a deterministic model to a stochastic one, the challenge may be to solve the model due to its size. This suggests that the number of scenarios should not be "too high". For a discussion of how many scenarios are needed in a stochastic programing context, see Kaut and Wallace (2007).

With our approach, there is no need for introducing expected values of variables in the first-order conditions. Moreover, the algorithm for the deterministic model can be used also for the stochastic model; no specialized algorithm is needed. The latter is in contrast to the numerical papers cited in Section 1. Hence, with our guide numerical deterministic models can rather easily be transformed into stochastic models that are in line with standard economic theory on uncertainty.

Does our transformation strategy work for large-scale computable energy market equilibrium models? In a companion paper it is illustrated that our transformation strategy works for a large-scale, numerical, deterministic multi-market equilibrium model of the Western European energy markets – LIBEMOD, see Aune et al. (2008). The guide is used to transform this deterministic model to a stochastic equilibrium model, and then to analyze the impact of economic uncertainty on the European energy markets. It is shown that replacing the uncertainty with expected values leads to large deviations from the optimal solution under uncertainty. Therefore, policy assessments should be based on stochastic equilibrium modeling.

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