Private Politics and Public Regulation

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Abstract

Public regulation is increasingly facing competition from "private politics" in the form of activism and corporate self-regulation. However, its effectiveness, welfare consequences, and interaction with public regulation are poorly understood. This paper presents a unified dynamic framework for studying the interaction between public regulation, self-regulation, and boycotts. We show that the possibility of self-regulation saves on administrative costs, but also leads to delays. Without an active regulator, firms self-regulate to preempt or end a boycott and private politics is beneficial for activists but harmful for firms. With an active regulator, in contrast, firms self-regulate to preempt public regulation and private politics is harmful for activists but beneficial for firms. Our analysis generates a rich set of testable predictions that are consistent with the rise of private politics over time and the fact that there is more self-regulation and activism in the US, while public regulation continues to be more common in Europe.

Keywords: Private politics, boycotts, war of attrition, activism, regulation, self-regulation, corporate social responsibility (CSR).
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1 Introduction

For many companies, their business models or practices result in negative externalities that markets fail to correct. These externalities may come in different forms. Some, like water and air pollution, are relatively easy to measure and quantify, while others are more intangible, such as the direct disutility that some people may experience if a company uses child labor, provides poor workplace conditions, or logs tropical forests, thereby endangering species and biodiversity. The traditional solution to such externalities involves the government in one way or another, but more recently, a phenomenon called *private politics* has started to receive more attention. It is now quite common for activist groups that seek to curb or limit certain practices to not necessarily engage in public channels like lobbying or political campaigns; instead, they often start activist campaigns and threaten to organize a boycott if their demands are not met.[1] The rise of activism has led to an increasing number of companies or even entire industries choosing to self-regulate and restrict their practices, which has put an end to the government's monopoly on regulation.

Textbook examples of effective and successful boycotts include Greenpeace's boycott of Shell in 1995 over Shell sinking the outdated offshore oil storage facility Brent Spar, and the boycott of Citigroup by the Rainforest Action Network (RAN) from 2000 to 2004 over Citigroup's loans to companies engaged in unsustainable mining and logging. The campaign against Shell included organizing a successful boycott in Germany where sales at Shell gas stations fell by as much as 40 percent and an occupation of Brent Spar by Greenpeace activists. Although Shell initially gave the impression that it had made a firm decision, the company gave in after two months of protests.[2] The campaign by RAN against Citigroup lasted much longer, involving episodes such as Columbia University students cutting their Citibank cards as well as picketing the residences of Citigroup's senior executives. However, although Citigroup was able to stand firm by its decision to not give in for several years, also this boycott was ultimately successful (Baron and Yurday, 2004).

Not all firms face the same regulatory environment, understood as the combination of presence or influence of a motivated government regulator and/or powerful activist groups; this varies substantially by industry and by jurisdiction. For example, producers of specialized medical equip-

[1] According to Glickman (2009:302-310), the late 1990s saw a big increase in boycotts. This was accompanied by a growing number of issues that captured the attention of activists. While the boycotts of the 1980s were primarily aimed at companies collaborating with the regime in South Africa, in the 1990s many new issues, such as animal rights, emerged.

[2] See Diermeier (1995). The statement released by Shell on June 20, 1995, included this: "Shell's position as a major European enterprise has become untenable. The Spar had gained a symbolic significance out of all proportion to its environmental impact. In consequence, Shell companies were faced with increasingly intense public criticism, mostly in continental northern Europe. Many politicians and ministers were openly hostile and several called for consumer boycotts."

ment (goods with few substitutes, supplied to hospitals) are likely to be concerned with government regulation only, whereas airlines (a competitive, consumer-oriented industry) should be concerned with both government regulation and consumer activism. The cross-country differences are also noteworthy: while the government has a more traditional regulatory role in Europe, self-regulation and activism are more common in the US. The reasons behind these stark differences, as well as the causes and consequences of the rise of private politics and the changing regulatory environment are poorly understood and barely discussed in the formal literature.

We seek to close this gap by studying public and private regulation within a single framework. This framework allows us to address a number of fundamental questions previously ignored in the literature, specifically:

(1) WHEN does private politics improve efficiency?

(2) WHO benefits and who loses from the emergence of private politics?

(3) WHAT is the relationship between private politics and public regulation, and do they substitute or complement each other?

(4) HOW can we explain the rise of private politics over time, and the differences between the US and Europe?

These questions are important for several reasons. If private politics becomes more common, we need to understand whether it can or even should replace public regulation, or whether private politics needs the presence of a government regulator instead. This concern is discussed by Doh and Guay (2006:51): "*Some observers now regard NGOs as a counterweight to business... others suggest that there may be risks of 'privatizing' public policies that deal with environmental, labour, and social issues.*" It is even more important to understand the interaction between the two in order to evaluate the total regulatory pressure on an industry. Such evaluations are not only important to appropriately choose domestic regulation, but also when comparing concessions, e.g., in trade or climate negotiations.

We provide a model that abstracts away from specific details about the country and the industry—such as the competitive environment or the nature of the good produced—and instead it focuses on the regulatory environment. More precisely, there is a firm (F) which faces a government regulator (R), or an activist group (A), or both. The firm produces and sells goods, but does so in a way that the activist group believes to be wrong or harmful. The firm is aware of the activists' concern and may decide to adjust its practice (i.e., self-regulate). Such self-regulation, however, is costly to the firm. As long as no regulation is in place, the activist that runs a campaign against the firm can decide to initiate a costly boycott. The boycott ends if the firm self-regulates or the activist gives up. To model boycotts, we develop a novel model of war-of-attrition with private information, where each player learns its cost parameter only at the beginning of the boycott. Thus, even though both players use pure strategies in equilibrium during the boycott, the time where either of them gives in during the boycott appears to be stochastic from the viewpoint of the other player, as well as from that of its earlier (before-the-boycott) self. In addition, the regulator may at any time impose regulation on the firm. Both self-regulation by F and regulation by R are irreversible actions that end the game.[3]

Our dynamic framework has a number of attractive features. First, by allowing the three players to act at any time, we can be agnostic about the exact sequence of moves. In finite multi-stage games, the outcome is typically sensitive to the order of moves, as will be explained below. Second, the difference between the regulator, who can use coercion, and the activist, who only can impose a cost without ending the game, is meaningful in a dynamic model only. Third, real-life boycotts are often quite long-lasting, so it is realistic to model them as wars of attritions. A dynamic model is also necessary to make predictions regarding delays and durations of boycotts. Furthermore, while self-regulation is always the first-best terminal outcome in our game, the possibility of delay shows that private politics come at a real cost, rather than in the form of an off-path threat. Finally, the model is rich enough to allow for a number of scenarios along the equilibrium paths. For example, the firm may withstand the activists' campaign, be forced to self-regulate, or end up regulated. Self-regulation or regulation may happen before a boycott starts, during the boycott, or even after the boycott ends. These possibilities are, of course, possible outcomes that can happen in reality.[4]

[3] This assumption is weak, since a reversal would never happen in equilibrium if there were a small positive cost of returning to no regulation. For the firm, abandoning self-regulation after the boycott is called off is a possibility, but then the activist would never stop the boycott, which eliminates the rationale to self-regulate in the first place. To avoid this unnatural feature, we assume that self-regulation is irreversible. In practice, there may be technological and/or institutional reasons that make reversals unlikely. E.g., a firm that invested in filters that reduce pollution will likely find it impractical to uninstall them, given the small savings and large reputational risk, or a regulator may come under scrutiny if it imposes and lifts requirements within a short timeframe. See also Besanko, Diermeier, and Abito (2011) for a different model in which activists constantly trash the firm's reputation in order to induce it to keep investing in self-regulation.

[4] In addition to the boycotts of Shell by Greenpeace and of Citigroup by RAN, which are examples of ultimately successful boycotts (though the first was very short and the second very long), there are many cases of boycotts that failed to result in self-regulation. Friedman (1985) noted that full-scale boycotts achieve success in only half of the cases. He also observed that "simply announcing that a boycott was under consideration was associated with success or partial success in about one third of the cases studied," (p.109) so self-regulation may materialize even before the boycott starts. In other cases, the outcome is public regulation. For example, a number of activist groups boycotted Nestlé over its practice of marketing infant formula to mothers in the 1980s and 1990s. They formed coalitions such as INFACT (Infant Formula Action Coalition) in the U.S. and Canada and IBFAN (International Baby Food Action Network) in other countries such as Sweden, India, and New Zealand. Several years of boycotts did not lead to any credible voluntary action by Nestlé, but the boycotts led to governmental interventions in different countries at different times. In India, for example, the government effectively banned Nestlé's promotions of breast-milk substitutes and feeding bottles in 2003 (see Saunders, 1996, and http://www.infactcanada.ca/The%20History%20of%20the%20Campaign.pdf). In other cases, regulation has come before boycotts. For example, in 2010, McDonald's Happy Meals were banned in San Francisco by the city Board of Supervisors on the grounds that including a free toy with an unhealthy meal pro-

Our analysis sheds light on the questions above. First, self-regulation may be more efficient than government regulation, but the firm takes advantage of this fact and will only self-regulate after a substantial delay. Second, the possibility to self-regulate benefits the firm but harms the activists if an active regulator is present; but the converse is true otherwise. Third, private politics and government regulation are strategic substitutes and crowd each other out.

In addition to these results, the model makes a large number of testable predictions—for example, regarding the duration of boycotts and their likelihood of success. Nevertheless, when we address the fourth question on the difference between the US and Europe, we focus on two simple parameters: the (expected) cost of running the boycott for the activist and for the firm. Section 4 argues that because the US has a larger market and smaller trade barriers on its continent, the market competition is stronger. This makes it more costly for a firm to be singled out in an campaign, and it also reduces the boycotters' cost of finding a substitute good to purchase instead. As we argue, these differences can explain the US-vs.-Europe puzzle.

The term "private politics" was coined by David Baron (2001; 2003) to describe nonmarket interactions between individuals, NGOs, and companies, and the term has since been at the center of a relatively small but growing literature. The puzzle of why firms *self-regulate* was addressed by Baron (2001), who assumed that a company's reputation positively affects demand for its product and thus is worth investing in. A different theory is presented in Feddersen and Gilligan (2001), who argue that self-regulation by one of the competing firms results in market segmentation that can benefit all firms. When investments in Corporate Social Responsibility (CSR) improve the firm's reputation (stock), activists can increase the firm's investment in CSR by occasionally destroying its reputation when it becomes too good (Besanko, Diermeier and Abito, 2011).[5]

The *activists* play a more central role in Baron (2003) and Baron and Diermeier (2007), where firms are faced with demands to adopt certain practices or else face a damaging campaign. The analysis is extended by Baron (2009), which studies two competing firms and allows the activist to be an (imperfect) agent of citizens.[6]

motes obesity in children. Attacking such practices by a large multinational corporation would be typical for activists, but in this case, boycotts never materialized (see also http://abcnews.go.com/blogs/health/2011/11/30/mcdonalds-skirts-ban-charges-10-cents-per-happy-meal-toy/).

[5] The idea that socially responsible actions of companies have a positive impact on their reputation and performance has found empirical support. For example, Dean (2004) finds that a pre-existing reputation at the time of crisis affects consumers' perception of a brand after the crisis, while Minor and Morgan (2011) document the fact that companies with a good reputation take a lower hit on their stock price as a result of a crisis. Bartling, Weber, and Yao (2015) use a series of laboratory experiments to study socially responsible behavior of firms in Switzerland and in China.

[6] Baron (2012) further develops this case by allowing for two activist groups, one more moderate and one more aggressive. It then makes sense for each of the two competing firms to cooperate with the moderate group, as it makes a boycott less likely. See also Baron (2010), which looks at cooperative arrangements in which various types of activist groups can enforce cooperative behavior.

The *boycott* itself has attracted quite a bit of attention, since it is one of the most typical, and certainly the most visible, implementations of private politics. Diermeier and Van Mieghem (2008) model boycotts as a dynamic process, in which each of the (infinitesimal) consumers decides whether to participate, depending on the number of other consumers boycotting the product. Delacote (2009) observes that, when consumers are heterogeneous, boycotts are less effective since consumers who buy a lot (and thus could hurt the firm most) are also the ones with the highest cost of boycotting. Innes (2006) builds a theory of boycotts under symmetric information, suggesting that an activist either targets a large firm with a short boycott that would show that the activist invested in preparation, or targets a small firm, in which case the boycott is persistent since the firm finds it too costly to satisfy the demands of the activist. In the latter case, the purpose of the boycott is to redistribute customers to a larger, more responsible firm. Baron (2014b) specializes the model to study multiple firms, multiple activists, and the matching between them. In contrast to the literature above, we model the boycott as a *war of attrition*, and to the best of our knowledge we are the first to do so.

Relatively few papers study self-regulation and/or activism in the shadow of the government. For example, Maxwell, Lyon, and Hackett (2000) let firms lobby for regulation in order to effectively restrict entry to the market in which they operate, and self-regulation allows the firm to stay in business. In Baron (2014a), the government as well as activists has preferences over the degree of the firm's self-regulation. In equilibrium, the firm will satisfy the demands of the government up to the point at which the government would reach gridlock if it attempted further regulation, but it might also put in place additional self-regulation in order to prevent an activist campaign. In Lyon and Salant (2013), activists target individual firms and force them to self-regulate in order to change their behavior in a subsequent lobbying game. For instance, a firm that has been forced to reduce its level of emissions will later prefer that other firms do the same; it thus supports rather than opposes public regulation. In another recent paper, Daubanes and Rochet (2013) study an environment in which regulators are perfectly informed about the social optimum but are captured by the industry, while activists are poorly informed but committed to their cause. The authors derive conditions under which the presence of activists improves social welfare. All these papers either involve a static model or assume a particular sequence of moves, and thus do not incorporate the dynamics of activists' campaigns or boycotts.[7]

[7] Our paper bridges the traditional literature on regulation and the more recent literature on private politics. The literature on public regulation is huge, and the works most closely related to our paper are those that attempt to compare different regulatory regimes. For example, Djankov et al. (2003) and Shleifer (2005) describe different regulatory regimes as loci on an Institutional Possibility Frontier. When choosing the extent of the regulatory state, as opposed to relying on market forces, the society trades off the costs of potential chaos (disorder) and of excess rigidity

Finally, our paper makes a theoretical contribution to the literature on wars of attrition. Since the two players, the firm and the activist, are fundamentally different, we cannot make the standard assumption that the types are drawn from the exact same distribution. To the best of our knowledge, the only paper permitting such heterogeneity is Ponsati and Sákovics (1995), where participants are uncertain about each other's *benefits from winning*. In our framework, their assumptions would lead to the possibility of signaling before the boycott begins and during the boycott, unnecessarily complicating the analysis. Thus, we assume instead that the players have private information about their *cost of continuing* the boycott. These costs are boycott-specific, so a player learns the cost only when the boycott starts. By assuming that types are distributed exponentially, we prove existence and uniqueness of an equilibrium that takes a very tractable form: while each type of firm and activist plays a pure strategy, for the other player, as well as outside observer, it appears as if the player acts at a *fixed* Poisson rate. This feature simplifies the characterization of comparative statics and payoffs. The tractability of our war-of-attrition model should make it useful for other settings as well, e.g., in entry/exit models of markets or political campaigns, such as the primaries in the US elections.

The next section describes a model with all three players: the firm, the activist, and the regulator. Section 3 analyzes the different regulatory environments: (1) the firm and the regulator only, (2) the firm and the activist only, and finally (3) the model with all three players in the same game. The analysis ends with a discussion of the role of commitments, and shows why our model naturally permits a unique equilibrium outcome. Section 4 explains how our theory can shed light on the rise of private politics over time, and why self-regulation and activism are more common in the US, while public regulation is more common in Europe. Section 5 concludes, while Appendix A contains all proofs. Appendix B (to be available online) is technical and contains a complete analysis of the war-of-attrition game.

2 The Model

The game allows for up to three players: the regulator R (she), the activist A (he), and the firm F (it). Time is continuous and infinite, and we do not impose any assumption on the sequence of moves. We proceed with introducing the (very simple) action sets of each of the players, one by

(dictatorship), and selects an efficient alternative on the frontier (Mulligan and Shleifer, 2005). Glaeser, Johnson, and Shleifer (2001) compare the efficiency of public regulators and independent judges in enforcing contracts. Other works have emphasized the problem of regulatory capture; see, for example, Laffont and Tirole (1991); Dal Bó (2006) provides an overview of both the theoretical and the empirical literature. The interaction between the firm and the regulator is very different in our paper (and more similar to the model of Harstad, 2016, studying rainforest conservation).

one. Flow payoffs in the status quo are normalized to zero, and r is the common discount rate.

The firm: At any point in time, F can end the game by self-regulating. For simplicity, we assume that this decision is binary: either the firm self-regulates or not. This dichotomy is natural in many situations, as when a firm must decide whether or not to use palm oil or child labor, or when Shell had to decide whether or not to sink Brent Spar. For these types of self-regulations, it is also natural to assume that the firm's action is observable by everyone. In the concluding section, we discuss how our results may continue to hold if self-regulation could be gradual.

The firm's flow *cost* of self-regulation is $c > 0$, so that F realizes the present-discounted payoff $-\frac{c}{r}$ at the moment it decides to self-regulate. The other parties (A and R) benefit from F self-regulating: the flow *benefit* equals $b > 0$ for A, while R gets flow *surplus* $s > 0$. We do not assume any relation between these parameters, although the case where $s = b - c$ could be a natural benchmark if the regulator fully internalized the payoffs of F and A but nothing else.

The regulator: Just as F can self-regulate at any moment, regulator R can at any moment decide whether or not to impose public regulation on F; we assume that this is also an irreversible decision that ends the game. Importantly, we assume that regulation is more expensive than self-regulation for both F and R. Specifically, the flow cost of F increases by $k > 0$ to $c + k$ if it is regulated by R, whereas for R the additional cost is $q \in (0, s)$, so that her benefit from regulating F is $s - q$. These assumptions are natural: R may need to monitor and frequently visit the firm, which involves both direct costs and opportunity costs, as it takes valuable resources from regulating other firms or industries. Similarly, the cost for F is likely to be higher because it must deal with red tape, documentation, paperwork, or bureaucratic rules. Furthermore, R may be 'clumsy' and unable to regulate F in the most efficient manner.[8]

The activist: Like the regulator, also the activist A is assumed to benefit from regulation. On the one hand, one might argue that also A ought to prefer that F self-regulates, since that saves on administrative costs. On the other hand, if F self-regulates, A may find it necessary to monitor the firm regularly herself, suggesting that A may prefer public regulation. For simplicity, we assume A's flow benefit to be b, regardless of whether regulation is public or private.

The activist A can impose a cost on F to motivate F to self-regulate. However, in contrast to R, A does not have the authority to impose regulation on F. Instead, A can try to pressure F to self-regulate by initiating and continuing a boycott. Once a boycott has started, the boycott can

[8] In industries where these assumptions are violated the equilibria are trivial. If R preferred to regulate, she would regulate right away and the game would be over. If F preferred government regulation, then R would know that F would never self-regulate, and would again have to regulate immediately. In either case the outcome would be a publicly regulated industry.

end in one of three possible ways: A can give up, F may self-regulate, or R might regulate; in the two latter cases, we call the boycott successful.[9]

The boycott: We provide a novel formalization of the boycott as a war of attrition between A and F. The boycott is costly for both A and F, and it is reasonable that the costs are increasing over time. For the firm, a short-lasting boycott might merely imply delayed consumption by the same buyers, whereas a longer boycott may prompt the buyers to permanently switch to a competitor.[10] For the activist, a longer boycott may be increasingly costly because of budget constraints, or because maintaining the interest of potential buyers may require carrying out costlier and more newsworthy activities. To capture this intuition, the marginal cost of a boycott after some time τ is assumed to equal $\frac{\tau}{\theta_F}$ and $\frac{\tau}{\theta_A}$ for F and A, respectively. Here, θ_A and θ_F are the *types* of A and F, and the type measures the player's ability to deal with the boycott. In reality, it is difficult for A and F to be certain about the other party's ability to deal with the boycott, so we assume the types are only privately known. Furthermore, since it is hard to predict in advance how costly (or effective) a boycott is going to be, we assume that a player learns its type only when the boycott starts (Baron, 2012, is similarly assuming that the effect of the boycott is drawn only when it starts). At that time, θ_i is independently drawn from an exponential distribution with expectation λ_i, so its probability density function is $f_i(\theta_i) = \frac{1}{\lambda_i}\exp\left(-\frac{\theta_i}{\lambda_i}\right)$, for $i \in \{\text{A,F}\}$. With this notation, λ_i measures how *cheap* the boycott is for $i \in \{\text{A,F}\}$.

When we discuss comparative static, we will emphasize the effects of the λ_i's for two reasons. First, the λ_i's vary between the countries in natural ways: Section 4 documents that there is more competition in the US than in Europe, which implies that it is more costly for a firm to be singled out in a campaign, and that it is easier for activists to find substitute products. Thus, λ_A should be larger and λ_F should be smaller in the US than in Europe. (We find it harder to argue that b and c vary systematically between countries.) Second, the comparative static with respect to the λ_i's is likely to be more robust, because they capture distributions rather than exact values, and are thus less sensitive to our assumption of common knowledge of other parameters.

Apart from the ongoing costs of boycott, it is realistic that the players also fear reputational losses. For example, a firm that has been targeted in a boycott may never fully recover its reputation, since some potential consumers may not pay attention when activists cease the boycott or the firm gives in to their demands, or their trust to the firm may be permanently undermined in either case.

[9] If actions by one or more participants are made at exactly the same time, then we assume that there is a lottery that determines which of the actions 'goes through', and each of the actions has a positive chance to have the impact (e.g., a fair lottery to save on notation). In equilibrium this will happen with probability zero.

[10] King and McDonnell (2012:22) find that "for every additional day of boycott media coverage the corporate target experiences greater damage to the market value."

As a short-cut for such a reputational cost, we assume that F, as soon it is publicly accused of wrongful activity (i.e., at the start of the boycott), pays the present-discounted cost of $\frac{h}{r} > 0$ or, equivalently, it is forever paying the flow cost h. Similarly, the activist may also fear reputation losses: if the boycott is unsuccessful and A gives in, it will be less feared by firms, less trusted by consumers, and perceived as ineffective by donors. As a short-cut for these costs, we assume that when A calls off an unsuccessful boycott, he faces the present-discounted damage $\frac{l}{r} > 0$ or, equivalently, the flow cost l.

For simplicity, we assume that R does not get any direct costs or benefits from the boycott, and that R's type is publicly known.[11] The following table summarizes the flow payoffs:

Payoffs	Status quo	Self-regulation	Regulation	Boycott	If started	If called off
Activist	0	b	b	$-\tau/\theta_A$	0	$-l$
Firm	0	$-c$	$-c-k$	$-\tau/\theta_F$	$-h$	0
Regulator	0	s	$s-q$	0	0	0

Commitments: While we allow for reputational losses, we assume that none of the players can *commit* to future actions. It is clearly reasonable that activists cannot pre-commit to boycott forever; if they could, firm would give in immediately. Similarly, firms cannot credibly promise to never give in to the activists' demands, for otherwise boycotts will never happen. Furthermore, firms are unable to commit to self-regulate at a given future date, since, when that time arrives, the firm would prefer to renege on the pledge. We treat the regulator in a similar way by assuming she cannot pre-commit to future actions, for example, because future regulation may be influenced by new politicians. Section 3 ends the analyses with a discussion of what the players would want to commit to, if they could.

At the same time, as mentioned above, we assume that F's and R's *actions*, once taken, are irreversible and therefore end the game. This assumption is weak, however, since neither F nor R could strictly benefit from reversing their decisions, even if they could. Furthermore, the costs of monitoring that the players stick to their commitments could be incorporated into the parameters; we omit these to save on notation. We also maintain the assumption that once the boycott has ended, it cannot be started again.[12] Thus, we can refer to the following parts of the game as *phases*: Phase 0 is the initial phase of the campaign where the boycott has not yet started; Phase 1 refers to an ongoing boycott; Phase 2 begins if A gives up on the boycott.

The equilibrium concept: Since a part of the game (Phase 1) features asymmetric information,

[11] This is intuitive, since R may be thought of as a long-term player that regularly engages in similar problems, thus it is natural to assume that R's type is common knowledge. In any case, it will become clear that small departures from these assumptions will not alter our results.

[12] Multiple boycotts are analyzed in our working paper version, Egorov and Harstad (2015).

the natural equilibrium concept is Perfect Bayesian equilibrium (PBE). It turns out that there is a unique PBE in Phase 1 in the game between F and A (a complete proof of this fact is given in Appendix B). For the other phases or games, we need a refinement. We thus assume that every player's strategy is a function of only those aspects of the (private) history that is payoff relevant to that player, as long as such a simple strategy is a best response to the strategies of the other players.[13] It follows that strategies must be stationary in Phase 0 and Phase 2 of the game, since calendar time itself is not payoff relevant. This refinement is in line with Maskin and Tirole's (2001) idea of Markov perfectness (typically defined for games with complete information), which requires strategies to be conditioned on payoff-relevant aspects of the history only.

3 Analysis

Before we analyze the game with all three players, we start by studying the interaction between only two of them. The game between the firm and the activist is relevant when the public regulator can be assumed to be rather passive, e.g., because she is convinced that not regulating is socially optimal, while the game between the firm and the regulator is interesting in contexts where activist groups are less effective, e.g., when the boycott is prohibitively costly or illegal. The insights generated by these games are helpful to understand the full game. Furthermore, comparing the outcomes of these games allows us to make conclusions about the importance of the regulatory regime.[14]

3.1 Public Regulation vs. Self-regulation

The game between the firm, F, and the regulator, R, is a simple stopping game: F can stop at any time by self-regulating and guarantee the payoff $-\frac{c}{r}$ to itself and $\frac{s}{r}$ to R, while R can regulate the firm at any time, giving payoffs $-\frac{c+k}{r}$ to F and $\frac{s-q}{r}$ to herself. Note that both players would prefer self-regulation to direct regulation when k and q are positive. Despite this alignment of interest, there is no equilibrium where F self-regulates immediately: if F did so, R would simply wait; but if R never imposed regulation, F would not self-regulate. Similarly, there is no equilibrium

[13] To be precise on what "payoff relevant" is, define the private history of player $i \in \{$A,F,R$\}$ at time t (if the game has not yet ended) as the actions played by any player or Nature and observed by player i by time t, as well as the time when these actions took place. Thus, for R, the history is either empty, or it consists of the time t_s the boycott started, or t_s as well as the time t_e the boycott was called off by A. For A and F, histories are similar, except that if the boycott has started, the history includes the choice of A**'s** and F's types, respectively, which is made by Nature when the boycott starts. For player i, we call two histories h_i^t and $\tilde{h}_i^{\tilde{t}}$ of player i *payoff equivalent* if the continuation payoffs of player i, computed from moment t for h_i^t and from moment $\tilde{t}$ for $\tilde{h}_i^{\tilde{t}}$, are the same for any combination of future actions by A, F, R, or Nature, such that for any action at time $t+\tau$ after h_i^t there is an identical action at time $\tilde{t}+\tau$ after $\tilde{h}_i^{\tilde{t}}$, and vice versa, for every $\tau \geq 0$. We restrict attention to PBEs for which the strategy for each player $i \in \{$A,F,R$\}$ is the same for any pair of payoff-equivalent histories.

[14] The game between the activist and the regulator only would be trivial, since the regulator would regulate immediately if the firm existed but could not self-regulate.

where R regulates immediately, as then F would self-regulate immediately, but then, again, R would prefer not to regulate. Thus, there is no equilibrium in pure strategies. However, there is a unique equilibrium in mixed strategies (where the players mix between times to act).

Since time is continuous, the stationary mixed-strategy equilibrium is characterized by two *Poisson rates*: For the firm, the Poisson rate $\phi \geq 0$ specifies the probability that F acts over the next marginal unit of time; for the regulator, the Poisson rate is $\gamma \geq 0$. For F, it is a best response to self-regulate if and only if γ is large. For R, it is a best response to regulate if and only if ϕ is small. The best-response curves cross exactly once, and this pins down the equilibrium rates. More precisely, there is a unique equilibrium, and in that equilibrium both F and R are indifferent between acting and waiting.

Proposition 1 *In the game between F and R, there is a unique equilibrium and it is in mixed strategies. The regulator introduces regulation at a larger Poisson rate if c is large and k is small, while F self-regulates at a larger Poisson rate if s is large while q is small:*

$$\begin{aligned} \phi &= r\frac{s-q}{q} \in (0,\infty)\,, \\ \gamma &= r\frac{c}{k} \in (0,\infty)\,. \end{aligned}$$

The comparative statics are intuitive. If self-regulation is costly for F, while public regulation is not much more costly, then F is relatively reluctant to self-regulate and R needs to step in sooner. If R faces a low cost and a large surplus when regulating F, then R is eager to act and F must in equilibrium self-regulate at a higher rate.

It is straightforward to derive additional comparative static on the probability of the various outcomes. The probability that eventual regulation is public is $\frac{\gamma}{\phi+\gamma} = \frac{1}{1+\frac{k}{c}\frac{s-q}{q}}$, which is increasing in q and c but decreasing in k and s. The expected delay before regulation or self-regulation equals $\frac{1}{\phi+\gamma} = \frac{1}{r}\frac{1}{\frac{c}{k}+\frac{s-q}{q}}$, which is increasing in q and k but decreasing in s and c. Naturally, the less is at stake, the slower the players will act.

The payoffs for the players are the following. Since acting and thereby ending the game is a best response to both players, the firm's payoff is $v = -\frac{c}{r}$, and the regulator's payoff is $w = \frac{s-q}{r}$. For a passive activist benefitting b from any type of regulation, the expected utility is $u = \frac{\phi+\gamma}{\phi+\gamma+r}\frac{b}{r} = \frac{\frac{s-q}{q}+\frac{c}{k}}{\frac{s-q}{q}+\frac{c}{k}+1}\frac{b}{r}$.

The benefits of private politics are the following. If the firm were unable to self-regulate, perhaps because such an action could not be monitored or verified, public regulation would be imposed immediately. Thus, the possibility to self-regulate leads to a more efficient outcome, since the firm may self-regulate at a lower cost, but the cost is that the regulatory outcome is delayed. Moreover, the firm captures the entire benefit of the possibility to self-regulate, since its expected cost decreases from $c+k$ to c. The regulator is indifferent and receives the payoff $s-q$ whether or not F can self-regulate. A passive activist benefitting b from any type of regulation is therefore directly harmed by the firm's ability to self-regulate, since regulation is then delayed.

3.2 Private Politics

This subsection analyzes the two-player game between the firm, F, and the activist, A. Under the assumption that A can boycott only once, the game can be solved by backwards induction. Consider Phase 2, the subgame *after* the boycott has ended. In this phase, only F is capable of taking an action. Since self-regulation is costly, F prefers to stick to the status-quo and not self-regulate:

$$\phi_2 = 0,$$

where the subscript refers to Phase 2. In other words, both players expect to receive a payoff of zero when entering Phase 2 (so $u_2 = v_2 = 0$). This outcome is anticipated during the boycott.

In Phase 1, the two players play a war of attrition: they face an ongoing cost of boycott which is private information, and each player wants the other to give in first. The moment F gives in, it receives a payoff equal to $-\frac{c}{r}$, while A gets $\frac{b}{r}$. If A gives in instead, then F gets away with 0, while A receives a payoff equal to $-\frac{l}{r}$.

As long as none of the players have ended the boycott, each player will over time become more pessimistic about the opponent's cost of boycott. Furthermore, the cost of the boycott increases over time. Thus, for any type θ_i of player $i \in \{\text{A,F}\}$, there is an optimal stopping time at which this particular type prefers to give in. In other words, each type plays a pure strategy and is determined to give in at this exact time, unless the opponent has already ended the game.

Since each player's type is private information, the stopping time looks uncertain and randomly distributed from the opponent's point of view. In fact, in the model we have specified above, it turns out that in the unique equilibrium, each player will end the game at a Poisson rate that is constant and independent of how long the boycott has lasted. The Poisson rate for the firm in Phase 1 will be referred to as ϕ_1, while the Poisson rate for the activist is named ρ. The fact that the equilibrium rates happen to be constant simplifies the analysis as well as the comparisons to the other phases and alternative regulatory environments.

Proposition 2 *In the game where A boycotts F, there is a unique equilibrium, and it is in pure strategies.*

(i) Both F and A play 'linear' strategies: F of type θ_F gives in after time $\tau = \frac{1}{\phi_1 \lambda_F}\theta_F$ from the start of the boycott and A of type θ_A gives in after time $\tau = \frac{1}{\rho \lambda_A}\theta_A$, where $\phi_1 > 0$ and $\rho > 0$ are the unique positive solutions to the pair of equations:

$$\begin{cases} \phi_1 = \frac{1}{c\lambda_F\left(1+\frac{\rho}{r}\right)}; \\ \rho = \frac{1}{\lambda_A\left(l+\phi_1\frac{b+l}{r}\right)}. \end{cases}$$

Consequently, for an outside observer, the times of concessions for F and A are distributed exponentially with expectations $\frac{1}{\phi_1}$ and $\frac{1}{\rho}$, respectively, and F and A concede at Poisson rates ϕ_1 and ρ, respectively.

(ii) If λ_F decreases, or λ_A increases, F self-regulates faster (ϕ_1 increases) and A ends the boycott at a slower rate (ρ decreases), so the boycott is more likely to succeed. This makes F worse off (v_1 decreases) and A better off (u_1 increases).

When the players take the equilibrium (ϕ_1, ρ) as given, a firm of type θ_F gives in by self-regulating exactly at time $\tau = \frac{1}{\phi_1 \lambda_F}\theta_F$, linearly increasing in its ability to deal with the boycott, as measured by its type, θ_F. The activist's time of ending the boycott is similarly increasing in θ_A. The equilibrium is the unique positive solution to the two best-response functions. With the exponential distributions, each player's choice of time becomes uncertain (given that its type is unknown to the other player), and the Poisson rate at which each player acts happens to be constant.[15]

The comparative static is natural. Although the players' actual costs of the boycott are private information, parameters λ_i that measure the expected costs are publicly known, for $i \in \{\text{A,F}\}$. If λ_F decreases, and the boycott becomes more expensive for F, then F self-regulates at a faster rate. This, in turn, encourages A to wait and A thus concedes at a lower rate. As a result, the boycott is unambiguously more likely to be successful. Furthermore, F is worse off both because of a higher cost of boycott, and also because A gives in later. For similar reasons, A becomes better off. An increase in λ_A has the same effects for similar reasons: When A finds the boycott inexpensive, he gives in later, and F gives in sooner as a response. This therefore benefits A and harms F. One can easily derive comparative statics results with respect to other parameters,[16] or with respect to the

[15] The proof that both A and F must play such strategies is much more complex, and is presented in full in Appendix B.

[16] For example, a higher c makes F less willing to concede, which in turn forces A to give up at a higher rate. Because of this, a higher c makes a boycott less likely to be successful; moreover, it makes A unambiguously worse off, whereas the direct negative effect on F is partly offset by the lower willingness of A to sustain a boycott. For similar reasons, a higher l makes A more committed to continuing the boycott, and thus boycotts are more likely to be successful which makes F is worse off, however, the effect on A is ambiguous, because its direct negative effect is offset by a less resolute F. A higher b leads to more successful boycotts, and makes A better off and F worse off.

duration of the boycott.[17]

In Phase 0, before the boycott has started, the players anticipate the equilibrium play in Phase 1. The players take their expected payoffs for Phase 1 (u_1 and v_1), and there is no asymmetric information, since the cost of the boycott is not known before it starts.

Just like in the game between F and R only, the stationary equilibrium for Phase 0 is unlikely to be in pure strategies: If F were to self-regulate immediately, A would never start the boycott, but then F would not self-regulate, contradicting the assertion. If we instead asserted that F would never self-regulate, then A would find it necessary to start a boycott to get u_1 rather than nothing, as long as $u_1 > 0$. However, if a boycott were eminent, F would prefer to act, provided that its payoff from boycott is sufficiently low, i.e., $v_1 - \frac{h}{r} < -\frac{c}{r}$. In what follows, we assume that this inequality holds, to simplify exposition and limit attention to the most interesting and reasonable cases.[18] Thus, if $u_1 > 0$, the equilibrium must be in mixed strategies.

The equilibrium rates, ϕ_0 and α, are defined by two indifference conditions. The activist is indifferent between starting and not starting a boycott if and only if $u_1 = \frac{\phi_0}{\phi_0 + r}\frac{b}{r}$, while the firm is indifferent between self-regulate and not if and only if $-\frac{c}{r} = \frac{\alpha}{\alpha + r}\left(v_1 - \frac{h}{r}\right)$. This gives us the following result for all generic cases.[19]

Proposition 3 *There is a unique equilibrium in the pre-boycott game between A and F.*

(i) If $u_1 > 0$, the equilibrium is in mixed strategies and given by the Poisson rates:

$$\begin{cases} \phi_0 = r^2 \frac{u_1}{b - r u_1}; \\ \alpha = \frac{c}{-\left(v_1 - \frac{h}{r}\right) - \frac{c}{r}}. \end{cases}$$

If λ_F decreases, or λ_A increases, the firm self-regulates at a faster rate and the boycott starts at a slower rate.

(ii) If $u_1 < 0$, then $\phi_0 = \alpha = 0$.

[17] Reducing the costs of the boycott by increasing either λ_F or λ_A leads to longer boycott, provided that boycotts are not too expensive (i.e., if $\lambda_A > \frac{b+l}{l^2 r}$ and $\lambda_F > \frac{1}{rc}$). The possible ambiguity in the effect of λ_F or λ_A is interesting. For example, an increase in λ_F may be thought of as having two effects on delay. The direct effect makes the boycott cheaper for F, thus making it less willing to give in and thereby prolonging the boycott. On the other hand, this makes A more willing to give in (the indirect effect), and this makes the boycott shorter. On balance, the direct effect tends to dominate, except for the case where F is highly unlikely to give in, so the duration of the boycott mainly depends on A.

[18] If this condition does *not* hold, even an eminent boycott would not make the firm self-regulate beforehand, because it values the delay that the boycott creates. The equilibrium would then require $\phi_0 = 0$, while $\alpha = \infty$ if $u_1 > 0$ and $\alpha = 0$ otherwise. Note that the condition $v_1 - h/r < -c/r$ is automatically satisfied if $h \geq c$, i.e., if the firm values reputation a lot and self-regulation is not too costly (note that v_1 is always negative); this is arguably the most interesting case if we are to study firms vulnerable to private politics.

[19] For simplicity, we ignore the nongeneric (borderline) cases, since they follow straightforwardly and intuitively from the propositions. For example, if $u_1 = 0$, then (ϕ_0, α) characterizes an equilibrium if and only if both $\phi_0 = 0$ and $\alpha \in \left[0, \frac{c}{-(v_1 - h/r) - c/r}\right]$.

The comparative static is interesting. A larger λ_A or a lower λ_F increases u_1, which makes it more likely that the equilibrium is of type (i). As $u_1 > 0$ increases further, A becomes more tempted to start the boycott, so the firm becomes more likely to self-regulate. Thus, ϕ_0 must increase if λ_A becomes larger or λ_F becomes smaller.[20]

The payoffs for Phase 0 will be a function of the payoffs for Phase 1, described by Proposition 2(ii). If $u_1 < 0$, A prefers to not start the costly boycott, so both players get zero. If $u_1 > 0$, A is just willing to start the boycott, so $u_0 = u_1$, while F is indifferent between self-regulating and not, so $v_0 = -\frac{c}{r}$. If λ_F decreases or λ_A decreases, A benefits more from the boycott (u_1 increases) and becomes more tempted to initiate it. As a response, the firm will be more likely to self-regulate, and A's expected payoff increases in Phase 0 as well. The same changes in parameters can make the firm worse off, but only at the point where u_1 switches sign from negative to positive, since then the firm will find it necessary to self-regulate at some rate.

The benefits of private politics are the following. If the firm could not self-regulate, perhaps because such an action could not be monitored or verified, nothing would happen. The same would hold if activism were impossible. Thus, the activist benefits from the fact that private politics is possible (its payoff is $\max(u_1, 0) \geq 0$), while the firm is harmed by this possibility. On the aggregate, the possibility of private politics leads to a more efficient outcome if $b > c$, but only after a delay and potentially after a costly boycott. Self-regulation is entirely driven by the possibility to boycott, while the activist is motivated by the firm's possibility to self-regulate. In this sense, self-regulation and activism are *strategic complements.*

As a final remark, note that there is a strong similarity between the situation in Phase 0 and the situation with only F and R, analyzed in Section 3.1. In both situations, F prefers the status quo, but F as well as F's opponent prefer that F acts before the other does. In fact, the two subgames are equivalent, except that the exact payoffs at the terminal nodes may differ. Therefore, if it turned out that $s = b$ and $s - q = u_1$, the firm would have to self-regulate with the same probability in the equilibria of the two subgames. If $c - k$ happened to equal v_1, we would have $\alpha = \gamma$. Thus, the qualitative difference between the two games is that when R acts, the game ends, while when A acts, we continue to Phase 1.

[20] In addition, note that a larger λ_A or a lower λ_F leads to a smaller v_1, which reduces α. The intuition is that if the firm fears the boycott because it is costly for the firm, but not for the activist, then the firm is willing to self-regulate in Phase 0 even when the likelihood for a boycott is small.

3.3 Public Regulation Meets Private Politics

Here, we finally consider the situation where A, F, and R are all present. The previous subsections are helpful as stepping stones: In Phase 2, once the boycott has ended, A is no longer capable of taking any action. The game is then between F and R only, and the outcome is exactly as described by Proposition 1. Thus, in Phase 2, there is a unique stationary equilibrium characterized by

$$\phi_2^* = r\frac{s-q}{q} \text{ and } \gamma_2^* = r\frac{c}{k},$$

where the asterisk in superscript is used throughout to refer to the game with all three players. The payoffs in Phase 2 are as described in Section 3.1: R gets $w_2^* = w = \frac{s-q}{r}$, F gets $v_2^* = v = -\frac{c}{r}$, and A gets $u_2^* = u = \frac{\frac{s-q}{q}+\frac{c}{k}}{\frac{s-q}{q}+\frac{c}{k}+1}\frac{b}{r}$. This outcome is anticipated during the boycott, where, if A gives in, he receives a payoff equal to $u_2^* - \frac{l}{r}$, while F gets $-\frac{c}{r}$, since self-regulation is a best response in Phase 2 for F. If F gives in during the boycott, F receives the payoff $-\frac{c}{r}$, while A gets $\frac{b}{r}$. To stay focused on the interesting case, we henceforth assume that $u_2^* < \frac{l}{r}$.[21]

Going back to Phase 1, we notice that there are two different scenarios. In one scenario, the boycott itself motivates F to regulate early, so that $\phi_1^* > \phi_2^*$ even without R intervening; in this case, R indeed prefers to wait and remain passive. In this situation, F self-regulates because the boycott becomes increasingly costly and because F is learning over time that A finds the boycott inexpensive. Naturally, this situation is reasonable if F's expected cost is large, i.e., if λ_F is smaller than some threshold, $\overline{\lambda}_F$. The smaller is $\lambda_F < \overline{\lambda}_F$, the faster the firm self-regulates even if R stays passive during boycotts. It is then beneficial for A to wait longer before ending the boycott, so ρ^* is lower. Similarly, if λ_A increases, A finds the boycott less expensive, he gives in later, and ρ^* declines.[22] Note the similarity between this situation and the boycott when R were not present, as described by Proposition 3.

The other scenario bears similarity to Proposition 1, and it arises when and λ_F is so large (namely, $\lambda_F > \overline{\lambda}_F$) that the F's cost of the boycott is sufficiently small, and the boycott on its own can only motivate F to self-regulate at rate $\phi_1^* < \phi_2^*$. This rate is insufficient to make R stay passive and wait, and R would instead prefere to regulate. Therefore, for $\lambda_F > \overline{\lambda}_F$, R must intervene in equilibrium. At the same time, R cannot act with a very large probability, since then F's best response would be to self-regulate immediately, and that would have induced R to wait. Thus, just like in the game between F and R only, R must play a mixed strategy by imposing regulation at

[21] The main insights continue to hold if $u_2^* > \frac{l}{r}$, but then there exist parameter values at which A would give in immediately, because his payoff from the post-boycott game is high enough. Notice that the assumption $u_2^* < \frac{l}{r}$ means that the reputational loss of A if it gives in is nontrivial.

[22] However, F does not respond to the decline ρ^*, because at the exact time when F gives in, F is indifferent between whether or not to end, so the rate ρ^* does not influence F's decision.

some Poisson rate, and the higher this rate is, the more F is willing to self-regulate. Furthermore, R is willing to randomize only if $\phi_1^* = \phi_2^*$. Thus, if λ_F increases, R must act at a higher rate to motivate F to self-regulate sufficiently soon. Since R acts at a larger rate when $\lambda_F > \overline{\lambda}_F$ increases, while F continues to self-regulate at the constant rate $\phi_1^* = \phi_2^*$, A benefits more from continuing the campaign, and ρ^* must decrease. In other words, u_1^* is U-shaped while ρ^* is hump-shaped in λ_F, so the effect of a larger λ_F on A's strategy and payoff is reversed when R becomes active in Phase 1.

Proposition 4 *There is a unique equilibrium in the boycott game with all three players.*

(i) If $\lambda_F < \overline{\lambda}_F \equiv \frac{q}{cr(s-q)}$, *then*

$$\begin{aligned}\phi_1 &= \frac{1}{c\lambda_F} > \phi_2^*,\\ \rho^* &= \frac{1}{\lambda_A\left(\frac{b+l-ru_2^*}{rc\lambda_F} + l - ru_2^*\right)}, \text{ and}\\ \gamma_1^* &= 0.\end{aligned}$$

A smaller λ_F *increases* ϕ_1^* *and decreases* ρ^**, making the boycott more likely to succeed. A larger* λ_A *reduces* ρ^* *and makes the boycott more likely to succeed.*

(ii) If $\lambda_F > \overline{\lambda}_F$, *then*

$$\begin{aligned}\phi_1^* &= \phi_2^* = \frac{r(s-q)}{q},\\ \rho^* &= \frac{1}{\lambda_A\left(\left(\frac{s-q}{q} + \frac{c}{k} - \frac{q}{rk(s-q)\lambda_F}\right)(b+l-ru_2^*) + l - u_2^*\right)}, \text{ and}\\ \gamma_1^* &= \frac{r}{k}\left(c - \frac{q}{r(s-q)\lambda_F}\right).\end{aligned}$$

A smaller λ_F *decreases* γ_1^* *and increases* ρ^**, making the boycotts less likely to succeed. A larger* λ_A *preserves* γ_1^* *and* ϕ_1^**, but decreases* ρ^**, making boycotts more likely to succeed.*

(iii) If λ_A *increases, then both A's expected payoff* u_1^* *and F's expected payoff* v_1^* *increase. A smaller* λ_F *makes A better off (*u_1^* *increases) if* $\lambda_F < \overline{\lambda}_F$*, but worse off if* $\lambda_F > \overline{\lambda}_F$*. Furthermore, there exists a threshold* $\widehat{\lambda}_F < \overline{\lambda}_F$*, decreasing in* λ_A*, such that* $u_1^* \geq u_2^*$ *if and only if* $\lambda_F \leq \widehat{\lambda}_F$*.*

The comparative static of λ_F and λ_A is discussed earlier. Figure 1 depicts the three equilibrium rates as a function of λ_F, and A's expected payoff u_1^* is also illustrated, following part (iii) of the proposition. Since the proposition states explicit equations for all equilibrium rates, it is easy for the interested reader to derive testable empirical predictions also of the other parameters of the

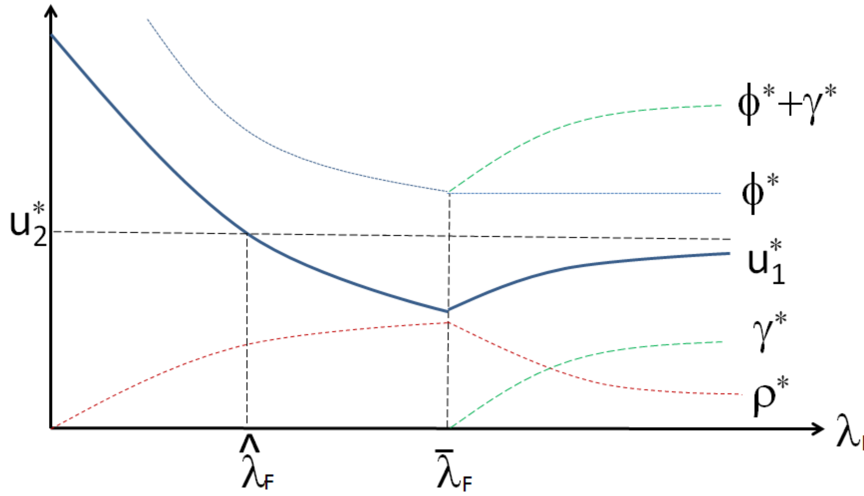

Figure 1: *The firms' expected ability to deal with the boycott is important for the equilibrium Poisson rates.*

model, and also for other aspects of the equilibrium, such as for the likely outcome or the expected boycott duration.[23]

In Phase 0, the players anticipate the effects of starting a boycott, as described in Proposition 4. Similarly to Phase 1, the equilibrium in Phase 0 may be with or without R as an active player. In particular, if the mere threat of the boycott is sufficient to motivate the firm to self-regulate at a fast rate, the regulator strictly prefers to wait. In this case, the equilibrium is just as described by Proposition 2 (except that F's and A's continuation payoffs are u_1^* and v_1^* instead of u_1 and v_1). Since u_1^* is at the largest when $\lambda_F \leq \widehat{\lambda}_F$ is very small, this type of equilibrium is likely to exist when λ_F is small, i.e., when the boycott is expected to be costly to the firm.

However, if $\lambda_F \leq \widehat{\lambda}_F$ increases, u_1^* decreases and F can reduce ϕ_0^* while still ensuring that A is willing to wait with the boycott. For a sufficiently small u_1^*, referred to as some threshold $\underline{u} \in [0, u_2^*)$, ϕ_0^* is reduced to the point where the regulator is not willing to wait. At that point, R must regulate at a positive Poisson rate for F to be willing to self-regulate so fast that R is willing to wait. Thus, for $u_1^* < \underline{u}$, the likelihood of regulation increases to the poing where A strictly prefers to abstain from initiating a boycott, Thus, for $u_1^* < \underline{u}$, the unique equilibrium is such that only F and R actively participate, just as described in Section 3.1.

The equilibrium with only F and R playing the game exists even when u_1^* is larger than $\underline{u}$.

[23] For example, a larger c (a higher cost of self-regulation for F) reduces ϕ_1^* for $\lambda_F < \overline{\lambda}_F$ and increases γ_1^* for $\lambda_F > \overline{\lambda}_F$, thus making the boycott longer and less effective in the former case and shorter and more effective in the latter case ($\overline{\lambda}_F$ is reduced as well). This again highlights that the consequences of changes in parameters critically depends on whether R is present and active. On the other hand, an increase in b decreases ρ^*, thereby making boycotts longer and more effective in both cases. A larger λ_A reduces ρ^*, so the boycott lasts longer and succeeds with a larger probability. A larger l has the same effect, while a smaller $\lambda_F \leq \overline{\lambda}_F$ makes the boycott more likely to succeed.

Indeed, A gets $u_2^* > \underline{u}$ when A stays passive while only R and F play, so A strictly prefers to enter the game (by initiating a boycott) only when $u_1^* > u_2^*$. Thus, for $u_1^* \in [\underline{u}, u_2^*]$, both equilibria exist.[24]

Proposition 5 *There exists a threshold $\underline{u} \in [0, u_2^*)$ such that:*

(i) If $u_1^* > u_2^*$, there is a unique equilibrium, given by

$$\phi_0^* = r\frac{u_1^*}{b/r - u_1^*} > \phi_2^*,\ \alpha^* = \frac{c}{\frac{h}{r} + (-v_1^*) - \frac{c}{r}},\ \textit{and } \gamma_0^* = 0;$$

(ii) If $u_1^* < \underline{u}$, the unique equilibrium is:

$$\phi_0^* = \phi_2^*,\ \alpha^* = 0,\ \textit{and } \gamma_0^* = \gamma_2^*.$$

(iii) If $u_1^* \in [\underline{u}, u_2^*]$, both equilibria (i) and (ii) exist. In this case, A prefers the equilibrium of type (i) whereas R prefers the equilibrium of type (ii).

In case (iii), when $u_1^* \in [\underline{u}, u_2^*]$, there also exist equilibria in which both A and R are active ($\alpha^* > 0$ and $\gamma_0^* > 0$), but these equilibria are unstable.[25] In any case, when $u_1^* < u_2^*$, A would prefer to not enter the game in the first place, and leave the floor for R and F, thereby securing $u_2^* > u_1^*$. Thus, if A were able to choose whether to enter the game or not, then there will be a unique equilibrium played in case (iii) as well—which would coincide with the equilibrium in case (ii). Section 3.4 discusses this situation further.

The comparative static is as expected. Since Proposition 4 states that $u_1^* \geq u_2^*$ if and only if $\lambda_F \leq \widehat{\lambda}_F$, the equilibrium of type (i) in Proposition 5 arises only when $\lambda_F < \widehat{\lambda}_F$. When this condition holds, a smaller λ_F increases u_1^* and thus increases ϕ_0^*; a larger λ_A will have the similar effect. In addition, and just as before, the above equations imply that a rich set of testable predictions can be derived by the interested reader.

The payoffs for the players are the following. For the equilibrium of type (i), both F and A are just willing to act, so F receives $-c/r$ while A receives u_1^*. Since R strictly prefers to remain passive, $w_0^* > w_2^*$. For the equilibrium of type (ii), both F and R are willing to act, so F receives $-\frac{c}{r}$ while R receives $\frac{s-q}{r}$, and as explained earlier, A receives $u_2^* > u_1^*$. This explains why both R

[24] As in the previous subsection, we assume that the firm is harmed from the boycott and would prefer to self-regulate rather than risking an immediate boycott with probability one. This implies $\frac{h}{r} - v_1^* > \frac{c}{r}$, which is satisfied if $h > c$, similarly to Subsection 3.1.

[25] When $u_1^* \in [\underline{u}, u_2^*]$, there is an equilibrium with $\alpha^* > 0$ and $\gamma_0^* > 0$, whereas ϕ_0^* may be zero or positive. However, such equilibrium will be unstable in the following sense. Since either all players or at least A and R randomize, they are indifferent between acting and not. If one strategy is slightly perturbed, the best responses of the other players will ensure that we end up with either R or A being passive. To see this, suppose γ_0^* increases (decreases) slightly from the equilibrium rate. Then A strictly prefers not to start a boycott (to start immediately). This change reinforces R's motivation to raise (reduce) γ_0^*, making the initial equilibrium locally unstable.

and A prefer the equilibrium where the other player is active whenever both equilibria exist. Both A and R can impose a cost on F and induce it to self-regulate, but if one of A and R becomes more active, the other prefers to remain passive. In other words, public regulation and private politics, in the form of activism, are *strategic substitutes.*

The benefits of private politics are therefore different from the situation without R in the game. The possibility to self-regulate is now beneficial for the firm but harmful for the activist, since the regulator would impose regulation immediately if F could not self-regulate.[26]

3.4 Commitments, Participation, and Uniqueness

Equilibria in dynamic games are often required to be subgame perfect in order to rule out non-credible strategies. In our setting, it is reasonable to assume that players cannot commit to future strategies. This desirable modelling feature is one motivation for why we chose a dynamic model in the first place.

Nevertheless, there are two reasons to return to the assumption on commitment in this subsection. First, by discussing the equilibrium if a player could hypothetically commit, we deepen our understanding of the inefficiencies discussed above. Second, the entry of the activists in the game should ideally be endogenous (unlike the regulator, who is best thought of as a long-term player). If the activist can decide whether to enter, we show below that the equilibrium outcome is unique. This uniqueness allows us to make sharp and testable empirical predictions, which we can use when comparing the US and Europe in the next section.

Consider the boycott game between A and F first. If a single player could commit during the boycott, that player would commit to *never act*, since the best response of the opponent would then be to give in immediately. The situation is completely different in Phase 0, before the boycott, and also in the game between F and R only. In both situations, F would like to promise to self-regulate at some future time. The firm generally prefers to delay the time of self-regulation, but its promise will only defer the other player from acting if the time is sufficiently close.[27] If instead F's opponent could commit, the opponent would commit to act soon, so that the firm would find it optimal to self-regulate even sooner. Any of these commitments would generate a Pareto improvement for the

[26] The possibility to *boycott* is irrelevant for the firm as long as the regulator is present, but the regulator may strictly benefit from activism, since then equilibrium (i) may be played instead of equilibrium (ii) in Proposition 5.

[27] The firm would like to promise self-regulation at some future date t, hoping that this will discourage A from initiating a boycott, or R from imposing public regulation. However, in the game between F and R, R is willing to wait only if $se^{-rt} \geq s - q$, which implies $t \leq -\ln(1 - s/q)/r$, so F's optimal promise would be to self-regulate at $t = -\ln(1 - s/q)/r$. In the game between F and A, A is willing to wait if and only if $be^{-rt} \geq u_1^*$, so $t \leq -\ln(u_1^*/b)/r$, implying that F's optimal promise would be to self-regulate at $t = -\ln(u_1^*/b)/r$. However, when these dates arrive, F would prefer to postpone self-regulation if it could.

two players, and the final outcome would be efficient (i.e., self-regulation).

The most interesting situation arises when all three players may be active, i.e., when both equilibria (i) and (ii) in Proposition 5 exist. In this case, A and R can be interpreted as two different principals who both would like to regulate the agent (i.e., the firm). Acting is costly, however, so each of A and R would prefer to commit to stay passive, since then the other player would pay the cost instead.

It seems unrealistic for the regulator to commit to be passive, given the number of regulatory tasks it has to deal with.[28] Activists, however, are making deliberate choices on whether to get organized or to reveal themselves to the firm and the regulator; even famous activists like Greenpeace have limited resources and may credibly signal to be busy with something else. Thus, we find it realistic to extend our framework by allowing A to decide whether to enter the game or not at the beginning.

In the game between A and F only, A is at least weakly better off by entering the game (strictly if $u_1 > 0$), since A captures the entire benefit of private politics when R is absent. With R present in the game, the activist receives the payoff u_2^* when leaving the scene to F and R. Anticipating this, A only wants to enter if $u_1^* > u_2^*$, which is true if and only if $\lambda_F < \widehat{\lambda}_F$, leading to equilibrium (i) in Proposition 5. Thus, whenever both equilibria exist, as in part (iii) of the proposition, A will not enter. This natural refinement leads to a unique equilibrium outcome in Phase 0 as well. Based on this refinement, we can summarize some of our most important results in the following way:

Corollary 1 *Suppose A can decide whether to enter the game at the beginning.*

(i) A enters only when $\lambda_F < \widehat{\lambda}_F$. In this case, F self-regulates at a fast rate ($\phi_1^* > \phi_0^* \geq \phi_2^* + \gamma_2^* > \phi_2^*$) while R stays passive until the boycott fails ($\gamma_0^* = \gamma_1^* = 0 < \gamma_2^*$). A decrease in λ_F or an increase in λ_A makes F self-regulate sooner (ϕ_1^* and ϕ_0^* increase), and leads to a decrease in ρ^*, thus making a boycott more likely to succeed. Furthermore, an increase in λ_A increases α^*, thus making the boycott more likely to begin with.

(ii) If $\lambda_F > \widehat{\lambda}_F$, A does not enter and the equilibrium is as in Proposition 1.

(iii) Since $\widehat{\lambda}_F$ increases in λ_A, there exists a threshold $\widehat{\lambda}_A(\lambda_F)$ such that A enters and the equilibrium is as described by Proposition 5(i) if and only if $\lambda_A > \widehat{\lambda}_A(\lambda_F)$.

As stated by the last part of the corollary, the comparative static can be expressed in terms of λ_A instead of λ_F: fixing λ_F, the equilibrium is more likely to be of type (i) if λ_A is large.

[28] In developing countries, however, the state capacities of governments are built gradually. In that context, interestingly, the government may benefit from not building its state capacity if there exist NGOs (perhaps international) that otherwise will provide public goods and pressure firms to self-regulate.

4 Application: Europe vs. the United States

Our theoretical framework provides a large number of empirically testable predictions. These predictions are important and of political relevance for how to understand governmental regulation as well as private politics. Thus, the results deserve a thorough and serious empirical investigation. Such an investigation requires the collection of new data sets that must be analyzed in detail.

Undertaking a serious test is certainly beyond the scope of this paper. However, to get some idea of what empirical regularities the theory may shed light on, we will here discuss a number of differences between the US and Europe and how our theory can explain the puzzles.

Let us start with the two stable equilibria in Phase 0, Section 3.3. In equilibrium (i) of Proposition 5, the activist was likely to initiate campaigns while the regulator was passive; in equilibrium (ii), the activist was passive but the regulator active. The activist's and the regulator's actions are strategic substitutes, and the presence of an active regulator, for example, discourages the activist from initiating costly campaigns. The two equilibria may coexist even if we fix a set of parameters. When they do, the likelihood of self-regulation has to be higher in equilibrium (i), where the activist is active, than in the equilibrium (ii), in which the regulator is active (otherwise the regulator would not be willing to remain passive in equilibrium (i)).

These characteristics describing regulatory environments fit well when we contrast the US vs. Europe: The US is more similar to equilibrium (i) on all of the following three dimensions than are most of the countries in Europe, as we will now argue.

Regulation: Matten and Moon (2008:5) write that "The key distinguishing feature of American and European political systems is the power of the state. This has tended to be greater in Europe than in the USA." Such characterizations are common in the literature.[29] The set of official and public regulations is clearly more elaborate in Europe, for example when it comes to the labor market, carbon dioxide emission standards, or GMO.[30]

Self-regulation: US companies more frequently participate in voluntary codes of conducts than their European counterparts, and they spend more than ten times as much as UK companies on "corporate community contributions."[31]

[29] Similarly, Vogel (2003:3) states that "EU regulations are more stringent, innovative and comprehensive than those adopted by the US." Matten and Moon (2008:5) also suggest that "European governments have been generally more engaged in economic and social activity" than that in the US.

[30] Davison (2010:94) compares regulation of GMOs (genetically modified organisms) between the US and Europe and notes that "the EU has the probably strictest regulations in the world for the presence of GMOs in food and feed", and: "in contrast to the EU, the USA has no GMO thresholds or obligatory GMO labeling" (p. 96). Löfstedt and Vogel (2001) present several other examples were US have taken less action with respect to potential regulation issues, e.g., plasticizers in toys and mobile phones for children. See also Belot (2007) on labor market regulations or Lynch and Vogel (2000) for more on GMO.

[31] Matten and Moon (2008:1) summarize the literature and find that voluntary codes of conducts is more common

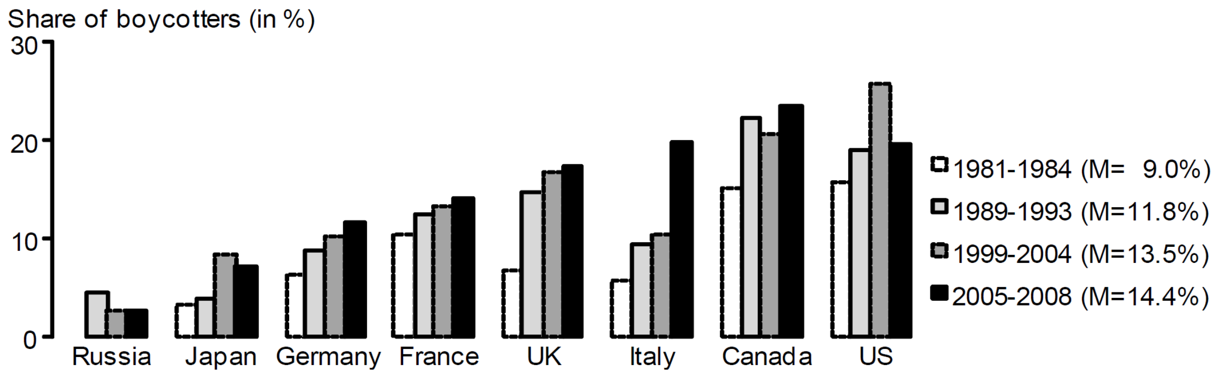

Figure 2: *Share of consumers that have already boycotted (The World Values Survey as illustrated in Hoffmann, 2014).*

Boycotts are described as an "American political tradition" and "American custom" by Glickman (2009), who provides a detailed historical overview of how boycotts have developed over time in the US. The World Values Survey provides more precise data on the fractions of consumers that have participated in boycotts. This fraction is larger in the US than in European countries (except for Sweden), according to the tables of Hoffmann (2014:149); see Figure 2 for a subset of the countries.

These large differences between the US and Europe over the several dimensions may appear puzzling, particularly because it is hard to point to a few fundamental aspects that may drive all the differences. Nevertheless, the differences can be rationalized by the theoretical predictions of our model described above. Whether we look at public regulation, activism or self-regulation by firms, the US environment is consistent with equilibrium (i) in Proposition 5, Section 3.3, while Europe is consistent with equilibrium (ii). The substitutability between activism and public regulation has also be recognized by observers: "In response to particular stakeholder pressure [US companies] assumed the explicit responsibility which most of their European counterparts left to regulators."[32]

Most importantly, our theory can also shed light on *why* the US and Europe may have ended up with different regulatory environments. Even though Europe is now a single market, thanks to the development of the European Union, trade on the continent has historically been hindered by

in the US than in Europe, and that "while 53% of US companies mention CSR explicitly on their websites only 29% of French and 25% of Dutch companies do the same." Kolk (2005) identifies a total of 15 corporate codes globally, of which only two were European (both by the same company Nestlé) while the remaining 13 codes were issued and adopted by exclusively US-American corporations. Bennett (1998) documents that corporate donations are smaller in Europe compared to the US.

[32] Matten and Moon (2008:13). The full quote is: "The US Food and Drug Administration and the Department of Agriculture operate a 'laisser-faire' approach releasing 58 GMOs until 2002 in which time the EU Commission legalized just 18... However, in response to substantial consumer activism some major US food companies (e.g. McDonalds, Gerber, McCain) publicly renounced ingredients made from genetically altered seeds. In response to particular stakeholder pressure they assumed the explicit responsibility which most of their European counterparts left to regulators."

borders and barriers to a much larger extent than the market in the US. The larger market in the US implied that local monopolists had a smaller chance to abuse market power, and competition was thus fiercer.[33] As a result, competition tends to be larger in the US than in most European countries. The Global Competitiveness Index (2016-17) ranks the US third in the world, only beaten by Switzerland and Singapore. All other European countries lag behind. Furthermore, the World Economic Forum ranks the US sixth in "intensity of local competition," and only the UK and Malta are ranked about the US in Europe.

When competition is tough, the cost of being singled out in an activist campaign is larger, and the cost of running the boycott is smaller (Garrett, 1987; Smith, 1990). Thus, it is reasonable that the firms' expected cost of boycotts is higher in the USA (implying a smaller λ_F). In addition, since competition is tougher and close substitutes are readily available, the activists' cost of running the boycott is probably smaller in the US (implying a larger λ_A).

The consequence of a smaller λ_F or a larger λ_A are spelled out in the propositions above, and they are summarized in Corollary 1 in Section 3.4: If λ_F decreases, the rate of self-regulation increases in Phase 1 as well as in Phase 0. In Phase 1, the rate of governmental regulation (weakly) decreases if λ_F falls, and the activists become less likely to give in by letting the boycott end in failure. The activists also become less likely to give in if λ_A increases. For all these reasons, if either λ_F decreases or λ_A increases, a boycott becomes more likely to be successful and the activist's payoff increases. The larger payoff raises the likelihood that the activists decide to enter the game in the first place. Thus, we are more likely to have a Phase 0 equilibrium of type (i), where A is active while R is passive in the case where λ_F is small and λ_A is large. As argued above, such parameter values fit better with the US than with Europe, and so do the equilibrium outcomes, namely likelihood of boycotts and the rates of regulation and self-regulation. The theory is thus consistent with the documented differences in regulatory environments between the US and Europe.

Over time, it is likely that λ_F decreases and λ_A increases in Europe as well as in the US. Indeed, trade liberalization reduces barriers to trade and lower transportation costs intensify competition, while the rise of the social media makes it easier for activists to organize campaigns. More intense competition raises the cost of being singled out in a campaign, and it reduces the cost of boycotting one brand and switching to a substitute. With these developments, our theory predicts that private politics should become more important over time, at the expense of public regulation: firms should become more likely to self-regulate, activists will be less willing to give in during boycotts, and

[33] In addition to geography, another reason for the difference in competitiveness may be due to stricter merger policies in the US. For example, Bergman et al. (2010:4) write that "the US actively enforced against mergers. . . while the EU has rarely brought action against oligopolies."

overall the societies should more frequently find themselves in equilibria where activists are more active, while regulators are less so.

We are the first to admit that the reasoning in this section is not perfectly conclusive and partly speculative. A careful test of our theory requires the collection of new data and statistical analysis. Doing this is beyond the scope of the present paper, although we hope that our theory may provide the foundation for a promising agenda of future research. Our brief look at the anecdotal evidence suggests that a thorough empirical investigation might be academically rewarding as well as politically important.

5 Concluding Remarks

Over the last decades, governments have lost their monopoly on regulation and faced competition from both sides of the market. Thanks to reduced transaction costs, consumers are increasingly able to influence or boycott firms, and firms are increasingly self-regulating and investing in CSR. The economic and social consequences of the rise of private politics are important but not well understood. According to Doh and Guay (2007:130), "*governments may choose to let NGOs and business resolve controversial issues voluntarily... This may not be optimal, since companies may choose to resist NGO pressure knowing that government regulation will not be forthcoming.*"

This paper started by raising four fundamental questions that the previous literature had not addressed in a satisfactory way. We asked *when* private politics is efficient, and described when the better terminal outcome comes at the cost of delay. We asked *who* benefits and who loses from private politics, and showed that the answer critically depends on whether the regulator is present. Regarding the interaction with *public* regulation, we emphasized that activism and public politics are strategic substitutes, while activism and corporate self-regulation are strategic complements. We also asked *how* one may explain the raise of private politics over time, and why there are large differences between the US and Europe, and concluded that our theoretical predictions can shed light on these puzzles.

The four questions are important. If the total regulatory pressure is coming from market participants as well as the government, we need to understand them all, as well as how they interact, to determine the appropriate level of governmental regulation. Legislators need to know whether private politics is sufficient to achieve efficiency, or whether private politics instead should be regulated, perhaps restricted or, to the contrary, subsidized. One also needs to know how private politics reacts to the introduction of governmental regulation. If we understand the difference between various regulatory environments, we can also understand how and why the business environment differs between, e.g., the US and Europe, thus enabling us to compare concessions when different countries negotiate trade or environmental agreements.

This paper is only one step toward a better understanding of private and public politics. A lot remains to be done. On the empirical side, the theoretical predictions ought to be taken to the data in a careful way; the anecdotal evidence discussed above is just indicative that such an effort may be fruitful. On the theoretical side, future research should relax the assumptions we have imposed. Several generalizations are feasible, we believe, since our model is relatively simple and tractable.

Regarding regulation and self-regulation, our analysis was simplified by assuming that regulation—whether private of public—was binary: the firm is either regulated, or it is not. This assumption fits well with some of the examples mentioned in the Introduction. In other cases, however, it may be possible for the firm to make partial concessions. There are multiple ways of generalizing our model in this direction. One possibility is that all parameters in the model, such as the benefits as well as the costs, are proportional to the extent of self-regulation. For example, the firm may produce several products (or in several geographical locations), and self-regulation of (or in) one of them may be done independently of the others. If the boycott is restricted to the product or the location in question, then all our results continue to hold without any change: if the firm self-regulates a certain fraction of its facilities, regulation of the remaining fraction becomes the subject of a new but similar subgame.[34]

Regarding activism, we have abstracted from several types of interactions between the activist and the regulator. Our present theory treats these players as two different principals that both seek to influence the same agent (i.e., the firm). In reality, the activist may consider an alternative strategy of influencing the regulator. For example, the activist may initiate a costly campaign against the government, hoping that such a pressure will prompt the government to act. Such a campaign may be analogous to a boycott; the activist's preferred strategy will then depend on the relative cost and effectiveness. Alternatively, the activist might attempt to provide information to the regulator, hoping that this will raise the likelihood for regulation. In both cases, the activist's two strategies may be strategic substitutes. While this paper has focused exclusively on the case where the activist opts to influence the firm, future research should study regulatory campaigns and the conditions under which they substitute or complement corporate boycotts.

[34] If there are decreasing returns to scale when it comes to self-regulation, however, the analysis would be less straightforward. In this case, it is likely that the firm may prefer to self-regulate up to the level at which the activist and/or the regulator are sufficiently satisfied. This level is smaller if a campaign is costly for the activist, and if direct regulation is costly to the regulator. Although a careful analysis must await future research, our conjecture is that the firm will never strictly prefer to self-regulate such as to satisfy only one of the regulator or the activist, since doing so would lead to one of the two-player games we have analyzed, and in both of them self-regulation is a best response.

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Appendix A: Proofs of Main Results

Proof of Proposition 1. The equilibrium is given by rates of self-regulation and regulation ϕ and γ, respectively, that are constant over time (stationarity requirement). Suppose $\phi = \infty$; then R's best response is to never regulate ($\gamma = 0$), in which case F should not self-regulate either ($\phi = 0$). But if $\phi = 0$, then R should regulate immediately ($\gamma = \infty$), in which case F's best response is also self-regulate immediately ($\phi = \infty$), given the positive probability that of the two immediate actions (by R and F), its action would take precedence. This proves that in equilibrium, ϕ and γ must lie strictly between 0 and ∞, which implies that both F and R must be indifferent whether to act or not at any moment in time. In particular, they should be indifferent between acting immediately and not acting ever.

Consider F. If it self-regulates (at time 0), it gets payoff

$$\int_0^\infty (-c) \exp(-r\tau)\, d\tau = \frac{c}{r} \exp(-r\tau)\Big|_{\tau=0}^{\tau=\infty} = -\frac{c}{r}.$$

If it never self-regulates, its payoff equals (t is the time of government regulation, which arrives at rate γ)

$$\int_0^\infty \gamma \exp(-\gamma t) \left(\int_0^t 0\,(-r\tau)\, d\tau + \int_t^\infty (-c-k) \exp(-r\tau)\, d\tau \right) dt$$
$$= \int_0^\infty \left(\frac{-c-k}{r} \right) \gamma \exp(-(\gamma + r)\, t)\, dt = -\frac{\gamma (c+k)}{r(\gamma + r)}.$$

Thus, F is indifferent if and only if $\gamma = r\frac{c}{k}$. Similarly, R is indifferent if and only if $\frac{s-q}{r} = \frac{\phi s}{r(\phi + r)}$, i.e., $\phi = r\frac{s-q}{q}$. Therefore, these ϕ and γ constitute the unique equilibrium of the game without A.

It is straightforward to see that γ is increasing in c and decreasing in k, whereas F is increaing in s and decreasing in q. Since the probability of F self-regulating before R regulates (the probability of self-regulation) is given by

$$\int_0^\infty \gamma \exp(-\gamma t) \int_0^t \phi \exp(-\phi\tau)\, d\tau dt = \int_0^\infty \gamma \exp(-\gamma t)(1 - \exp(-\phi t))\, dt = 1 - \frac{\gamma}{\phi + \gamma} = \frac{\phi}{\phi + \gamma},$$

and the expected duration of boycott is

$$\int_0^\infty t d(1 - \exp(-\gamma t)\exp(-\phi t)) = \int_0^\infty t(\phi + \gamma)\exp(-(\phi + \gamma)\, t)\, dt = \frac{1}{\phi + \gamma},$$

the comparative statics results discussed in the text follow immediately. ■

Proof of Proposition 2. Here, we prove a weaker version of the claim: namely, if F and A play linear strategies, then these strategies satisfy the properties stated in the Proposition. The fact that F and A will, in fact, play linear strategies is proved in Appendix B.

In what follows, the time where boycott began is normalized to 0 to save on notation. Suppose that A plays linear strategy $t(\theta_A) = \frac{1}{\rho\lambda_A}\theta_A$ for some ρ. Then the share of A that have given up by time t is $\Pr(\theta_A < \rho\lambda_A t) = 1 - e^{-\rho t}$, which implies that A stops the boycott at rate equal to ρ. For F, the cost of boycott of duration τ equals $\frac{1-e^{-r\tau}-r\tau e^{-r\tau}}{\theta_F r^2}$, as follows from Lemma 1 of Appendix B. Then the expected payoff of F if it concedes at time t is given by Lemma B4:

$$V_F(t) = \int_0^t \left(-\frac{1-e^{-r\tau}-r\tau e^{-r\tau}}{\theta_F r^2}\right)\rho e^{-\rho\tau}d\tau - e^{-\rho t}\left(\frac{c}{r}e^{-rt} + \frac{1-e^{-rt}-rte^{-rt}}{\theta_F r^2}\right).$$

Differentiating with respect to t, we get, after simplification,

$$\frac{dV_F(t)}{dt} = e^{-(\rho+r)t}\left(\rho\frac{c}{r} - \left(c + \frac{t}{\theta_F}\right)\right).$$

The first-order condition is therefore satisfied at $t = c\left(1+\frac{\rho}{r}\right)\theta_F$, which is a linear strategy. Furthermore, at this point,

$$\frac{d^2V_F(t)}{dt^2} = -(\rho+r)e^{-(\rho+r)t}\left(\rho\frac{c}{r} - \left(c+\frac{t}{\theta_F}\right)\right) - e^{-(\rho+r)t}\frac{1}{\theta_F} = -e^{-(\rho+r)t}\frac{1}{\theta_F} < 0,$$

so this is a global maximum. Thus, if all types of F follow this best-response strategy, F's rate of self-regulation is given by $\phi_1 = \frac{1}{\lambda_F c\left(1+\frac{\rho}{r}\right)}$, which gives us the first equation.

Similarly, if F plays linear strategy $t(\theta_F) = \frac{1}{\phi_1\lambda_F}\theta_F$ for some ϕ_1, then A's best response should also be linear. Indeed, the expected payoff of A if it concedes at time t is given by

$$V_A(t) = \int_0^t \left(\frac{b}{r}e^{-r\tau} - \frac{1-e^{-r\tau}-r\tau e^{-r\tau}}{\theta_A r^2}\right)\phi_1 e^{-\phi_1\tau}d\tau - e^{-\phi_1 t}\left(\frac{l}{r}e^{-rt} + \frac{1-e^{-rt}-rte^{-rt}}{\theta_A r^2}\right).$$

We thus have

$$\frac{dV_A(t)}{dt} = e^{-(\phi_1+r)t}\left(\phi_1\frac{b+l}{r} - \left(-l+\frac{t}{\theta_A}\right)\right),$$

and the first-order condition is satisfied at $t = \left(l + \phi_1\frac{b+l}{r}\right)\theta_A$ (and, as before, the second-order condition is also satisfied). Therefore, A's optimal rate of self-regulation is given by $\rho = \frac{1}{\lambda_A\left(l+\phi_1\frac{b+l}{r}\right)}$, which gives the second equation.

These two equations have a unique positive solution (ϕ_1, ρ). The simplest way to see this is notice that plugging ρ from the second equation into the first results in a quadratic equation, thus there are at most two values of ρ that solve the equation, and for each of them ϕ_1 is determined uniquely. Furthermore, the equations may be rewritten as

$$\begin{aligned}\phi_1(\rho+r) &= \frac{r}{\lambda_F c};\\ \left(\phi_1 + \frac{lr}{b+l}\right)\rho &= \frac{r}{\lambda_A(b+l)}.\end{aligned}$$

The first defines a hyperbola with asymptotes given by $\phi_1 = 0$, $\rho = -r$, whereas the second defines one with asymptotes $\phi_1 = -\frac{lr}{b+l}$, $\rho = 0$. Thus, these hyperbolas have one intersection with positive (ϕ_1, ρ) and one with negative (ϕ_1, ρ), which proves uniqueness of the solution.

Consider an increase in λ_F. This does not affect the second equation, while the first hyperbola moves down and to the left. The intersection point therefore moves along the second curve up and to the left. Thus, equilibrium value of ϕ_1 decreases and equilibrium value of ρ increases. Since the probability that the boycott ends in self-regulation equals $\frac{\phi_1}{\phi_1+\rho}$, an increase in λ_F makes the boycott less likely to be successful. The comparative statics results with respect to other parameters are considered similarly.

Let us prove that an increase in λ_F makes F better off and A worse off. Suppose that $\lambda_F > \lambda_F'$, and the corresponding equilibrium rates are (ϕ_1, ρ) and $\left(\phi_1', \rho'\right)$. Since $\rho' > \rho$, Lemma B24 in Appendix B implies that F is better off if the rates are (ϕ_1, ρ') than if they are (ϕ_1, ρ), since this is true for any fixed ϕ_1. Furthermore, F is better off under $\left(\phi_1', \rho'\right)$ than under (ϕ_1, ρ'), because ϕ_1' is F's best response to A regulating at rate ρ', and ϕ_1 is not a best response to this ρ'. This proves that F is better off. To show that A is worse off under λ_F', we can proceed similarly. Indeed, A is better off under (ϕ_1, ρ') than under $\left(\phi_1', \rho'\right)$ by Lemma B24, because $\phi_1' < \phi_1$, and it is better off under (ϕ_1, ρ) than under (ϕ_1, ρ'), because ρ is a best response to ϕ_1 and ρ' is not. This proves the payoff effect of an increase in λ_F. The other comparative statics results, specifically, that an increase in λ_A makes A better off and F worse off, are proved analogously, and the explicit form of payoffs may be found using the same Lemma B24:

$$\begin{aligned} u_1^{FA} &= \frac{b}{r}\frac{\phi_1}{\phi_1+\rho+r} - \frac{l}{r}\frac{\rho}{\phi_1+\rho+r} + \frac{1-\left(1+\frac{\rho}{\phi_1+r}\right)\ln\left(1+\frac{\phi_1+r}{\rho}\right)}{(\phi_1+r)(\phi_1+\rho+r)\lambda_A}; \\ v_1^{FA} &= -\frac{c}{r}\frac{\phi_1}{\phi_1+\rho+r} + \frac{1-\left(1+\frac{\phi_1}{\rho+r}\right)\ln\left(1+\frac{\rho+r}{\phi_1}\right)}{(\rho+r)(\phi_1+\rho+r)\lambda_F}. \end{aligned}$$

Lastly, let us prove the result on duration. The expected duration of the boycott is given by $\frac{1}{\phi_1+\rho}$ (see the proof of Proposition 1 for an explicit calculation). Thus, a marginal increase in λ_F makes the boycott shorter if and only if at (ϕ_1, ρ), the slope of the curve $\rho = \frac{r}{\lambda_A(lr+(b+l)\phi_1)}$ is between -1 and 0, i.e., if $\frac{r(b+l)}{\lambda_A(lr+(b+l)\phi_1)^2} < 1$ (or, substituting ϕ_1 for ρ, if $\rho^2 < \frac{r}{\lambda_A(b+l)}$). This condition holds for all ϕ_1 if and only if it (weakly) holds at $\phi_1 = 0$, i.e., if $\lambda_A > \frac{b+l}{l^2 r}$. The result on the effect of an increase in λ_A on duration is proved similarly. This completes the proof. ■

Proof of Proposition 3. Suppose first that $u_1 > 0$. Then if F never self-regulates, A would start a boycott immediately. In this case, F would self-regulate immediately as well, because there is a positive probability that its action would take precedence. But if F self-regulates immediately, A would not start a boycott, because $u_1 < \frac{b}{r}$ (the self-regulation is delayed, there is a positive chance of having to suffer $-\frac{l}{r}$, and the expected cost of boycott is positive). In that case, F would not self-regulate. This proves that an equilibrium must take the form of rates (ϕ_0, α), with $\phi_0, \alpha \in (0, +\infty)$.

Therefore, both A and F are indifferent whether to act (start a boycott and self-regulate, respectively) immediately, or never. For A, never acting yields $\frac{\phi_0}{\phi_0+r}\frac{b}{r}$, and starting a boycott yields u_1. These are equal if and only if $\phi_0 = \frac{ru_1}{\frac{b}{r}-u_1}$. For F, never acting yields $\frac{\alpha}{\alpha+r}\left(v_1 - \frac{h}{r}\right)$, and self-regulating immediately yields $-\frac{c}{r}$; these are equal if $\alpha = \frac{c}{\frac{h}{r}-v_1-\frac{c}{r}}$. Under the assumption that $v_1 - \frac{h}{r} < -\frac{c}{r}$, the denominator is positive, and then these α and ϕ_0 yield an equilibrium.

By Proposition 2, if λ_F decreases or λ_A increases, then u_1 increases and v_1 decreases. Since ϕ_0 is increasing in u_1 and does not depend on v_1, it increases as a result. Similarly, since α is incresing in v_1, it decreases as a result of this change. This proves the comparative statics results.

Now suppose that $u_1 < 0$. Then there is no equilibrium where A starts a boycott, because if it never does so, its expected payoff is nonnegative. At the same time, if $\alpha = 0$, then F must choose $\phi_0 = 0$. It is trivial to verify that $\alpha = 0$, $\phi_0 = 0$ constitutes an equilibrium in this case.

Lastly, suppose $u_1 = 0$, then the same $\alpha = 0$, $\phi_0 = 0$ constitutes an equilibrium. Notice that if $\phi_0 > 0$ (and if it equals ∞ in particular), then A gets a positive expected payoff from not starting a boycott, thus $\alpha = 0$ in equilibrium, in which case $\phi_0 = 0$, a contradiction. If $\phi_0 = 0$, then A is indifferent between starting a boycott and not. If $\alpha = \infty$, then F would prefer to self-regulate, for finite α, not self-regulating is a best response for F if and only if $\frac{\alpha}{\alpha+r}\left(v_1 - \frac{h}{r}\right) \geq -\frac{c}{r}$, so $\alpha \leq \frac{c}{-\left(v_1 - \frac{h}{r}\right) - \frac{c}{r}}$. ■

Proof of Proposition 4. Notice first that $\gamma_1^* = \infty$ is impossible: in that case, F would self-regulate immediately, and if so, R would prefer to wait instead. Thus $\gamma_1^* \in [0, +\infty)$. Let us start by analyzing the game during-the-boycott game between F and A if R's strategy is fixed at some γ_1^*.

Let us start with the best response of F (to any strategy of A), and let us show that for F of type θ_F, conceding at time $t(\theta_F) = \max\left\{\left(c - k\frac{\gamma_1^*}{r}\right)\theta_F, 0\right\}$ is a dominant strategy. To do so, consider A that gives up at time T. Notice that if F gives up at some time τ, it gets $-\frac{c}{r}$, and if A calls the boycott off at time τ, F gets the same amount. Consequently, the payoff of F if it gives up at time t is given by

$$\begin{aligned} V_F^*(t) \quad &= \quad \int_0^{\min\{t,T\}} \left(-\frac{c+k}{r}e^{-r\tau} - \frac{1-e^{-r\tau} - r\tau e^{-r\tau}}{\theta_F r^2}\right)\gamma_1^* e^{-\gamma_1^*\tau} d\tau \\ &\quad -e^{-\gamma_1^* \min(t,T)} \left(\frac{c}{r}e^{-r\min(t,T)} + \frac{1-e^{-r\min(t,T)} - r\min(t,T)\,e^{-r\min(t,T)}}{\theta_F r^2}\right). \end{aligned}$$

Notice that the function

$$\tilde{V}_F^*(t) = \int_0^t \left(-\frac{c+k}{r}e^{-r\tau} - \frac{1-e^{-r\tau} - r\tau e^{-r\tau}}{\theta_F r^2}\right)\gamma_1^* e^{-\gamma_1^*\tau} d\tau - e^{-\gamma_1^* t}\left(\frac{c}{r}e^{-rt} + \frac{1-e^{-rt} - rte^{-rt}}{\theta_F r^2}\right)$$

is single-peaked in t and has a unique maximum achieved at $t^*(\theta_F) = \max\left\{\left(c - k\frac{\gamma_1^*}{r}\right)\theta_F, 0\right\}$ (similar to the proof of Proposition 2); indeed,

$$\frac{d\tilde{V}_F^*(t)}{dt} = e^{-(\gamma_1^*+r)t}\left(c - k\frac{\gamma_1^*}{r} - \frac{t}{\theta_F}\right),$$

which is positive for $t < t^*(\theta_F)$ and negative for $t > t^*(\theta_F)$. This implies that for $T > t^*(\theta_F)$, function $V_F^*(t)$ has a unique maximum at $t = t^*(\theta_F)$, and for $T \leq t^*(\theta_F)$, $V_F^*(t)$ is maximized at any $t \geq T$, and in particular at $t^*(\theta_F)$. This implies that playing $t(\theta_F) = t^*(\theta_F)$ is a dominant strategy for F of type θ_F.

Let us now prove that $\gamma_1^* \geq \frac{rc}{k}$ is impossible in equilibrium. Indeed, if this is not the case, then $t^*(\theta_F) = 0$ for all θ_F. However, in this case, at time 0, R is strictly better off waiting rather than regulating, in which case $\gamma_1^* \geq \frac{rc}{k}$ is impossible. This proves that only $\gamma_1^* < \frac{rc}{k}$ is possible in equilibrium, in which case $t^*(\theta_F) = \left(c - k\frac{\gamma_1^*}{r}\right)\theta_F > 0$.

Notice that for any $T < \infty$, a positive share of types of A concede at or later than T in any equilibrium. Suppose not, then there is some $T \in [0, +\infty)$ such that all types of A concede on or before T. Without loss of generality, we can assume that T is the smallest T that satisfies this property (it exists because the set of such T is closed). This means that for types of F that satisfy $\theta_F > \frac{T}{\left(c-k\frac{\gamma_1^*}{r}\right)\theta_F}$, it is not a best response to concede earlier than T, as we proved earlier, because there is a positive share of A that concede after $T-\varepsilon$ for any $\varepsilon > 0$. If so, the types of A with $\theta_A > \frac{T}{l-ru_2^*}$ would strictly prefer to concede later than T than at T or earlier, because the marginal cost of waiting an extra period, $\frac{T}{\theta_A}$, is smaller than the benefit of getting $\frac{l}{r} - u_2^*$ earlier (and, furthermore, F may concede or R may regulate, leading to an even higher payoff. This contradiction proves that a positive share of types of A concede later than T for any given $T \in [0,\infty)$. But if A plays such strategy, then for F of type θ_F, the only best response is to play $t(\theta_F) = t^*(\theta_F)$ found earlier, because it is the unique best response if A plays arbitrarily high T.

We have thus proved that in any equilibrium, F must play $t(\theta_F) = t^*(\theta_F) = \left(k\frac{\gamma_1^*}{r} + c\right)\theta_F$. Therefore, F's rate of self-regulation must equal $\phi_1^* = \frac{1}{\lambda_F\left(c-k\frac{\gamma_1^*}{r}\right)}$ in any equilibrium where R regulates at rate $\gamma_1^* < \frac{rc}{k}$. Let us now write down the payoff of A of type θ_A given these γ_1^* and ϕ_1^*; since it gets the same payoffs from self-regulation by F and from regulation by R, it equals

$$\begin{aligned} V_A^*(t) &= \int_0^t \left(\frac{b}{r}e^{-r\tau} - \frac{1-e^{-r\tau}-r\tau e^{-r\tau}}{\theta_A r^2}\right)(\phi_1^*+\gamma_1^*)\, e^{-(\phi_1^*+\gamma_1^*)\tau} d\tau \\ &\quad - e^{-(\phi_1^*+\gamma_1^*)t}\left(\left(\frac{l}{r} - u_2^*\right)e^{-rt} + \frac{1-e^{-rt}-rte^{-rt}}{\theta_A r^2}\right). \end{aligned}$$

Similarly to the proof of Proposition 2, we can show that A also follows a linear strategy $t = \left(l - ru_2^* + (\phi_1^*+\gamma_1^*)\left(\frac{b+l}{r} - u_2^*\right)\right)\theta_A$, which implies the rate with which A calls the boycott off $\rho^* = \frac{1}{\lambda_A\left(l-ru_2^*+(\phi_1^*+\gamma_1^*)\left(\frac{b+l}{r}-u_2^*\right)\right)}$. We have thus proved that if R regulates with rate γ_1^*, then F must self-regulate at rate $\phi_1^* = \frac{1}{\lambda_F\left(c-k\frac{\gamma_1^*}{r}\right)}$, and A must stop the boycott at rate $\rho^* = \frac{1}{\lambda_A\left(l-ru_2^*+\left(\frac{1}{\lambda_F\left(c-k\frac{\gamma_1^*}{r}\right)}+\gamma_1^*\right)\left(\frac{b+l}{r}-u_2^*\right)\right)}$.

Consider two possibilities. First, suppose that $\lambda_F \leq \overline{\lambda}_F = \frac{q}{cr(s-q)}$; this implies that $\frac{1}{\lambda_F c} \geq \phi_2^*$, where $\phi_2^* = r\frac{s-q}{q}$. Then for $\gamma_1^* = 0$, F self-regulates at rate $\phi_1^* \geq \phi_2^*$, and given that at rate ϕ_2^*, R is indifferent between regulating and not, then not regulating (playing $\gamma_1^* = 0$) is a best response for R. This implies that for such parameter values, $\gamma_1^* = 0$, $\phi_1^* = \frac{1}{\lambda_F c}$, $\rho^* = \frac{1}{\lambda_A\left(l-ru_2^*+\frac{b+l-ru_2^*}{\lambda_F c}\right)}$ is an equilibrium, because R plays a best response, and F and A play best responses to $\gamma_1^* = 0$ and to each other's strategies. On the other hand, there is no equilibrium with $\gamma_1^* > 0$, as in this case, $\phi_1^* > \frac{1}{\lambda_F c} \geq \phi_2^*$, and thus R is strictly better off not regulating, which contradicts $\gamma_1^* > 0$.

Second, suppose that $\lambda_F > \overline{\lambda}_F = \frac{q}{cr(s-q)}$. In this case, there is no equilibrium where $\gamma_1^* = 0$, because in this case, $\phi_1^* = \frac{1}{\lambda_F c} < \phi_2^*$, and R would be strictly better off regulating, which contradicts $\gamma_1^* = 0$. Thus, any equilibrium in this case must feature $\gamma_1^* \in \left(0, \frac{rc}{k}\right)$, so R must be indifferent between regulating and not. This is only possible if $\phi_1^* = \phi_2^*$, i.e., if $\frac{1}{\lambda_F\left(c-k\frac{\gamma_1^*}{r}\right)} = \phi_2^*$, which implies $\gamma_1^* = \frac{r}{k}\left(c - \frac{1}{\phi_2^*\lambda_F}\right) \in \left(0, \frac{rc}{k}\right)$. For this γ_1^*, $\phi_1^* = \phi_2^* = r\frac{s-q}{q}$ and $\rho^* = \frac{1}{\lambda_A\left(l-ru_2^*+\left(\frac{s-q}{q}+\frac{1}{k}\left(c-\frac{1}{r\frac{s-q}{q}\lambda_F}\right)\right)\left(b+l-ru_2^*\right)\right)}$. These values thus constitute a unique equilibrium in this case.

Let us now prove the comparative statics results. If $\lambda_F < \overline{\lambda}_F$, then an increase in λ_A reduces ρ^* and does not affect ϕ_1^*, thus making the boycott longer in expectation and more likely to succeed. An increase in λ_F decreases ϕ_1^* and increases ρ^*, making the boycott less likely to succeed. If $\lambda_F > \overline{\lambda}_F$, then an increase in λ_A reduces ρ^* and does not affect ϕ_1^* or γ_1^*, which again makes the boycott longer in expectation and more likely to succeed. At the same time, an increase in λ_F makes γ_1^* higher and ρ^* lower, which makes the boycott more likely to succeed.

Now consider the effect of parameters on the payoffs of F and A. Suppose that λ_A increases. This does not change $\overline{\lambda}_F$, and thus does not affect whether R is active during the boycott; furthermore, it does not change γ_1^* and ϕ_1^*, but only decreases ρ^*. This means that for any θ_A, the stopping time of A, $t(\theta_A)$, is higher, which increases the payoff of types of F with $t^*(\theta_F) > t(\theta_A)$ without affecting those with $t^*(\theta_F) < t(\theta_A)$; thus, the expected payoff of F is higher for any realization of θ_A. Consequently, a higher λ_A implies that the expected payoff of F, v_1^* is higher. On the other hand, the effect of an increase in λ_A on u_1^* is also positive: indeed, γ_1^* and ϕ_1^* do not change, and if we take any θ_A, A of type θ_A would be strictly better off from higher λ_A even if he did not change his equilibrium behavior (because of lower cost of boycott), and he may do even better by reoptimizing. Thus, u_1^* also increases in λ_A.

Suppose that λ_F increases. This makes $\phi_1^* + \gamma_1^*$ lower as long as $\lambda_F < \overline{\lambda}_F$, but makes it higher if $\lambda_F > \overline{\lambda}_F$. Similarly to Lemma B23 we can show that $V_A^*(t)$ is increasing in $\phi_1^* + \gamma_1^*$ for any strategy of A of type θ_A, and similarly to Lemma B24 we can deduce from that that u_1^* is increasing in $\phi_1^* + \gamma_1^*$. Therefore, u_1^* is decreasing in λ_F for $\lambda_F < \overline{\lambda}_F$ and increases if $\lambda_F > \overline{\lambda}_F$.

Lastly, let us describe the set of parameter values such that $u_1^* \geq u_2^*$. We have proved that $u_1^*(\lambda_F)$ as a function of λ_F is U-shaped with a minimum achieved at $\lambda_F = \overline{\lambda}_F$. We prove the following two results: that $\lim_{\lambda_F \to 0} u_1^*(\lambda_F) > u_2^*$ and that $\lim_{\lambda_F \to \infty} u_1^*(\lambda_F) \leq u_2^*$. The latter would imply that $u_1^*\left(\overline{\lambda}_F\right) < u_2^*$, and thus $u_1^*(\lambda_F) < u_2^*$ for any $\lambda_F \geq \overline{\lambda}_F$. These two results (together with continuity) would imply that there is $\widehat{\lambda}_F \in \left(0, \overline{\lambda}_F\right)$ such that $u_1^*\left(\widehat{\lambda}_F\right) = u_2^*$, and $u_1^*(\lambda_F) > u_2^*$ for $\lambda_F < \widehat{\lambda}_F$ and $u_1^*(\lambda_F) < u_2^*$ for $\lambda_F \in \left(\widehat{\lambda}_F, \overline{\lambda}_F\right)$, which together would yield the desired property of $\widehat{\lambda}_F$. Thus, let us show that $\lim_{\lambda_F \to 0} u_1^*(\lambda_F) > u_2^*$. Indeed, as for $\lambda_F < \overline{\lambda}_F$, $\gamma_1^* = 0$, and as $\lambda_F \to 0$, $\phi_1^* \to \infty$ and $\rho^* \to 0$. This implies that the boycott ends with F self-regulating with arbitrarily high probability and arbitrarily fast, while the rate of A calling the boycott off remains bounded away from ∞.

Thus, $\lim_{\lambda_F \to 0} u_1^*(\lambda_F) = \frac{b}{r} > \frac{\frac{s-q}{q}+\frac{c}{k}}{\frac{s-q}{q}+\frac{c}{k}+1}\frac{b}{r} = u_2^*$. To prove that $\lim_{\lambda_F \to \infty} u_1^*(\lambda_F) \leq u_2^*$, notice that for $\lambda_F > \overline{\lambda}_F$, $\phi_1^* = \phi_2^*$, and as $\lambda_F \to \infty$, $\gamma_1^* \to \frac{rc}{k} = \gamma_2^*$, and $\rho^* \to \frac{1}{\lambda_A\left(l - ru_2^* + \left(\frac{s-q}{q}+\frac{c}{k}\right)\left(b+l-ru_2^*\right)\right)}$. This implies that the total rate of regulation $\phi_1^* + \gamma_1^*$ tends to $\phi_2^* + \gamma_2^*$, but since ρ^* is bounded away from both 0 and ∞, $\lim_{\lambda_F \to \infty} u_1^*(\lambda_F) < u_2^*$ because of positive expected cost of boycott, as well as positive probability of having to face the loss $-\frac{l}{r}$.

As we just showed, these results imply existence of cutoff $\widehat{\lambda}_F$. It remains to prove that this cutoff is increasing in λ_A. This immediately follows from that $u_1^* - u_2^*$ is increasing in λ_A and locally decreasing in λ_F (because u_2^* does not depend on either λ_A or λ_F), which completes the proof. ■

Proof of Proposition 5. Notice that R's payoff during the boycott is given by $w_1^* = \frac{s-q}{r}$ if $\lambda_F \geq \overline{\lambda}_F$ and by $w_1^* = \frac{\phi_1^*}{\phi_1^*+\rho^*+r}\frac{s}{r} + \frac{\rho^*}{\phi_1^*+\rho^*+r}\frac{s-q}{r}$ if $\lambda_F < \overline{\lambda}_F$, in which case $w_1^* > \frac{s-q}{r}$ as follows from Proposition 4. We define $\underline{u}$ as follows:

$$\underline{u} = \begin{cases} \frac{s-q}{s}\frac{b}{r} & \text{if } w_1^* = \frac{1}{r}(s-q); \\ \frac{s-q\left(1+\frac{1}{\frac{s\left(-\left(v_1^*-\frac{h}{r}\right)-\frac{c}{r}\right)}{\left(w_1^*-\frac{s-q}{r}\right)c}-1}\right)}{s}\frac{b}{r} & \text{if } w_1^* \in \left(\frac{1}{r}(s-q), \left(\frac{1}{r}+\frac{-\left(v_1^*-\frac{h}{r}\right)-\frac{c}{r}}{c}\right)(s-q)\right); \\ 0 & \text{if } w_1^* > \left(\frac{1}{r}+\frac{-\left(v_1^*-\frac{h}{r}\right)-\frac{c}{r}}{c}\right)(s-q); \end{cases}$$

then it satisfies $0 \leq \underline{u} \leq \frac{s-q}{s}\frac{b}{r} = \frac{\frac{s-q}{q}}{\frac{s-q}{q}+1}\frac{b}{r} < \frac{\frac{s-q}{q}+\frac{c}{k}}{\frac{s-q}{q}+\frac{c}{k}+1}\frac{b}{r} = u_2^*$. Notice that $\underline{u}$ is well-defined, because $w_1^* \geq \frac{s-q}{r}$, as R can always guarantee this payoff at the beginning of boycott. We now prove that the proposition holds for this value $\underline{u}$. Before proceding, we also define

$$\tilde{u} = \begin{cases} 0 & \text{if } w_1^* \in \left[\frac{1}{r}(s-q), \left(\frac{1}{r}+\frac{-\left(v_1^*-\frac{h}{r}\right)-\frac{c}{r}}{c}\right)(s-q)\right); \\ \frac{1}{1+\frac{k}{c-\left(-\left(v_1^*-\frac{h}{r}\right)-\frac{c}{r}\right)\frac{s-q}{w_1^*-\frac{s-q}{r}}}}\frac{b}{r} & \text{if } w_1^* > \left(\frac{1}{r}+\frac{-\left(v_1^*-\frac{h}{r}\right)-\frac{c}{r}}{c}\right)(s-q); \end{cases}$$

in this case, $\tilde{u} > 0$ if and only if $\underline{u} = 0$, and it is straightforward to verify that $\tilde{u} < u_2^*$.

We start by describing possible equilibria as well as conditions for their existence. First, suppose that A does not start a boycott ($\alpha^* = 0$). In this case, the game is between F and R, which is identical to the game studied in Subsection 3.2. By Proposition 1, if such equilibrium exists, it must have $\gamma_0^* = r\frac{c}{k}$ and $\phi_0^* = r\frac{s-q}{q}$. This is an equilibrium if and only if not starting a boycott is indeed a best response for A under such rates (indeed, F and R are playing best responses by Proposition 1). If A starts a boycott, its expected discounted payoff is u_1^*; if it does not, then since the game is isomorphic to Phase 2, its payoff is u_2^*. Therefore, there exists an equilibrium with $\alpha^* = 0$, $\gamma_0^* = r\frac{c}{k}$ and $\phi_0^* = r\frac{s-q}{q}$ if and only if $u_1^* \leq u_2^*$.

Second, suppose that $\alpha^* = \infty$, so A starts a boycott immediately. Since we assumed that $v_1^* - \frac{h}{r} < -\frac{c}{r}$, this cannot be an equilibrium, because in this case, F would self-regulate immediately, but then A would prefer not to start a boycott.

Third, suppose that $\phi_0^* = 0$, but $\alpha^* \in (0, \infty)$ (the case where α^* is extreme was already analyzed). This is only possible where A is indifferent between starting the boycott and not, and since $\phi_0^* = 0$, it must be that $\gamma_0^* = r^2 \frac{u_1^*}{b - r u_1^*}$. This is only possible if $u_1^* \geq 0$. Thus, if $u_1^* = 0$, we have equilibrium $\phi_0^* = \gamma_0^* = 0$ as long as α^* is such that $\phi_0^* = 0$ is optimal for F and $\gamma_0^* = 0$ is optimal for R. The former is true if $\frac{\alpha^*}{\alpha^* + r}\left(v_1^* - \frac{h}{r}\right) \geq -\frac{c}{r}$, i.e., for $\alpha^* \leq \alpha_1^* = \frac{c}{-\left(v_1^* - \frac{h}{r}\right) - \frac{c}{r}}$, where the denominator is positive by assumption. The latter can only be true if $\lambda_F < \overline{\lambda}_F$ (so F self-regulates at a faster rate during the boycott than ϕ_2^* that would make R indifferent), and then the condition is $\frac{\alpha^*}{\alpha^* + r}\left(\frac{\phi_1^*}{\phi_1^* + \rho^* + r}\frac{s}{r} + \frac{\rho^*}{\phi_1^* + \rho^* + r}\frac{s-q}{r}\right) \geq \frac{s-q}{r}$, which is equivalent to $\alpha^* \geq \alpha_2^* = \frac{r(\phi_1^* + \rho^* + r)}{\frac{\phi_1^* q}{s-q} - 1} = \frac{s-q}{w_1^* - \frac{s-q}{r}}$. Thus, if $u_1^* = 0$, such equilibrium exists whenever $\lambda_F < \overline{\lambda}_F$ and $\alpha_1^* \geq \alpha_2^*$, where the latter is equivalent to $u_1^* < \tilde{u}$. Now consider the case where $u_1^* > 0$, in which case $\gamma_0^* > 0$. For R to be indifferent, it must be that $\lambda_F < \overline{\lambda}_F$ and $\alpha^* = \alpha_2^* = \frac{s-q}{w_1^* - \frac{s-q}{r}}$. This constitutes an equilibrium if and only if for these values of γ_0^* and α^*, F prefers to not self-regulate, the condition for which is the following: $\frac{\alpha^*}{\alpha^* + \gamma_0^* + r}\left(v_1^* - \frac{h}{r}\right) + \frac{\gamma_0^*}{\alpha^* + \gamma_0^* + r}\left(-\frac{c+k}{r}\right) \geq -\frac{c}{r}$, which is equivalent to $\gamma_0^* \leq r\frac{c}{k} - \frac{r\alpha^*}{k}\left(-\left(v_1^* - \frac{h}{r}\right) - \frac{c}{r}\right)$, which after plugging in γ_0^* and α^* becomes $r^2 \frac{u_1^*}{b - r u_1^*} \leq r\frac{c}{k} - \frac{r}{k}\left(-\left(v_1^* - \frac{h}{r}\right) - \frac{c}{r}\right)\frac{s-q}{w_1^* - \frac{s-q}{r}}$. This condition simplifies as $u_1^* \leq \tilde{u}$; this implies that equilibrium with $\phi_0^* = 0$ and $\alpha^* \in (0, \infty)$ exists if $u_1^* \in [0, \tilde{u}]$ and $\lambda_F < \overline{\lambda}_F$.

Fourth, suppose that $\phi_0^* = \infty$, but $\alpha^* \in (0, \infty)$. Then R strictly prefers not to regulate ($\gamma_0^* = 0$) and A strictly prefers not to start a boycott, because $u_1^* < \frac{b}{r}$. This contradicts $\alpha^* \in (0, \infty)$, so $\phi_0^* = \infty$ and $\alpha^* \in (0, \infty)$ cannot happen in equilibrium.

Fifth, suppose that $\gamma_0^* = 0$, but $\alpha^*, \phi_0^* \in (0, \infty)$. Then both A and F are indifferent between acting and not; A is indifferent if and only if $\phi_0^* = r^2 \frac{u_1^*}{b - r u_1^*}$, whereas F is indifferent if and only if $\alpha^* = \frac{c}{-\left(v_1^* - \frac{h}{r}\right) - \frac{c}{r}}$. Thus, this equilibrium exists if and only if $u_1^* > 0$ and R does not want to intervene for such ϕ_0^* and α^*. If $\lambda_F \geq \overline{\lambda}_F$, then this last condition is equivalent to $\phi_0^* \geq \phi_2^*$, which is equivalent to $r^2 \frac{u_1^*}{b - r u_1^*} \geq r\frac{s-q}{q}$, which holds if and only if $u_1^* \geq \frac{s-q}{s}\frac{b}{r} = \underline{u}$ (indeed, $\lambda_F \geq \overline{\lambda}_F$ implies $w_1^* = \frac{s-q}{r}$). If $\lambda_F < \overline{\lambda}_F$, not regulating is a best response for R if and only if $\frac{\phi_0^*}{\phi_0^* + \alpha^* + r}\frac{s}{r} + \frac{\alpha^*}{\phi_0 + \alpha^* + r} w_1^* \geq \frac{s-q}{r}$, where $w_1^* = \frac{\phi_1^*}{\phi_1^* + \rho^* + r}\frac{s}{r} + \frac{\rho^*}{\phi_1^* + \rho^* + r}\frac{s-q}{r}$ satisfies $w_1^* \in \left(\frac{s-q}{r}, \frac{s}{r}\right)$, since $\lambda_F < \overline{\lambda}_F$. The inequality is equivalent to $\phi_0^* \geq \frac{r\alpha^*}{q}\left(\frac{s-q}{r} - w_1^*\right) + r\frac{s-q}{q}$, which is automatically satisfied if the numerator is nonpositive (this is true if and only if $w_1^* \geq \frac{s-q}{r} + \frac{s-q}{\alpha^*}$), and otherwise is equivalent to $u_1^* \geq \frac{b}{r}\frac{\min\left(\frac{r\alpha^*}{q}\left(\frac{s-q}{r} - w_1^*\right) + r\frac{s-q}{q}, 0\right)}{\min\left(\frac{r\alpha^*}{q}\left(\frac{s-q}{r} - w_1^*\right) + r\frac{s-q}{q}, 0\right) + r}$. Since $\alpha^* = \frac{c}{-\left(v_1^* - \frac{h}{r}\right) - \frac{c}{r}}$, we find that such equilibrium exists if and only if $u_1^* \geq \underline{u}$ in all cases.

Sixth, suppose that $\gamma_0^* = \infty$, but $\alpha^*, \phi_0^* \in (0, \infty)$. In this case, it is a best response for F to try to preempt regulation by R, so ϕ_0^* must equal ∞, which is a contradiction. Thus, there are no equilibria satisfying this property.

Seventh and last, suppose that $\alpha^*, \phi_0^*, \gamma_0^* \in (0, \infty)$. Since A is indifferent, it must be that $\frac{\phi_0^* + \gamma_0^*}{\phi_0^* + \gamma_0^* + r}\frac{b}{r} = u_1^*$, which uniquely pins down $\phi_0^* + \gamma_0^* = r^2 \frac{u_1^*}{b - r u_1^*}$; thus, such equilibria exist only if $u_1^* > 0$. Suppose $\lambda_F \geq \overline{\lambda}_F$; then R is indifferent if and only if $\phi_0^* = \phi_2^*$ (because after the boycott starts, the rate of F self-regulating is $\phi_1^* = \phi_2^*$), which pins down $\gamma_0^* = r^2 \frac{u_1^*}{b - r u_1^*} - \phi_2^*$, and this implies that such equilibrium can exist only if $u_1^* > \frac{s-q}{s}\frac{b}{r}$, which equals $\underline{u}$ in this case. In this case, F being indifferent implies that $\frac{\alpha^*}{\alpha^*+\gamma_0^*+r}\left(v_1^*-\frac{h}{r}\right)+\frac{\gamma_0^*}{\alpha^*+\gamma_0^*+r}\left(-\frac{c+k}{r}\right)=-\frac{c}{r}$; here, the left-hand side equals $\frac{\gamma_0^*}{\gamma_0^*+r}\left(-\frac{c+k}{r}\right)$ if $\alpha^*=0$, which exceeds $-\frac{c}{r}$ if and only if $\gamma_0^*<\gamma_2^*$, and if $\alpha^*=\infty$ then it equals $v_1^*-\frac{h}{r}<-\frac{c}{r}$ by assumption. Consequently, a value $\alpha^*\in(0,\infty)$ for which F is indifferent exists if and only if $\phi_0^*+\gamma_0^*<\phi_2^*+\gamma_2^*$, which is equivalent to $r^2\frac{u_1^*}{b-ru_1^*}<r^2\frac{u_2^*}{b-ru_2^*}$, i.e., $u_1^*<u_2^*$. Thus, in this case, a fully mixed equilibrium exists if and only if $u_1^*\in(\underline{u},u_2^*)$.

Now suppose $\lambda_F<\overline{\lambda}_F$; in this case, R is indifferent if and only if $\frac{\phi_0^*}{\phi_0^*+\alpha^*+r}\frac{s}{r}+\frac{\alpha^*}{\phi_0^*+\alpha^*+r}\left(\frac{\phi_1^*}{\phi_1^*+\rho^*+r}\frac{s}{r}+\frac{\rho^*}{\phi_1^*+\rho^*+r}\frac{s-q}{r}\right)=\frac{s-q}{r}$ and F is indifferent if and only if $\frac{\alpha^*}{\alpha^*+\gamma_0^*+r}\left(v_1^*-\frac{h}{r}\right)+\frac{\gamma_0^*}{\alpha^*+\gamma_0^*+r}\left(-\frac{c+k}{r}\right)=-\frac{c}{r}$. The first of these equations defines ϕ_0^* as an implicit function of α^*, namely, $\phi_0^*(\alpha^*)=r\frac{s-q}{q}-\frac{\phi_1^*-r\frac{s-q}{q}}{\phi_1^*+\rho^*+r}\alpha^*$, so it is monotonically decreasing in α^*, since in this case $\phi_1^*>\phi_2^*=r\frac{s-q}{q}$, and achieving 0 at $\alpha^*=r\left(\phi_1^*+\rho^*+r\right)\frac{\frac{s-q}{q}}{\phi_1^*-r\frac{s-q}{q}}\in(0,\infty)$. The second equation also defines γ_0^* implicitly as a function of α^*; specifically, $\gamma_0^*(\alpha^*)=\frac{rc}{k}+\frac{r}{k}\left(v_1^*-\frac{h}{r}+\frac{c}{r}\right)\alpha^*$, which is decreasing in α^*, since $v_1^*-\frac{h}{r}<-\frac{c}{r}$; furthermore, it equals $\gamma_0^*=\frac{rc}{k}$ for $\alpha^*=0$ and monotonically decreases to 0 at $\alpha^*=\frac{c}{-\left(v_1^*-\frac{h}{r}\right)-\frac{c}{r}}$. Thus, at $\alpha^*=0$, total rate of regulation $r\frac{s-q}{q}+r\frac{c}{k}$ would give A utility u_2^*, and since it is decreasing in α, such equilibrium may exist only if $u_1^*<u_2^*$. In this case, it exists if and only if for $\alpha^*=\min\left(r\left(\phi_1^*+\rho^*+r\right)\frac{\frac{s-q}{q}}{\phi_1^*-r\frac{s-q}{q}},\frac{c}{-\left(v_1^*-\frac{h}{r}\right)-\frac{c}{r}}\right)$, the total rate of regulation $\phi_0^*(\alpha^*)+\gamma_0^*(\alpha^*)<r^2\frac{u_1^*}{b-ru_1^*}$. This condition holds if and only if $\alpha^*=r\left(\phi_1^*+\rho^*+r\right)\frac{\frac{s-q}{q}}{\phi_1^*-r\frac{s-q}{q}}$, $\gamma_0^*(\alpha^*)<r^2\frac{u_1^*}{b-ru_1^*}$, and for $\alpha^*=\frac{c}{-\left(v_1^*-\frac{h}{r}\right)-\frac{c}{r}}$, $\phi_0^*(\alpha^*)<r^2\frac{u_1^*}{b-ru_1^*}$. The former condition is equivalent to $\frac{rc}{k}+\frac{r}{k}\left(v_1^*-\frac{h}{r}+\frac{c}{r}\right)r\left(\phi_1^*+\rho^*+r\right)\frac{\frac{s-q}{q}}{\phi_1^*-r\frac{s-q}{q}}>r^2\frac{u_1^*}{b-ru_1^*}$, or equivalently $\frac{rc}{k}+\frac{r}{k}\left(v_1^*-\frac{h}{r}+\frac{c}{r}\right)\frac{s-q}{w_1^*-\frac{s-q}{r}}>r^2\frac{u_1^*}{b-ru_1^*}$, which is equivalent to $u_1^*>\tilde{u}$. The latter condition is equivalent to $r\frac{s-q}{q}-\frac{\phi_1^*-r\frac{s-q}{q}}{\phi_1^*+\rho^*+r}\alpha^*<r^2\frac{u_1^*}{b-ru_1^*}$, or equivalently $r\frac{s-q}{q}-\frac{r}{q}\left(w_1^*-\frac{s-q}{r}\right)\alpha^*<r^2\frac{u_1^*}{b-ru_1^*}$, which, after simplification, is equivalent to $u_1^*>\underline{u}$. Thus, in this case, this equilibrium exists if and only if $u_1^*\in\left(\max\left\{\underline{u},\tilde{u}\right\},u_2^*\right)$.

We therefore have the following result. For $u_1^*>u_2^*$, the equilibrium is unique, with $\alpha^*=\frac{c}{-\left(v_1^*-\frac{h}{r}\right)-\frac{c}{r}}$, $\phi_0^*=r^2\frac{u_1^*}{b-ru_1^*}$, $\gamma_0^*=0$; it is straightforward to check that it is stable. For $u_1^*\in\left(\max\left\{\underline{u},\tilde{u}\right\},u_2^*\right)$, there are three equilibria: the previous one, one given by $\alpha^*=0$, $\phi_0^*=r\frac{s-q}{q}$, $\gamma_0^*=r\frac{c}{k}$ (both stable), and a fully mixed, which is unstable. For $u_1^*\in(0,\tilde{u})$, there are the two stable equilibria as in the previous case, and $\alpha=\frac{s-q}{w_1^*-\frac{s-q}{r}}$, $\phi_0^*=0$, $\gamma_0^*=r^2\frac{u_1^*}{b-ru_1^*}$, which is unstable. For $u_1^*<\max\left(\underline{u},0\right)$, the equilibrium is unique, given by $\alpha^*=0$, $\phi_0^*=r\frac{s-q}{q}$, $\gamma_0^*=r\frac{c}{k}$, and it is stable. The borderline cases are straightforward, except for the case $\underline{u}=u_1^*=0<\tilde{u}$, where there is a continuum of equilibria given by $\frac{c}{-\left(v_1^*-\frac{h}{r}\right)-\frac{c}{r}}\le\alpha^*\le\frac{s-q}{w_1^*-\frac{s-q}{r}}$, $\phi_0^*=0$, $\gamma_0^*=0$, with only the one with the highest α^* being stable. This proves that the sets of stable equilibria are as described in the Proposition.

Lastly, if $\underline{u}<u_1^*<u_2^*$, A prefers the equilibrium where $\alpha^*=0$; indeed, in this case his payoff is higher than u_1^*, whereas in the other equilibria he starts the boycott with a positive probability, so

his payoff is u_1^*. Similarly, R prefers the equilibrium where $\gamma_0^* = 0$, as its payoff is then higher than w_1^*. This completes the proof. ■

Proof of Corollary 1. If following entrance by A, the equilibrium where A starts a boycott with a positive probability is played, then the expected payoff of A from entering is u_1^*, whereas if it does not enter it gets u_2^*. In the proof of Proposition 4, we showed that $u_1^* > u_2^*$ if and only if $\lambda_F < \widehat{\lambda}_F$, where $\widehat{\lambda}_F < \overline{\lambda}_F$. Consider the stable equilibrium $\alpha^* = \frac{c}{-(v_1^* - \frac{h}{r}) - \frac{c}{r}}$, $\phi_0^* = r^2 \frac{u_1^*}{b - r u_1^*}$, $\gamma_0^* = 0$ played after such entrance. Since $\lambda_F < \overline{\lambda}_F$, $\gamma_1^* = \gamma_1^* = 0$, whereas $\gamma_2^* > 0$.

Notice that in this case, $\phi_1^* > \phi_2^* + \gamma_2^*$, because $\phi_1^* \leq \phi_2^* + \gamma_2^*$ would imply that $u_1^* < \frac{\phi_1^*}{\phi_1^* + \rho^* + r} \frac{b}{r} < \frac{\phi_1^*}{\phi_1^* + r} \frac{b}{r} \leq \frac{\phi_2^* + \gamma_2^*}{\phi_2^* + \gamma_2^* + r} \frac{b}{r} = u_2^*$, a contradiction. Suppose, to obtain a contradiction, that $\phi_0^* \geq \phi_1^*$; then the total rate of regulation after A starts a boycott will be weakly lower before it ends and strictly lower after that, because we proved that $\phi_1^* > \phi_2^* + \gamma_2^*$. Taking account the cost of boycott and the possible loss of $\frac{l}{r}$, we find that A would be better off if it did not start a boycott, which contradicts $\alpha^* > 0$. Thus, $\phi_0^* < \phi_1^*$. Now suppose to obtain a contradiction that $\phi_0^* < \phi_2^* + \gamma_2^*$. But $u_1^* = \frac{\phi_0^*}{\phi_0^* + r} \frac{b}{r}$, and $\phi_0^* < \phi_2^* + \gamma_2^*$, would imply $u_1^* = \frac{\phi_0^*}{\phi_0^* + r} \frac{b}{r} < \frac{\phi_2^* + \gamma_2^*}{\phi_2^* + \gamma_2^* + r} \frac{b}{r} = u_2^*$, a contradiction. We have thus proved that $\phi_1^* > \phi_0^* \geq \phi_2^* + \gamma_2^* > \phi_2^*$. The comparative statics results follow from Proposition 4. Indeed, both u_1^* and v_1^* increase in λ_A, and thus ϕ_0^* and α^* increase in λ_A. Furthermore, since $\lambda_F < \overline{\lambda}_F$, then u_1^* is decreasing in λ_F, and therefore so does ϕ_0^*.

Consider the opposite case, where $\lambda_F > \widehat{\lambda}_F$. In this case, $u_1^* < u_2^*$, so A does not enter and the *equilibrium* $\phi_0^* = \phi_2^*$ *and* $\gamma_0^* = \gamma_2^*$ is played; thus ϕ_0^* and γ_0^* are not affected by λ_A or λ_F. This completes the proof. ■

Appendix B: Analysis of the boycott game (online appendix)

This Appendix contains a detailed analysis of war of attrition with private information about the ongoing cost of staying in the game. The setting is different from Ponsati and Sákovics (1995) because (a) it assumes that private information affects the ongoing cost rather than benefit of winning; (b) it assumes increasing, rather than constant, marginal cost of boycott; (c) parameters are such that there is a unique equilibrium, in which the two players concede at a constant rate. Furthermore, we characterize some comparative statics results and explicitly compute expected payoffs. Some notation and some lines of argument are inspired by those in Ponsati and Sákovics (1995); at the same time, the models are sufficiently different so that most lemmas in that paper are not directly applicable. The boycott game between firm and activist in the main part of the paper is a particular case of the game solved here; but the analysis of this game seems to be of sufficient independent interest to warrant an independent treatment in this online Appendix.

B1 Setup

The game is played by two agents, A and F. In what follows, we let i denote a generic agent, and then j denotes the other agent. The agents have private types, θ_A and θ_F, which are both positive numbers. Their distributions are common knowledge: θ_A is distributed exponentially with expectation λ_A, and θ_F is distributed exponentially with expectation λ_F. Thus, their cumulative and partial distribution functions are given by:

$$\begin{aligned} F_A(x) &= 1-\exp\left(-\frac{x}{\lambda_A}\right), f_A(x)=\frac{1}{\lambda_A}\exp\left(-\frac{x}{\lambda_A}\right), \\ F_F(x) &= 1-\exp\left(-\frac{x}{\lambda_F}\right), f_F(x)=\frac{1}{\lambda_F}\exp\left(-\frac{x}{\lambda_F}\right). \end{aligned}$$

The marginal cost of boycott at time τ is assumed to equal $\frac{\tau}{\theta_i}$.

Each player i has a single strategy, namely, given his type θ_i, he decides on the time where he concedes to the other player. We denote this strategy by $\sigma_i(\theta_i)$. If one of the players concedes, the game ends. Agents have discount factors r_A and r_F, respectively. The payoffs of A and F depend on the event that happened first and the time of the event, and are the following (in case of a draw, there is a lottery such that both events that occurred at the moment have positive probabilities of determining the payoffs).

If $\min(\sigma_A,\sigma_F)=\sigma_A=t$ (the game stops at time t due to decision of player A), then payoffs are:

$$\begin{aligned} U_A &= L_A\exp(-r_At)-\int_0^t\frac{\tau}{\theta_A}\exp(-r_A\tau)\,d\tau; \\ U_F &= W_F\exp(-r_Ft)-\int_0^t\frac{\tau}{\theta_F}\exp(-r_F\tau)\,d\tau; \end{aligned}$$

if $\min(\sigma_A, \sigma_F) = \sigma_F = t$ (the game stops at time t due to decision of player F), then payoffs are:

$$
\begin{aligned}
U_A &= W_A \exp(-r_A t) - \int_0^t \frac{\tau}{\theta_A} \exp(-r_A \tau)\, d\tau; \\
U_F &= L_F \exp(-r_F t) - \int_0^t \frac{\tau}{\theta_F} \exp(-r_F \tau)\, d\tau.
\end{aligned}
$$

In what follows, we maintain the assumption that for each player $i \in \{A, F\}$, $W_i \geq 0 > L_i$.

B2 Analysis

To analyze the game, we need to solve for the strategies of different types of players A and F. Strictly speaking, a pure strategy of player $i \in \{A, F\}$ with type θ_i is the time t at which the player concedes (if the game is not over yet), and a mixed strategy is a probability distribution over possible times (i.e., over $\mathbb{R}^+$). We capture this distribution with its c.d.f.: For player i with type θ_i playing profile σ, let $H_i^\sigma(\theta_i; t)$ be the probability that this type concedes no later than time t. The following notation is also helpful: If the players play strategy profile σ, then for $i \in \{A, F\}$, let $H_i^\sigma(t)$ denote the probability that player i concedes not later than t:

$$
H_i^\sigma(t) = \int_0^\infty H_i^\sigma(\theta_i; t) f_i(\theta_i)\, d\theta_i;
$$

in other words, this is the expectation that player i concedes no later than time t taken over possible realizations of this player's type, θ_i.

We start with the following mathematical result (this was indirectly used in Ponsati and Sákovics, 1995, without proof; we complete this gap).

Lemma B1 *Suppose that function f monotonically increasing and right-continuous on some interval (a, b). Suppose, furthermore, that for some $K > 0$ the following is true: for any $q \in (a, b)$ and for any $\varepsilon > 0$ there is $p \in (q - \varepsilon, q)$ such that*

$$
f(q) - f(p) \leq K(q - p).
$$

Then f is Lipschitz-continuous with parameter K.

Proof. Take any x, y such that $a < x < y < b$. We need to prove that $f(y) - f(x) \leq K(y - x)$. Consider the set X defined by

$$
X = \{\xi \in (x, y) : f(y) - f(\xi) \leq K(y - \xi)\}.
$$

By the condition of the Lemma, X is nonempty. Let us prove that $\inf X = x$; in this case, right-continuity of $f(\cdot)$ would imply that $f(y) - f(x) \leq K(y - x)$, which would prove Lipschitz-continuity.

Suppose not, i.e., $\inf X = l > x$. By right-continuity of $f(\cdot)$, $l \in X$. But since $l \in (a,b)$, then if we take $\varepsilon = l - x$, there would exist $p \in (l-\varepsilon, l)$ such that $f(l) - f(p) \le K(l-p)$. But this would imply that

$$f(y) - f(p) = (f(y) - f(l)) + (f(l) - f(p)) \le K(y-l) + K(l-p) = K(y-p),$$

where we used that $l \in X$. However, this inequality implies that $p \in X$, contradicting that $l = \inf X$. This contradiction proves that $\inf X = x$, which proves Lipschitz-continuity of f. ■

The following lemma defines and studies the properties of a certain function $z(t)$.

Lemma B2 *Define*

$$z(t) = z(t; r_i) = 1 - e^{-r_i t} - r_i t e^{-r_i t}. \tag{B1}$$

Then for any $r_i > 0$, $z(t)$ is (strictly) monotonically increasing for $t \in [0, +\infty)$, and $z(0) = 0$, $\lim_{t\to\infty} z(t) = 1$. *Furthermore, for any $a < b$ there is some $c \in (a,b)$ such that*

$$z(b) - z(a) = \left(e^{-r_i a} - e^{-r_i b}\right) r_i c. \tag{B2}$$

Proof. We have

$$\frac{dz}{dt} = t r_i^2 e^{-t r_i} > 0,$$

and the results for $z(0)$ and $\lim_{t\to\infty} z(t)$ are straightforward. Finally notice that $z(t) = f\left(e^{-r_i t}\right)$, where $f(y) = 1 - y + y \ln y$. We have

$$\frac{df}{dy} = \ln y,$$

which implies, by Lagrange theorem, that for any $p < q$,

$$f(q) - f(p) = (\ln y)(q-p)$$

for some $y \in (p,q)$. If we let $q = e^{-r_i a}$ and $p = e^{-r_i b}$ (so $p < q$), we have

$$z(a) - z(b) = (\ln y)\left(e^{-r_i a} - e^{-r_i b}\right)$$

If we let $c = -\frac{\ln y}{r_i}$, then $c \in (a,b)$, and (B2) follows. ■

The next lemma rules out the possibility that two players concede at once with a positive probability.

Lemma B3 *If $H_j^\sigma(t)$ is discontinuous at some time t (i.e., there is a positive probability of conceding by player j at time t), then for any type θ_i of player i, conceding at time t is not a best response and thus happens with probability* 0. *Furthermore, if $t > 0$, then there is $\varepsilon > 0$ such that $H_i^\sigma(\theta_i; t) = H_i^\sigma(\theta_i; t-\varepsilon)$ (this ε may depend on θ_i).*

Proof. Let p be the probability that player j concedes at time t, so $p = H_j^\sigma(t) - \lim_{\varepsilon \searrow 0} H_j^\sigma(t-\varepsilon)$ if $t > 0$, and $p = H_j^\sigma(0)$ if $t = 0$. If player i concedes at t, then there is a lottery, which he may lose with a positive probability α. By conceding at $t' = t + \delta$ for arbitrarily small δ, he wins with additional probability $p\alpha$, and thus his expected utility is increased by at least $p\alpha e^{-r_i t}(W_i - L_i) - \int_t^{t+\delta} e^{-r_i \tau} \frac{\tau}{\theta_i} d\tau$. For sufficiently small δ this is positive, thus deviation to t' is profitable. This contradicts that σ is an equilibrium, proving that player i cannot concede at t.

If $t > 0$, then similar reasoning proves that player i with type θ_i cannot concede at $t - \delta$ (again, for δ sufficiently small) either, thus there is $\varepsilon > 0$ such that $H_i^\sigma(\theta_i; t) = H_i^\sigma(\theta_i; t - \varepsilon)$. ∎

We next show the role of $z(t)$ in calculating the cost of boycott.

Claim 1 *The discounted net present cost of a game up until time t for a player of type θ_i with discount factor r_i is given by*

$$\int_0^t e^{-r_i \tau} \frac{\tau}{\theta_i} d\tau = \frac{z(t)}{\theta_i r_i^2},$$

where $z(t)$ is defined by (B1).

Proof. We have

$$\begin{aligned}
\int_0^t e^{-r_i \tau} \frac{\tau}{\theta_i} d\tau &= \frac{1}{\theta_i} \int_0^t e^{-r_i \tau} \tau d\tau \\
&= \frac{1}{\theta_i r_i^2} \int_0^{r_i t} e^{-r_i \tau} (r_i \tau) \, d(r_i \tau) \\
&= \frac{1}{\theta_i r_i^2} \int_0^{r_i t} e^{-y} y dy \\
&= -\frac{1}{\theta_i r_i^2} e^{-y}(y+1) \Big|_{y=0}^{y=r_i t} \\
&= \frac{1}{\theta_i r_i^2} - \frac{1}{\theta_i r_i^2} e^{-r_i t}(r_i t + 1) \\
&= \frac{1}{\theta_i r_i^2} \left(1 - e^{-r_i t} - r_i t e^{-r_i t}\right).
\end{aligned}$$

The last expression equals $\frac{z(t)}{\theta_i r_i^2}$ by definition of $z(t)$. ∎

The following lemma defines the payoff of player i of type θ_i if he concedes at time t, where $H_j^\sigma(t)$ is continuous.

Lemma B4 *If player i concedes at time $t \geq 0$ at which $H_j^\sigma(t)$ is continuous, then his payoff (net of cost of boycott) is given by*

$$V_i(t) = V_i(t; \theta_i) = \int_0^t \left(W_i e^{-r_i \tau} - \frac{z(\tau)}{\theta_i r_i^2} \right) dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(t)\right) \left(L_i e^{-r_i t} - \frac{z(t)}{\theta_i r_i^2} \right). \tag{B3}$$

Proof. Since under the stated conditions the two players concede at once with probability 0, the result follows immediately. ■

The next lemma suggests a (relatively) simple formula for the increment of $V_i(t)$ between two time points.

Lemma B5 *The function $V_i(t)$ satisfies, for any a, b:*

$$\begin{aligned} V_i(b) - V_i(a) &= \int_a^b \left(e^{-r_i\tau} W_i - e^{-r_i a} L_i\right) dH_j^\sigma(\tau) - \left(1 - H_j^\sigma(b)\right)\left(e^{-r_i a} - e^{-r_i b}\right) L_i \\ &\quad - \frac{1}{\theta_i r_i^2}\left(\int_a^b (z(\tau) - z(a))\, dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(b)\right)(z(b) - z(a))\right). \end{aligned} \tag{B4}$$

Proof. This follows from

$$\begin{aligned} V_i(b) - V_i(a) &= \int_0^b e^{-r_i\tau} W_i dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(b)\right) e^{-r_i b} L_i - \frac{1}{\theta_i r_i^2}\left(\int_0^b z(\tau)\, dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(b)\right) z(b)\right) \\ &\quad - \int_0^a e^{-r_i\tau} W_i dH_j^\sigma(\tau) - \left(1 - H_j^\sigma(a)\right) e^{-r_i a} L_i + \frac{1}{\theta_i r_i^2}\left(\int_0^a z(\tau)\, dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(a)\right) z(a)\right) \\ &= \int_a^b e^{-r_i\tau} W_i dH_j^\sigma(\tau) - \left(1 - H_j^\sigma(b)\right)\left(e^{-r_i a} - e^{-r_i b}\right) L_i - \left(H_j^\sigma(b) - H_j^\sigma(a)\right) e^{-r_i a} L_i \\ &\quad - \frac{1}{\theta_i r_i^2}\left(\int_a^b z(\tau)\, dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(b)\right)(z(b) - z(a)) - \left(H_j^\sigma(b) - H_j^\sigma(a)\right) z(a)\right) \\ &= \int_a^b \left(e^{-r_i\tau} W_i - e^{-r_i a} L_i\right) dH_j^\sigma(\tau) - \left(1 - H_j^\sigma(b)\right)\left(e^{-r_i a} - e^{-r_i b}\right) L_i \\ &\quad - \frac{1}{\theta_i r_i^2}\left(\int_a^b (z(\tau) - z(a))\, dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(b)\right)(z(b) - z(a))\right). \end{aligned}$$

■

We now prove that neither player concedes too fast (i.e., we establish a lower bound on concession time).

Lemma B6 *Player i with type θ_i concedes no earlier than $\breve{t}(\theta_i) = r_i |L_i| \theta_i$; in other words, $\lim_{\varepsilon \searrow 0} H_i^\sigma\left(\theta_i; \breve{t}_i(\theta_i) - \varepsilon\right) = 0$. Furthermore, $H_i^\sigma(0) = 0$ (so there is no atom in distribution $H_i^\sigma(\cdot)$ at zero) and $H_i^\sigma(t) < 1$ for all $t < \infty$ (so player i does not concede with probability 1 by some fixed time t).*

Proof. We start by proving a weaker version of the statement. For $i \in \{A, F\}$, define $T_i = \inf\{t : H_i^\sigma(t) = 1\}$ if the latter set is nonempty and $T_i = \infty$ otherwise. Let us prove that player i with type θ_i concedes no earlier than $\hat{t}(\theta_i) = \min\left(\breve{t}(\theta_i), T_j\right)$.

Suppose that this is not the case, and some type θ_i concedes at $t_0 < \min\left(\breve{t}(\theta_i), T_j\right)$. We then have $H_j^\sigma(t_0) < 1$ by definition of T_j; furthermore, by Lemma B3, we have that $H_j^\sigma(\cdot)$ is continuous at t_0. Let $\check{t} = \check{t}(\theta_i)$; we then have, by (B4), that

$$\begin{aligned}
V_i\left(\check{t}\right) - V_i\left(t_0\right) &= \int_{t_0}^{\check{t}} \left(e^{-r_i\tau}W_i - e^{-r_i t_0}L_i\right) dH_j^\sigma\left(\tau\right) - \left(1 - H_j^\sigma\left(\check{t}\right)\right)\left(e^{-r_i t_0} - e^{-r_i\check{t}}\right)L_i \\
&\quad - \frac{1}{\theta_i r_i^2}\left(\int_{t_0}^{\check{t}} \left(z\left(\tau\right) - z\left(t_0\right)\right) dH_j^\sigma\left(\tau\right) + \left(1 - H_j^\sigma\left(\check{t}\right)\right)\left(z\left(\check{t}\right) - z\left(t_0\right)\right)\right) \\
&\geq -\left(H_j^\sigma\left(\check{t}\right) - H_j^\sigma\left(t_0\right)\right)e^{-r_i t_0}L_i - \left(1 - H_j^\sigma\left(\check{t}\right)\right)\left(e^{-r_i t_0} - e^{-r_i\check{t}}\right)L_i \\
&\quad - \frac{1}{\theta_i r_i^2}\left(\left(H_j^\sigma\left(\check{t}\right) - H_j^\sigma\left(t_0\right)\right)\left(z\left(\check{t}\right) - z\left(t_0\right)\right) + \left(1 - H_j^\sigma\left(\check{t}\right)\right)\left(z\left(\check{t}\right) - z\left(t_0\right)\right)\right) \\
&= -\left(1 - H_j^\sigma\left(t_0\right)\right)e^{-r_i t_0}L_i + \left(1 - H_j^\sigma\left(\check{t}\right)\right)e^{-r_i\check{t}}L_i \\
&\quad - \frac{1}{\theta_i r_i^2}\left(1 - H_j^\sigma\left(t_0\right)\right)\left(z\left(\check{t}\right) - z\left(t_0\right)\right) \\
&\geq -\left(1 - H_j^\sigma\left(t_0\right)\right)\left(e^{-r_i t_0} - e^{-r_i\check{t}}\right)L_i - \frac{1}{\theta_i r_i^2}\left(1 - H_j^\sigma\left(t_0\right)\right)\left(z\left(\check{t}\right) - z\left(t_0\right)\right) \\
&> \left(1 - H_j^\sigma\left(t_0\right)\right)\left(e^{-r_i t_0} - e^{-r_i\check{t}}\right)\left(-L_i - \frac{1}{\theta_i r_i^2}r_i\check{t}\right) = 0,
\end{aligned}$$

since $-L_i - \frac{1}{\theta_i r_i^2}r_i\check{t} = |L_i| - \frac{1}{\theta_i r_i}r_i|L_i|\theta_i = 0$. If $H_j^\sigma(\cdot)$ is continuous at $\check{t}$, then $V_i\left(\check{t}\right)$ is the payoff of player i from deviating to $\check{t}$, which proves that type θ_i has a profitable deviation, which is a contradiction. Otherwise, if $H_j^\sigma(\cdot)$ has an atom at $\check{t}$, then we can find $t' < \check{t}$ arbitrarily close to $\check{t}$ and such that $H_j^\sigma(\cdot)$ is continuous at t'; if so, the calculation above would prove that $V_i(t') > V_i(t_0)$, and then type θ_i would have a profitable deviation to t', again a contradiction. This proves teh weaker version of the statement.

To prove the stronger version, it suffices to prove that $T_i = T_j = \infty$. Notice that we must have $T_i = T_j$. Indeed, suppose not; without loss of generality, $T_i < T_j$. If so, then the result that we proved shows that types $\theta_i > \frac{T_i}{r_i|L_i|}$ must concede not earlier than $\min(r_i|L_i|\theta_i, T_j)$, which satisfies $\min(r_i|L_i|\theta_i, T_j) > \min(T_i, T_j) = T_i$, which contradicts the definition of T_i. Thus, suppose, to obtain a contradiction, that $T_i = T_j = T < \infty$. We have that types of player i that satisfy $\theta_i \geq \frac{T}{r_i|L_i|}$ must concede no earlier than T, which implies that $H_i^\sigma(\cdot)$ has an atom at T. Similarly, types of player j that satisfy $\theta_j \geq \frac{T}{r_j|L_j|}$ must concede no earlier than T as well, which implies that $H_j^\sigma(\cdot)$ has an atom at T. However, this contradicts Lemma B3. Thus, we must have $T_i = T_j = \infty$, in which case the statement is equivalent to the version that was already proved, which completes the proof. ■

We next establish an upper bound for the concession time of each player of each type.

Lemma B7 *For almost all θ_i player i with type θ_i concedes no later than time* $\bar{t}_i(\theta_i) = 4\sqrt{\theta_i(W_i - L_i)\max\left(\theta_i r_i^2(W_i - L_i), 1\right)}$. *In other words,* $H_i^\sigma(\theta_i; \bar{t}_i(\theta_i)) = 1$.

Proof. Suppose not, then a positive measure of types of player i, θ_i it is a best response to concede at some time $\tilde{t}(\theta_i) > \bar{t}_i(\theta_i)$. From Lemma B3 it follows that $H_j^\sigma(\cdot)$ is continuous at $\tilde{t}(\theta_i)$ for all such θ_i. Notice that we can always pick θ_i so that $H_j^\sigma(\cdot)$ is continuous at $\frac{\bar{t}_i(\theta_i)}{2}$, since the set where $H_j^\sigma(\cdot)$ is discontinuous is at most countable. Denote $t_0 = \frac{\bar{t}_i(\theta_i)}{2}$ and let $\bar{t} = \bar{t}_i(\theta_i)$ and $\tilde{t} = \tilde{t}_i(\theta_i)$ for this particular value of θ_i.

By Lemma B6, we have $H_j^\sigma(\bar{t}) < 1$. By (B4), we have

$$\begin{aligned} V_i(t_0) - V_i(\tilde{t}) &= \int_{t_0}^{\tilde{t}} \left(e^{-r_i t_0} L_i - e^{-r_i \tau} W_i\right) dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(\tilde{t})\right)\left(e^{-r_i t_0} - e^{-r_i \tilde{t}}\right) L_i \\ &\quad + \frac{1}{\theta_i r_i^2}\left(\int_{t_0}^{\tilde{t}} \left(z(\tau) - z(t_0)\right) dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(\tilde{t})\right)\left(z(\tilde{t}) - z(t_0)\right)\right). \end{aligned}$$

Take the first term; since $W_i \geq 0 > L_i$, we have

$$\begin{aligned} 0 &\geq \int_{t_0}^{\tilde{t}} \left(e^{-r_i t_0} L_i - e^{-r_i \tau} W_i\right) dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(\tilde{t})\right)\left(e^{-r_i t_0} - e^{-r_i \tilde{t}}\right) L_i \\ &\geq -\left(H_j^\sigma(\tilde{t}) - H_j^\sigma(t_0)\right) e^{-r_i t_0} W_i + \left(1 - H_j^\sigma(t_0)\right) e^{-r_i t_0} L_i \\ &\geq -\left(1 - H_j^\sigma(t_0)\right) e^{-r_i t_0}\left(W_i - L_i\right). \end{aligned}$$

Now consider the last term. Suppose first that $H_j^\sigma(\tilde{t}) \leq \frac{1+H_j^\sigma(t_0)}{2}$, so $1 - H_j^\sigma(\tilde{t}) \geq \frac{1-H_j^\sigma(t_0)}{2}$. In this case,

$$\begin{aligned} &\frac{1}{\theta_i r_i^2}\left(\int_{t_0}^{\tilde{t}} \left(z(\tau) - z(t_0)\right) dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(\tilde{t})\right)\left(z(\tilde{t}) - z(t_0)\right)\right) \\ \geq\ &\frac{1}{\theta_i r_i^2}\left(1 - H_j^\sigma(\tilde{t})\right)\left(z(\tilde{t}) - z(t_0)\right) \geq \frac{1}{\theta_i r_i^2}\frac{1}{2}\left(1 - H_j^\sigma(t_0)\right)\left(z(\tilde{t}) - z(t_0)\right). \end{aligned}$$

We have

$$z(\tilde{t}) - z(t_0) \geq z(2t_0) - z(t_0) = e^{-r_i t_0}\left(1 + r_i t_0 - e^{-r_i t_0} - 2 r_i t_0 e^{-r_i t_0}\right) > e^{-r_i t_0} r_i t_0 \times \min\left(\frac{r_i t_0}{2}, 1\right);$$

the latter inequality follows from $1 + x - e^{-x} - 2xe^{-x} > \min\left(x, \frac{x^2}{2}\right)$ for all $x > 0$. Consequently, in this case (since $H_j^\sigma(t_0) \leq H_j^\sigma(\bar{t}) < 1$),

$$\frac{1}{\theta_i r_i^2}\left(\int_{t_0}^{\tilde{t}} \left(z(\tau) - z(t_0)\right) dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(\tilde{t})\right)\left(z(\tilde{t}) - z(t_0)\right)\right) \tag{B5}$$

$$> \left(1 - H_j^\sigma(t_0)\right) e^{-r_i t_0} \frac{1}{\theta_i r_i^2} \min\left(\frac{r_i t_0}{2}, \left(\frac{r_i t_0}{2}\right)^2\right). \tag{B6}$$

with a strict inequality.

Now suppose that $H_j^\sigma(\tilde{t}) > \frac{1+H_j^\sigma(t_0)}{2}$, so $H_j^\sigma(\tilde{t}) - H_j^\sigma(t_0) > \frac{1-H_j^\sigma(t_0)}{2}$. Then

$$\begin{aligned} &\frac{1}{\theta_i r_i^2}\left(\int_{0}^{\tilde{t}} \left(z(\tau) - z(0)\right) dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(\tilde{t})\right)\left(z(\tilde{t}) - z(0)\right)\right) \\ \geq\ &\frac{1}{\theta_i r_i^2}\int_{t_0}^{\tilde{t}} \left(z(t_0) - z(0)\right) dH_j^\sigma(\tau) \geq \frac{1}{\theta_i r_i^2}\left(H_j^\sigma(\tilde{t}) - H_j^\sigma(t_0)\right) z(t_0) \\ \geq\ &\frac{1}{\theta_i r_i^2}\frac{1}{2}\left(1 - H_j^\sigma(t_0)\right) z(t_0) \end{aligned}$$

But we have $z(t_0) > e^{-r_i t_0} \min\left((r_i t_0), \frac{(r_i t_0)^2}{2}\right)$, which is true because $e^x(1 - e^{-x} - xe^{-x}) = e^x - 1 - x > \frac{x^2}{2} \geq \min\left(x, \frac{x^2}{2}\right)$. Thus, in this case, the same inequality (B5) holds.

Consider again

$$V_i(t_0) - V_i(\tilde{t}) > -\left(1 - H_j^\sigma(t_0)\right) e^{-r_i t_0}(W_i - L_i) + \left(1 - H_j^\sigma(t_0)\right) e^{-r_i t_0} \frac{1}{\theta_i r_i^2} \min\left(\frac{r_i t_0}{2}, \left(\frac{r_i t_0}{2}\right)^2\right). \tag{B7}$$

Let us show that the right-hand side equals 0. We have, by definition of t_0, $t_0 = \frac{\bar{t}}{2} = 2\sqrt{\theta_i(W_i - L_i)\max\left(\theta_i r_i^2(W_i - L_i), 1\right)}$. Consider two cases. First, if $\theta_i r_i^2(W_i - L_i) \geq 1$, then $t_0 = 2\theta_i r_i(W_i - L_i)$, and furthermore $\frac{r_i t_0}{2} = \theta_i r_i^2(W_i - L_i) \geq 1$, which implies $\frac{r_i t_0}{2} \leq \left(\frac{r_i t_0}{2}\right)^2$. This means that the right-hand side of (B7) equals

$$\begin{aligned}
& \left(1 - H_j^\sigma(t_0)\right) e^{-r_i t_0}\left(-(W_i - L_i) + \frac{1}{\theta_i r_i^2}\frac{r_i t_0}{2}\right) \\
= \quad & \left(1 - H_j^\sigma(t_0)\right) e^{-r_i t_0}\left(-(W_i - L_i) + \frac{1}{\theta_i r_i^2}\theta_i r_i^2(W_i - L_i)\right) = 0.
\end{aligned}$$

Second, if $\theta_i r_i^2(W_i - L_i) < 1$, then $t_0 = 2\sqrt{\theta_i(W_i - L_i)}$, and furthermore $\frac{r_i t_0}{2} = \sqrt{\theta_i r_i^2(W_i - L_i)} < 1$, which implies $\frac{r_i t_0}{2} > \left(\frac{r_i t_0}{2}\right)^2$. In this case, the right-hand side of (B7) equals

$$\begin{aligned}
& \left(1 - H_j^\sigma(t_0)\right) e^{-r_i t_0}\left(-(W_i - L_i) + \frac{1}{\theta_i r_i^2}\left(\frac{r_i t_0}{2}\right)^2\right) \\
= \quad & \left(1 - H_j^\sigma(t_0)\right) e^{-r_i t_0}\left(-(W_i - L_i) + \frac{1}{\theta_i r_i^2}\theta_i r_i^2(W_i - L_i)\right) = 0.
\end{aligned}$$

We have thus proved that $V_i(t_0) - V_i(\tilde{t}) > 0$. But this implies that conceding at $\tilde{t}$ cannot be a best response for player i with type θ_i. This is a contradiction that completes the proof. ■

The following is a simple corollary, which documents the types that must concede before given time t.

Lemma B8 *Take some $t > 0$; then all types $\theta_i \leq \hat{\theta}$, where $\hat{\theta} = \frac{1}{W_i - L_i}\frac{t}{4}\min\left(\frac{t}{4}, \frac{1}{r_i}\right)$, concede on or before time t.*

Proof. Suppose first that $\frac{t}{4} \leq \frac{1}{r_i}$. Then $\hat{\theta} = \frac{1}{W_i - L_i}\frac{t^2}{16}$, and $\hat{\theta} r_i^2(W_i - L_i) = \left(\frac{t r_i}{4}\right)^2 \leq 1$.In that case,

$$\begin{aligned}
4\sqrt{\theta_i(W_i - L_i)\max\left(\theta_i r_i^2(W_i - L_i), 1\right)} \quad & \leq \quad 4\sqrt{\hat{\theta}(W_i - L_i)\max\left(\hat{\theta} r_i^2(W_i - L_i), 1\right)} \\
& = \quad 4\sqrt{\hat{\theta}(W_i - L_i)} = t,
\end{aligned}$$

and thus such type θ concedes no later than time t.

Now suppose $\frac{t}{4} > \frac{1}{r_i}$. Then $\hat{\theta} = \frac{1}{W_i - L_i}\frac{t}{4 r_i}$, and $\hat{\theta} r_i^2(W_i - L_i) = \frac{t r_i}{4} > 1$.In that case,

$$\begin{aligned}
4\sqrt{\theta_i(W_i - L_i)\max\left(\theta_i r_i^2(W_i - L_i), 1\right)} \quad & \leq \quad 4\sqrt{\hat{\theta}(W_i - L_i)\max\left(\hat{\theta} r_i^2(W_i - L_i), 1\right)} \\
& = \quad 4\hat{\theta}(W_i - L_i) r_i = t,
\end{aligned}$$

and thus such type θ concedes no later than time t in this case as well. ■

We now use these results to show that if an opponent does not concede on a certain interval, then the types of the player that concede on this interval have a particular form.

Lemma B9 *Suppose that for some $a < b$, player j concedes on (a, b) with probability 0. Then if player i with type θ_i concedes on this interval with a positive probability, then $\theta_i \in \left(\frac{a}{r_i|L_i|}, \frac{b}{r_i|L_i|}\right)$, so $\lim_{\varepsilon \searrow 0} H_i^\sigma(b - \varepsilon) - H_i^\sigma(a) \leq \exp\left(-\frac{a}{\lambda_i r_i |L_i|}\right) - \exp\left(-\frac{b}{\lambda_i r_i |L_i|}\right)$. Furthermore, if some type θ_i concedes at $t \in (a, b)$, then $\theta_i = \frac{t}{r_i|L_i|}$.*

Proof. Consider the case $L_i = 0$. Suppose, to obtain a contradiction, that some type θ_i concedes at $\tilde{t} \in (a, b)$. By Lemma B3, $H_j^\sigma(\cdot)$ is continuous at $\tilde{t}$. Take $t_0 \in (a, \tilde{t})$ such that $H_j^\sigma(\cdot)$ is continuous at t_0. By Lemma B5, we have

$$V_i(t_0) - V_i(\tilde{t}) = \frac{1}{\theta_i r_i^2}\left(\int_{t_0}^{\tilde{t}} (z(\tau) - z(t_0))\, dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(\tilde{t})\right)\left(z(\tilde{t}) - z(t_0)\right)\right).$$

Since $b < T_j$, we have $H_j^\sigma(\tilde{t}) \leq H_j^\sigma(b) < 1$ by definition of T_j (see Lemma B6). Thus, the latter term is positive, which implies that $V_i(t_0) - V_i(\tilde{t}) > 0$, a contradiction.

Now suppose $L_i < 0$, and some type θ_i concedes at $\tilde{t} \in (a, b)$. Again, H_j^σ is continuous at $\tilde{t}$ and we can pick $t_0 \in (a, \tilde{t})$ such that it is also continuous at t_0. We have, by (B4), that

$$\begin{aligned}
V_i(t_0) - V_i(\tilde{t}) &= \int_{t_0}^{\tilde{t}} \left(e^{-r_i t_0} L_i - e^{-r_i \tau} W_i\right) dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(\tilde{t})\right)\left(e^{-r_i t_0} - e^{-r_i \tilde{t}}\right) L_i \\
&\quad + \frac{1}{\theta_i r_i^2}\left(\int_{t_0}^{\tilde{t}} (z(\tau) - z(t_0))\, dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(\tilde{t})\right)\left(z(\tilde{t}) - z(t_0)\right)\right) \\
&= \left(1 - H_j^\sigma(\tilde{t})\right)\left(\left(e^{-r_i t_0} - e^{-r_i \tilde{t}}\right) L_i + \frac{1}{\theta_i r_i^2}\left(z(\tilde{t}) - z(t_0)\right)\right) \\
&\geq \left(1 - H_j^\sigma(\tilde{t})\right)\left(e^{-r_i t_0} - e^{-r_i \tilde{t}}\right)\left(L_i + \frac{1}{\theta_i r_i^2} r_i a\right),
\end{aligned}$$

where we used Lemma B2 to establish the last inequality. If $\theta_i < \frac{a}{r_i|L_i|}$, then the latter factor satisfies

$$\begin{aligned}
L_i + \frac{1}{\theta_i r_i^2} r_i a &> L_i + |L_i| + \frac{1}{\theta_i r_i^2}\left(-e^{-r_i \tilde{t}} - r_i \tilde{t} e^{-r_i \tilde{t}} + e^{-r_i t_0} + r_i t_0 e^{-r_i t_0}\right) \\
&= \left(e^{-r_i t_0} - e^{-r_i \tilde{t}}\right) L_i + \frac{1}{\theta_i r_i^2}\left(e^{-r_i t_0} - e^{-r_i \tilde{t}}\right) r_i a \\
&> \left(e^{-r_i t_0} - e^{-r_i \tilde{t}}\right)(L_i + |L_i|) = 0,
\end{aligned}$$

so there is a profitable deviation.

Now take t_1 arbitrarily close to $\frac{b+\tilde{t}}{2}$ so that H_j^σ is continuous at t_1; we have

$$\begin{aligned} V_i(t_1) - V_i(\tilde{t}) &= \int_{\tilde{t}}^{t_1} \left(e^{-r_i\tau}W_i - e^{-r_i\tilde{t}}L_i\right) dH_j^\sigma(\tau) - \left(1 - H_j^\sigma(t_1)\right)\left(e^{-r_i\tilde{t}} - e^{-r_it_1}\right) L_i \\ &\quad - \frac{1}{\theta_i r_i^2}\left(\int_{\tilde{t}}^{t_1}\left(z(\tau) - z(\tilde{t})\right) dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(t_1)\right)\left(z(t_1) - z(\tilde{t})\right)\right) \\ &= \left(1 - H_j^\sigma(t_1)\right)\left(-\left(e^{-r_i\tilde{t}} - e^{-r_it_1}\right) L_i - \frac{1}{\theta_i r_i^2}\left(z(t_1) - z(\tilde{t})\right)\right) \\ &\geq \left(1 - H_j^\sigma(t_1)\right)\left(e^{-r_i\tilde{t}} - e^{-r_it_1}\right)\left(-L_i - \frac{1}{\theta_i r_i^2} r_i b\right), \end{aligned}$$

again using Lemma B2. Notice that if $\theta_i > \frac{b}{r_i|L_i|}$, then the latter factor satisfies

$$-L_i - \frac{1}{\theta_i r_i^2} r_i b > -L_i - |L_i| = 0,$$

so again there is a profitable deviation.

Thus, if θ_i concedes at some $\tilde{t} \in (a, b)$, then $\theta_i \in \left(\frac{a}{r_i|L_i|}, \frac{b}{r_i|L_i|}\right)$. Notice that by the same argument, since $\tilde{t} \in \left(\tilde{t} - \varepsilon, \tilde{t} + \varepsilon\right)$ for small ε, it must be that $\theta_i \in \left(\frac{\tilde{t}-\varepsilon}{r_i|L_i|}, \frac{\tilde{t}+\varepsilon}{r_i|L_i|}\right)$, so $\theta_i = \frac{\tilde{t}}{r_i|L_i|}$. Lastly, the share of types $\theta_i \in \left(\frac{a}{r_i|L_i|}, \frac{b}{r_i|L_i|}\right)$ is $\exp\left(-\frac{a}{\lambda_i r_i|L_i|}\right) - \exp\left(-\frac{b}{\lambda_i r_i|L_i|}\right)$, and therefore the share of types who concede on (a, b) cannot be larger than that. This completes the proof. ■

We now have enough results to establish that $H_j^\sigma(\cdot)$ is Lipschitz continuous.

Lemma B10 *On any interval* $0 < a < b < T_j$, $H_j^\sigma(\cdot)$ *satisfies a Lipschitz condition with*

$$K = K_j(a, b) = \max\left(\frac{1}{\lambda_j r_j |L_j|} e^{-\frac{a}{\lambda_j r_j |L_j|}}, \frac{e^{r_i b - 1}}{\frac{a}{4}\min\left(\frac{a r_i}{4}, 1\right)}\right).$$

Proof. Suppose not, then by Lemma B1 there is $q \in (a, b)$ and $\varepsilon \in (0, q - a)$ such that for any $p \in (q - \varepsilon, q)$, $H_j^\sigma(q) - H_j^\sigma(p) > K(q - p)$. Since $p > a$, we have that only types with $\theta_i > \hat{\theta}$, where

$\hat{\theta} = \frac{1}{W_i - L_i}\frac{a}{4}\min\left(\frac{a}{4}, \frac{1}{r_i}\right)$ can concede on or after time p. For such θ_i, we use Lemma B5 to consider

$$\begin{aligned}
V_i(q) - V_i(p) &= \int_p^q \left(e^{-r_i\tau}W_i - e^{-r_ip}L_i\right) dH_j^\sigma(\tau) - \left(1 - H_j^\sigma(q)\right)\left(e^{-r_ip} - e^{-r_iq}\right) L_i \\
&\quad - \frac{1}{\theta_i r_i^2}\left(\int_p^q (z(\tau) - z(p))\, dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(q)\right)(z(q) - z(p))\right) \\
&\geq \left(e^{-r_iq}W_i - e^{-r_ip}L_i\right)\left(H_j^\sigma(q) - H_j^\sigma(p)\right) - \left(1 - H_j^\sigma(q)\right)\left(e^{-r_ip} - e^{-r_iq}\right) L_i \\
&\quad - \frac{1}{\theta_i r_i^2}\left((z(q) - z(p))\left(H_j^\sigma(q) - H_j^\sigma(p)\right) + \left(1 - H_j^\sigma(q)\right)(z(q) - z(p))\right) \\
&= e^{-r_iq}\left(H_j^\sigma(q) - H_j^\sigma(p)\right) W_i - \left(e^{-r_ip}\left(1 - H_j^\sigma(p)\right) L_i + e^{-r_iq}\left(1 - H_j^\sigma(q)\right)\right) L_i \\
&\quad - \frac{1}{\theta_i r_i^2}(z(q) - z(p))\left(1 - H_j^\sigma(p)\right) \\
&\geq e^{-r_iq}\left(H_j^\sigma(q) - H_j^\sigma(p)\right) W_i - \left(e^{-r_iq}\left(1 - H_j^\sigma(p)\right) L_i + e^{-r_iq}\left(1 - H_j^\sigma(q)\right)\right) L_i \\
&\quad - \frac{1}{\theta_i r_i^2}(q - p)\frac{r_i}{e}\left(1 - H_j^\sigma(p)\right) \\
&\geq e^{-r_iq}\left(\left(H_j^\sigma(q) - H_j^\sigma(p)\right)\right)(W_i - L_i) - \frac{1}{\theta_i r_i e}(q - p) \\
&> e^{-r_iq}K(q - p)(W_i - L_i) - \frac{1}{\hat{\theta} r_i e}(q - p) \\
&\geq e^{-r_iq}K(q - p)(W_i - L_i) - \frac{1}{\frac{a}{4}\min\left(\frac{a}{4}, \frac{1}{r_i}\right) r_i e}(q - p)(W_i - L_i) \\
&\geq (q - p)(W_i - L_i)\left(e^{-r_iq}\frac{e^{r_ib-1}}{\frac{a}{4}\min\left(\frac{ar_i}{4}, 1\right)} - \frac{1}{\frac{a}{4}\min\left(\frac{a}{4}, \frac{1}{r_i}\right) r_i e}\right) \geq 0;
\end{aligned}$$

here, we used $q < b$ for the last inequality, and also Lagrange theorem to get $z(q) - z(p) = (q - p)\, x r_i^2 e^{-xr_i}$ for some $x \in (p, q)$, and then $ye^{-y} \leq \frac{1}{e}$ for all y, in particular $y = xr_i$. Thus, no type of player i concedes on (p, q).

On the other hand, since player i does not concede on (p, q), then by Lemma B9 (and using that H_j^σ is continuous on (p, q)), we have

$$\begin{aligned}
H_j^\sigma(q) - H_j^\sigma(p) &\leq \exp\left(-\frac{p}{\lambda_j r_j |L_j|}\right) - \exp\left(-\frac{q}{\lambda_j r_j |L_j|}\right) \\
&\leq \frac{1}{\lambda_j r_j |L_j|}\exp\left(-\frac{p}{\lambda_j r_j |L_j|}\right)(q - p) \leq K(q - p).
\end{aligned}$$

This, however, contradicts the choice of p and q. This contradiction proves that $H_j^\sigma(\cdot)$ is Lipschitz continuous on (a, b) with parameter $K_j(a, b)$. ■

The previous lemma allows us to write the best response mapping in a simple way.

Lemma B11 *In equilibrium strategy profile σ, for type θ_i of player i,*

$$\begin{aligned}
\sigma_i(\theta_i) &\in \arg\max_{t \geq 0} V_i(t; \theta_i) \\
&= \arg\max_{t \geq 0}\left(\int_0^t \left(W_i e^{-r_i\tau} - \frac{z(\tau)}{\theta_i r_i^2}\right) dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(t)\right)\left(e^{-r_it}L_i - \frac{z(t)}{\theta_i r_i^2}\right)\right). \quad \text{(B8)}
\end{aligned}$$

Proof. By Lemma B10, function $H_j^\sigma(\cdot)$ is continuous on $t \in (0, \infty)$. Furthermore, by Lemma B6, it is continuous at 0, and thus it is continuous on $(-\infty, +\infty)$. Thus, the payoff of player i from conceding at time t is given by the expression under the integral in (B8), as follows from Lemma B4, which proves that the optimal strategy must satisfy (B8). ■

We now show that $V_i(t; \theta_i)$ satisfies increasing differences property.

Lemma B12 *$V_i(t; \theta_i)$ satisfies increasing differences property: if $\theta_i' > \theta_i$ and $t' > t$, then $V_i(t'; \theta_i') - V_i(t; \theta_i') > V_i(t'; \theta_i) - V_i(t; \theta_i)$.*

Proof. Consider the difference

$$
\begin{aligned}
& \left(V_i\left(t'; \theta_i'\right) - V_i\left(t; \theta_i'\right)\right) - \left(V_i\left(t'; \theta_i\right) - V_i\left(t; \theta_i\right)\right) \\
= \; & \frac{1}{r_i^2}\left(\frac{1}{\theta_i} - \frac{1}{\theta_i'}\right)\left(\int_0^{t'} z(\tau)\, dH_j^\sigma(\tau) + \left(1 - H_j^\sigma\left(t'\right)\right) z\left(t'\right)\right) \\
& - \frac{1}{r_i^2}\left(\frac{1}{\theta_i} - \frac{1}{\theta_i'}\right)\left(\int_0^{t} z(\tau)\, dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(t)\right) z(t)\right) \\
= \; & \frac{1}{r_i^2}\left(\frac{\theta_i' - \theta_i}{\theta_i'\theta_i}\right)\left(\int_t^{t'} \left(z(\tau) - z(t)\right) dH_j^\sigma(\tau) + \left(1 - H_j^\sigma\left(t'\right)\right)\left(z\left(t'\right) - z(t)\right)\right) > 0,
\end{aligned}
$$

because all the factors are positive. ■

We use the previous Lemma to establish that each player plays monotone strategies.

Lemma B13 *For any θ_i and θ_i', suppose $t \in \sigma_i(\theta_i)$ and $t' \in \sigma_i\left(\theta_i'\right)$. Then $\theta_i' > \theta_i$ implies $t' \geq t$.*

Proof. By Lemma B11, we have $V_i(t; \theta_i) - V_i(t'; \theta_i) \geq 0$. Suppose, to obtain a contradiction, that $t > t'$; then by Lemma B12, $V_i\left(t; \theta_i'\right) - V_i\left(t'; \theta_i'\right) > 0$. However, this contradicts that t' is a best response for type θ_i'. This contradiction completes the proof. ■

We are now able to prove that the intervals of time where either player does not concede coincide for the two players.

Lemma B14 *If player i does not concede on (a, b), so $H_i^\sigma(a) = H_i^\sigma(b)$, then the same is true for player j, so $H_j^\sigma(a) = H_j^\sigma(b)$.*

Proof. Denote $H_i^\sigma(a) = H_i^\sigma(b) = h_i$; since $H_i^\sigma(\cdot)$ is continuous, we can without loss of generality assume that $a = \inf\{t \geq 0 : H_i^\sigma(t) = h_i\}$ and $b = \sup\{t : H_i^\sigma(t) = h_i\}$. In that case, it must be that $a > 0$. Indeed, if $a = 0$, then Lemma B6 would imply that $h_i = H_i^\sigma(0) = 0$, in which case there are no types conceding up until time b, but this would contradict Lemma B7, because $\lim_{\theta_i \to 0} \bar{t}_i(\theta_i) = 0$.

Suppose that the statement is not true, then some type of player j, θ, concedes on (a, b), say at time t_0. If so, by Lemma B9, θ must concede at time $\breve{t}_j(\theta) = r_j |L_j| \theta$, so $t_0 = \breve{t}_j(\theta)$. By

Lemma B13, all types $\theta_j > \theta$ concede at $\sigma_j(\theta_j) \geq t_0$. On the other hand, all types $\theta_j \in \left(\frac{a}{r_j|L_j|}, \theta\right)$ must concede no earlier than time a, and by Lemma B13 they concede no later than t_0. This means, again by Lemma B9, that every $\theta_j \in \left(\frac{a}{r_j|L_j|}, \theta\right)$ concedes at $\breve{t}_j(\theta_j) = r_j|L_j|\theta_j$. Now, Lemma B13 implies that no other type of player j concedes on the interval (a, t_0). From this (and continuity), it follows that on $[a, t_0]$, the distribution of concession times of player j, $H_j^\sigma(\cdot)$, is given by $H_j^\sigma(t) = \Pr\left(\theta_j \leq \frac{t}{r_j|L_j|}\right) = 1 - e^{-\frac{t}{\lambda_j r_j |L_j|}}$.

Plugging this expression for $H_j^\sigma(t)$ into (B8), we see that on $t \in [a, t_0]$, $V_i(t, \theta_i)$ is given by

$$V_i(t, \theta_i) = \int_0^t \left(W_i e^{-r_i\tau} - \frac{1 - e^{-r_i\tau} - r_i\tau e^{-r_i\tau}}{\theta_i r_i^2}\right) \frac{e^{-\frac{\tau}{\lambda_j r_j|L_j|}}}{\lambda_j r_j|L_j|} d\tau + e^{-\frac{t}{\lambda_j r_j|L_j|}} \left(e^{-r_i t} L_i - \frac{1 - e^{-r_i t} - r_i t e^{-r_i t}}{\theta_i r_i^2}\right).$$

Differentiating, we get

$$\begin{aligned} \frac{\partial V_i(t, \theta_i)}{\partial t} &= \left(W_i e^{-r_i t} - \frac{1 - e^{-r_i t} - r_i \tau e^{-r_i t}}{\theta_i r_i^2}\right) \frac{e^{-\frac{t}{\lambda_j r_j|L_j|}}}{\lambda_j r_j|L_j|} \\ &\quad - \frac{e^{-\frac{t}{\lambda_j r_j|L_j|}}}{\lambda_j r_j|L_j|} \left(e^{-r_i t} L_i - \frac{1 - e^{-r_i t} - r_i t e^{-r_i t}}{\theta_i r_i^2}\right) + e^{-\frac{t}{\lambda_j r_j|L_j|}} \left(-r_i e^{-r_i t} L_i - \frac{r_i^2 t e^{-r_i t}}{\theta_i r_i^2}\right) \\ &= (W_i - L_i) \frac{e^{-r_i t} e^{-\frac{t}{\lambda_j r_j|L_j|}}}{\lambda_j r_j|L_j|} + e^{-r_i t} e^{-\frac{t}{\lambda_j r_j|L_j|}} \left(-r_i L_i - \frac{t}{\theta_i}\right). \end{aligned}$$

Now, by definition of a (and the fact that $a > 0$) there is some type of player i, $\theta_i = \theta_i(\varepsilon)$, that concedes at $t \in [a - \varepsilon, a]$ for any $\varepsilon > 0$. Take $\hat{\theta} = \sup_{\varepsilon>0} \theta_i(\varepsilon)$; by Lemma B13, type $\hat{\theta}$ of player i concedes at time $t \geq a$ and, by continuity of $V_i(t, \theta_i)$, conceding at a is also a best response for him, and Lemma B6 then implies that $\hat{\theta} \leq \frac{a}{r_i|L_i|}$. This, however, implies that $-r_i L_i - \frac{t}{\hat{\theta}} = r_i|L_i|\left(1 - \frac{t}{a}\right)$, and thus, since $W_i - L_i > 0$, $\frac{\partial V_i(t, \hat{\theta})}{\partial t} > 0$ at $t = a$. This means, however, that conceding at a is not a best response for type $\hat{\theta}$. This contradiction completes the proof. ■

The next results shows that, in fact, there are no gaps, i.e., no intervals where a player does not concede for any type realization.

Lemma B15 *For any* $0 \leq a < b \leq T_i$, $H_i^\sigma(a) < H_i^\sigma(b)$.

Proof. Suppose not, so there are $a < b$ such that $H_i^\sigma(a) = H_i^\sigma(b) = h_i$. Since $H_i^\sigma(\cdot)$ is continuous, we can without loss of generality assume that $a = \inf\{t > 0 : H_i^\sigma(t) = h_i\}$ and $b = \sup\{t : H_i^\sigma(t) = h_i\}$. By Lemma B14, $H_j^\sigma(a) = H_j^\sigma(b)$, and by the same Lemma B14, $a = \inf\left\{t > 0 : H_j^\sigma(t) = h_j\right\}$ and $b = \sup\left\{t : H_j^\sigma(t) = h_j\right\}$.

Let $\hat{\theta}$ be the infimum of the set of types of player i for which it is a best response to concede at time $t \geq b$ (this set is nonempty by Lemma B6). This latter set is closed by continuity of $V_i(t, \theta_i)$, and thus $\hat{\theta}$ belongs to this set. We must have $\hat{\theta} \leq \frac{a}{r_i|L_i|}$; indeed, if instead $\hat{\theta} > \frac{a}{r_i|L_i|}$, then for all types $\theta \in \left(\frac{a}{r_i|L_i|}, \theta'\right)$, it is not a best response to concede at $t \geq b$, but by Lemma B6 it is not a best response for them to concede at $t \leq a$ either, which means that their best responses lie on (a, b), and given that there is a positive measure of these types, this would contradict the assumption $H_i^\sigma(a) = H_i^\sigma(b)$.

Consider the difference

$$\begin{aligned}
V_i\left(b, \hat{\theta}\right) - V_i\left(a, \hat{\theta}\right) &= \int_a^b \left(e^{-r_i \tau} W_i - e^{-r_i a} L_i\right) dH_j^\sigma(\tau) - \left(1 - H_j^\sigma(b)\right)\left(e^{-r_i a} - e^{-r_i b}\right) L_i \\
&\quad - \frac{1}{\hat{\theta} r_i^2}\left(\int_a^b (z(\tau) - z(a))\, dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(b)\right)(z(b) - z(a))\right) \\
&= \left(1 - H_j^\sigma(b)\right)\left(\left(e^{-r_i a} - e^{-r_i b}\right)|L_i| - \frac{z(b) - z(a)}{\hat{\theta} r_i^2}\right) \\
&= \left(1 - H_j^\sigma(b)\right)\left(e^{-r_i a} - e^{-r_i b}\right)\left(|L_i| - \frac{c}{\hat{\theta} r_i}\right) \\
&< \left(1 - H_j^\sigma(b)\right)\left(e^{-r_i a} - e^{-r_i b}\right)\left(|L_i| - \frac{a}{\hat{\theta} r_i}\right) \leq 0,
\end{aligned}$$

where we used $z(b) - z(a) = \left(e^{-r_i a} - e^{-r_i b}\right) r_i c$ for some $c \in (a, b)$, which is true by (B2). This proves that $V_i\left(a, \hat{\theta}\right) > V_i\left(b, \hat{\theta}\right)$, which implies that b is not a best response of type $\hat{\theta}$, and then some $b' > b$ is a best response. However, then no type of player i would concede on the interval $[b, b')$, which contradicts the assumption $b = \sup\{t : H_i^\sigma(t) = h_i\}$. This contradiction that completes the proof. ■

The previous results implies that the equilibrium must be in pure strategies.

Lemma B16 *For any θ, the set of best responses of player i with type θ is a singleton. In particular, this player plays a pure strategy.*

Proof. Suppose that a, b are in best response set of type θ. Then all players with $\theta_i < \theta$ concede no later than a and all players with $\theta_i > \theta$ concede no earlier than b. This implies $H_i^\sigma(a) = H_i^\sigma(b)$, which, however, contradicts Lemma B15. This completes the proof. ■

Now we can prove that $V_i(t; \theta_i)$ is single-peaked in time t.

Lemma B17 *For any θ_i, $V_i(t; \theta_i)$ is single-peaked in t.*

Proof. Suppose not, then $V_i(t; \theta_i)$ has a local minimum at some t_0. By Lemma B15, there is type θ_0 that concedes at t_0. Consider the case $\theta_0 > \theta_i$. By definition of a local minimum, $V_i(t'; \theta_i) - V_i(t_0; \theta_i) \geq 0$ for some $t' > t_0$. Then $\theta_0 > \theta_i$ implies $V_i(t'; \theta_0) - V_i(t_0; \theta_0) > 0$ by Lemma B12, which contradicts that t_0 is a best response for type θ_0. If, on the other hand, $\theta_0 < \theta_i$, then, since $V_i(t''; \theta_i) - V_i(t_0; \theta_i) \geq 0$ for some $t'' < t_0$, then Lemma B12 implies $V_i(t''; \theta_0) - V_i(t_0; \theta_0) > 0$, which again yields a contradiction.The only remaining case is $\theta_0 = \theta_i$, but then $V_i(t; \theta_i)$ must have both a local maximum and a local minimum at t_0, i.e., it must be locally constant. However, this would violate Lemma B16. This contradiction completes the proof. ■

We use the previous result to show that $H_j^\sigma(\cdot)$ is differentiable.

Lemma B18 *In every equilibrium σ, $H_j^\sigma(\cdot)$ is differentiable at all $t > 0$.*

Proof. Notice that $H_j^\sigma(\cdot)$ is differentiable if and only if $V_i(t;\theta_i)$ is differentiable in t for any θ_i. Indeed, by (B4), we have, for every t,

$$\begin{aligned} V_i(t;\theta_i) &= \int_0^t \left(W_i - \frac{z(\tau)}{\theta_i r_i^2}\right) e^{-r_i\tau} dH_j^\sigma(\tau) + \left(1 - H_j^\sigma(t)\right) e^{-r_i t}\left(L_i - \frac{z(t)}{\theta_i r_i^2}\right) \\ &= e^{-r_i t}\left(L_i - \frac{z(t)}{\theta_i r_i^2}\right) + \int_0^t \left(W_i e^{-r_i\tau} - L_i e^{-r_i t} - \frac{1}{\theta_i r_i^2}\left(e^{-r_i\tau} z(\tau) - e^{-r_i t} z(t)\right)\right) dH_j^\sigma(\tau). \end{aligned}$$

Then differentiability of $H_j^\sigma(\cdot)$ at t implies differentiability of $V_i(t;\theta_i)$ ath that point. The reverse is also true, provided that the expression under the integral (which is a continuous function of τ) does not turn to 0 at $\tau = t$. The latter is true, as for $\tau = t$ it equals $(W_i - L_i)e^{-r_i\tau}$. Thus, to prove that $H_j^\sigma(\cdot)$ is differentiable at some t_0, it suffices to prove that $V_i(t;\theta_0)$ is differentiable at $t = t_0$ for some θ_0, in particular, for type θ_0 that finds it optimal to concede at t_0 (such type exists by Lemma B15).

By our choice of θ_0,$V_i(t;\theta_0)$ achieves a global maximum at t_0. Then if a derivative exists, then it must be 0. Suppose, to obtain a contradiction, that the derivative does not exist, then either left derivative does not exist or is positive, or right derivative does not exist or is negative. Consider the former case (the latter case is similar). Then it must be that there exists some $\varepsilon > 0$ and some sequence $\{\mu_n\}$ of positive numbers that converges to zero such that

$$\frac{V_i(t_0 - \mu_n;\theta_0) - V_i(t_0;\theta_0)}{-\mu_n} \geq \varepsilon.$$

This implies that

$$\frac{V_i(t_0 - \mu_n;\theta) - V_i(t_0;\theta)}{-\mu_n} > 0$$

for $\theta \in (\theta_0 - \delta, \theta_0 + \delta)$ for some $\delta > 0$, where δ may potentially depend on n. However, taking into account (B4) and the fact that $H_j^\sigma(\cdot)$ is Lipschitz-continuous by Lemma B10, one can take δ to be the same for all n. If so, there exists $\theta' < \theta$ for which

$$\frac{V_i\left(t_0 - \mu_n;\theta'\right) - V_i\left(t_0;\theta'\right)}{-\mu_n} > 0$$

for all n, so $V_i\left(t_0;\theta'\right) > V_i\left(t_0 - \mu_n;\theta'\right)$ for all n. Since $V_i\left(t;\theta'\right)$ is single-peaked by Lemma B17, this implies that the time of concession of type θ', denoted by t', satisfies $t' \geq t_0$. On the other hand, since $\theta' < \theta$, then $t' \leq t_0$ by Lemma B13. This implies $t' = t_0$, and therefore, again by Lemma B13, all types $\theta \in \left[\theta', \theta\right]$ concede at the same moment t_0. Consequently, $H_i^\sigma(\cdot)$ is discontinuous at t_0, which violates Lemma B10. This contradiction completes the proof that $H_i^\sigma(\cdot)$ is differentiable. ■

Given differentiability, it is relatively straightforward to show that in equilibrium, certain differential equations must be satisfied.

Lemma B19 *Player i makes optimal decision if and only if the following differential equation is satisfied:*

$$\theta_j'(t) = \frac{\lambda_j}{W_i - L_i}\left(r_i L_i + \frac{t}{\theta_i(t)}\right) \tag{B9}$$

Proof. The optimal decision of player i of type θ is given by (B8). Differentiating and simplifying, we get

$$(W_i - L_i)\frac{dH_j^\sigma(t)}{dt} = \left(1 - H_j^\sigma(t)\right)\left(r_i L_i + \frac{t}{\theta_i}\right).$$

Notice that if at time t, $H_j^\sigma(t)$ types conceded, then $H_j^\sigma(t) = 1 - e^{-\frac{\theta_j(t)}{\lambda_j}}$, where $\theta_j(t)$ is the marginal type that concedes at time t. This implies that $1 - H_j^\sigma(t) = e^{-\frac{\theta_j(t)}{\lambda_j}}$ and $\frac{dH_j^\sigma(t)}{dt} = \frac{1}{\lambda_j}e^{-\frac{\theta_j(t)}{\lambda_j}}\theta_j'(t)$ (the fact that $\theta_j(t)$ is differentiable follows from $\theta_j(t) = -\lambda_j \ln(1 - H_j(t))$; this also implies that its derivative is everywhere has the same sign as $\frac{dH_j^\sigma(t)}{dt}$). Thus, we have

$$(W_i - L_i)\frac{1}{\lambda_j}e^{-\frac{\theta_j(t)}{\lambda_j}}\theta_j'(t) = e^{-\frac{\theta_j(t)}{\lambda_j}}\left(r_i L_i + \frac{t}{\theta_i(t)}\right),$$

which is eqivalent to (B9). This completes the proof. ■

The following Lemma takes the system of equations that must be satisfied in equilibrium, and shows that it has a unique solution in the feasible domain.

Lemma B20 *The system of differential equations*

$$\begin{aligned}\theta_i'(t) &= A\frac{t}{\theta_j(t)} - B;\\ \theta_j'(t) &= C\frac{t}{\theta_i(t)} - D,\end{aligned}$$

where $A, B, C, D > 0$, has a unique solution among functions $(\theta_i(t), \theta_j(t))$ defined on $t \in (0, \infty)$ and such that for all such t, $\theta_i(t) > 0$, $\theta_j(t) > 0$, and θ_i, θ_j is non-decreasing. This solution is given by $\theta_i(t) = \kappa_i t$ and $\theta_j(t) = \kappa_j t$, where κ_i, κ_j are positive constants satisfying

$$\begin{aligned}\kappa_i &= \frac{A}{\kappa_j} - B;\\ \kappa_j &= \frac{C}{\kappa_i} - D.\end{aligned}$$

These constants are equal to:

$$\begin{aligned}\kappa_i &= \frac{C - A - BD + \sqrt{(A + C + BD)^2 - 4AC}}{2d};\\ \kappa_j &= \frac{A - C - BD + \sqrt{(A + C + BD)^2 - 4AC}}{2B}.\end{aligned}$$

Proof. We start by verifying that the proposed solution indeed solves the system of differential equations (this is straightforward).

Conversely, suppose a pair of functions $(\theta_i(t), \theta_j(t))$ satisfies the stated conditions and solves the system of differential equations. Consider pair of functions $\eta_i(t) = \frac{\theta_i(t)}{t}, \eta_j(t) = \frac{\theta_j(t)}{t}$; then this pair satisfies

$$\begin{aligned}
\eta_i'(t)\,t + \eta_i(t) &= \frac{A}{\eta_j(t)} - B; \\
\eta_j'(t)\,t + \eta_j(t) &= \frac{C}{\eta_i(t)} - D;
\end{aligned}$$

the conditions on $\theta_i(\cdot)$ and $\theta_j(\cdot)$ imply $\theta_i(t) > 0$, $\theta_j(t) > 0$, and furthermore, $\eta_i(t) \leq \frac{C}{D}$, $\eta_j(t) \leq \frac{A}{B}$ (the latter conditions follow from $\theta_j'(t) \geq 0$ and $\theta_i'(t) \geq 0$, respectively. Finally, define $\tau = \ln t \in (-\infty, \infty)$, and let $x(\tau) = \eta_i(e^\tau)$; $y(\tau) = \eta_j(e^\tau)$; then the functions $x(\cdot), y(\cdot)$ satisfy

$$\begin{aligned}
x'(\tau) &= -x(\tau) + \frac{A}{y(\tau)} - B; \\
y'(\tau) &= -y(\tau) + \frac{C}{x(\tau)} - D.
\end{aligned}$$

Furthermore, these functions must satisfy $0 < x(\tau) \leq \frac{C}{D}$, $0 < y(\tau) \leq \frac{A}{B}$. It now suffices to prove that this last stationary system of differential equations does not have solutions that lie entirely within the rectangle $R = \left[0, \frac{C}{D}\right] \times \left[0, \frac{A}{B}\right]$ for all $\tau \in (-\infty, +\infty)$ other than the stationary solution $x(\tau) = \kappa_i$, $y(\tau) = \kappa_j$.

It is straightforward to verify that (κ_i, κ_j) is the only critical point lying in the rectangle R (there is another critical point outside this rectangle). Indeed, substituting $\kappa_j = \frac{C}{\kappa_i} - D$ into $\kappa_i = \frac{A}{\kappa_j} - B$, we get the following equation on κ_i:

$$D\kappa_i^2 + (A - C + BD)\,\kappa_i - BC = 0.$$

It therefore has a unique positive root, given by the formula above, whereas the other root is negative. Plugging it back into $\kappa_j = \frac{C}{\kappa_i} - D$, we get the formula for κ_j. Finally, it is straightforward to check that $\kappa_i < \frac{C}{D}$ is satisfied: indeed,

$$\frac{C}{D} - \frac{C - A - BD + \sqrt{(A + C + BD)^2 - 4AC}}{2D} = \frac{C + A + BD - \sqrt{(A + C + BD)^2 - 4AC}}{2D} > 0;$$

similarly, $\kappa_j < \frac{A}{B}$ also holds.

Our next step is to prove that this is a saddle point. To verify the latter claim, consider the linearization of the system around this point:

$$\begin{aligned}
x'(\tau) &= -\kappa_i - (x(\tau) - \kappa_i) + \frac{A\kappa_j}{\kappa_j^2} - \frac{A}{\kappa_j^2}(y(\tau) - \kappa_j) - B + \cdots = -(x(\tau) - \kappa_i) - \frac{A}{\kappa_j^2}(y(\tau) - \kappa_j); \\
y'(\tau) &= -\kappa_j - (y(\tau) - \kappa_j) + \frac{C\kappa_i}{\kappa_i^2} - \frac{C}{\kappa_i^2}(x(\tau) - \kappa_i) - D + \cdots = -\frac{C}{\kappa_i^2}(x(\tau) - \kappa_i) - (y(\tau) - \kappa_j).
\end{aligned}$$

Consider the determinant

$$\begin{vmatrix} -1 & -\frac{A}{\kappa_j^2} \\ -\frac{C}{\kappa_i^2} & -1 \end{vmatrix} = 1 - \frac{AC}{\kappa_i^2 \kappa_j^2} < 0;$$

the latter holds because $\kappa_i = \frac{A}{\kappa_j} - B$ implies $\kappa_i < \frac{A}{\kappa_j}$ and $\kappa_j = \frac{C}{\kappa_i} - D$ implies $\kappa_j < \frac{C}{\kappa_i}$, and multiplying the two inequalities yields $\kappa_i^2 \kappa_j^2 < AC$. This already implies that the critical point is a saddle point (it is straightforward to check that the eigenvalues of the corresponding characteristic polynomial are $-1 \pm \sqrt{\frac{AC}{\kappa_i^2 \kappa_j^2}}$, which thus have different signs.

Now suppose, to obtain a contradiction, that a different solution to the system of differential equation that entirely lies within the rectangle R exists. By Poincaré-Bendixson theorem (applied to the open subset $(0, +\infty) \times (0, +\infty)$), the limit set for the corresponding trajectory at $-\infty$ or $+\infty$ must be either the critical point, or a periodic orbit, or the critical point together with an orbit connecting it to itself (a homoclinic orbit). If it is the critical point (κ_i, κ_j), then the trajectory connects this point to itself, in which case there must be another critical point in the interior of this trajectory, which as we know is not the case. For the same reason the latter case is ruled out. Finally, if the limit is a periodic orbit, then this periodic orbit must include include a critical point in its interior. However, since the only criticaly point is a saddle point, this is impossible, as follows trivially from considering index of the vector field (the periodic orbit must have index $+1$ whereas the saddle point has index -1). This proves that there is no nonstationary solution to the dynamic system lying within R, which proves that the original system of differential equations has a unique solution. ■

From the previous two lemmas, we find that the equilibrium must satisfy a certain system of equations, and vice versa, that the solution to this system indeed corresponds to an equilibrium.

Lemma B21 *There exists a unique equilibrium, given by $t_i(\theta_i) = \frac{\theta_i}{\kappa_i}$, $t_j(\theta_j) = \frac{\theta_j}{\kappa_j}$, where κ_i and κ_j are computed for $A = \frac{\lambda_i}{W_j - L_j}$, $B = -\frac{\lambda_i}{W_j - L_j} r_j L_j$, $C = \frac{\lambda_j}{W_i - L_i}$, $D = -\frac{\lambda_j}{W_i - L_i} r_i L_i$.*

Proof. From Lemma B20 it follows that there are no other solutions that satisfy (B9) for both i and j, and by Lemma B19 no other strategies can be equilibrium. It remains to verify that these strategies are indeed best responses to one another. By symmetry, it suffices to do so for player i only.

For player j, we have $H_j(t) = \Pr\left(\frac{\theta_j}{\kappa_j} \le t\right) = \Pr(\theta_j \le \kappa_j t) = 1 - \exp\left(-\frac{\kappa_j t}{\lambda_j}\right)$. The payoff of player i of type θ_i from conceding at time t is therefore, as follows from (B3),

$$V_i(\theta_i, t) = \int_0^t \left(W_i e^{-r_i \tau} - \frac{z(\tau)}{\theta_i r_i^2}\right) \frac{\kappa_j}{\lambda_j} \exp\left(-\frac{\kappa_j \tau}{\lambda_j}\right) d\tau + \exp\left(-\frac{\kappa_j t}{\lambda_j}\right) \left(e^{-r_i t} L_i - \frac{z(t)}{\theta_i r_i^2}\right).$$

Differentiating, we get

$$\begin{aligned}
\frac{\partial}{\partial t}V_i\left(\theta_i,t\right) &= \left(W_i e^{-r_i t}-\frac{z\left(t\right)}{\theta_i r_i^2}\right)\frac{\kappa_j}{\lambda_j}\exp\left(-\frac{\kappa_j t}{\lambda_j}\right)-\frac{\kappa_j}{\lambda_j}\exp\left(-\frac{\kappa_j t}{\lambda_j}\right)\left(e^{-r_i t}L_i-\frac{z\left(t\right)}{\theta_i r_i^2}\right) \\
&\quad +\exp\left(-\frac{\kappa_j t}{\lambda_j}\right)\left(-r_i e^{-r_i t}L_i-\frac{te^{-r_i t}}{\theta_i}\right) \\
&= \left(\left(W_i-L_i\right)\frac{\kappa_j}{\lambda_j}-r_iL_i-\frac{t}{\theta_i}\right)\exp\left(-\left(\frac{\kappa_j}{\lambda_j}+r_i\right)t\right).
\end{aligned}$$

Notice that $\left(W_i-L_i\right)\frac{\kappa_j}{\lambda_j}-r_iL_i-\frac{t}{\theta_i}$ may be rewritten, in our notation, as $\frac{1}{C}\left(\kappa_j-C\frac{t}{\theta_i}+D\right)$. By definition of κ_i and κ_j, $\kappa_j=\frac{C}{\kappa_i}-D$, and thus $\frac{\partial}{\partial t}V_i\left(\theta_i,t\right)=0$ if and only if $t=\frac{\theta_i}{\kappa_i}$. Thus, there is a unique point on $(0,+\infty)$ where $\frac{\partial}{\partial t}V_i\left(\theta_i,t\right)=0$. Thus, to verify that this point $t=\frac{\theta_i}{\kappa_i}$ is a global maximum it suffices to check the second-order condition. We have

$$\frac{\partial^2}{\partial t^2}V_i\left(\theta_i,t\right)=-\frac{1}{\theta_i}\exp\left(-\left(\frac{\kappa_j}{\lambda_j}+r_i\right)t\right)-\left(\left(W_i-L_i\right)\frac{\kappa_j}{\lambda_j}-r_iL_i-\frac{t}{\theta_i}\right)\left(\frac{\kappa_j}{\lambda_j}+r_i\right)\exp\left(-\left(\frac{\kappa_j}{\lambda_j}+r_i\right)t\right);$$

if $t=\frac{\theta_i}{\kappa_i}$ then the second term vanishes (as follows from $\frac{\partial}{\partial t}V_i\left(\theta_i,t\right)=0$), and therefore $\frac{\partial^2}{\partial t^2}V_i\left(\theta_i,t\right)=-\frac{1}{\theta_i}\exp\left(-\left(\frac{\kappa_j}{\lambda_j}+r_i\right)\frac{\theta_i}{\kappa_i}\right)<0$. This proves that conceding at $\frac{\theta_i}{\kappa_i}$ is a best response for player i. This proves that the pair of functions form an equilibrium. ■

The next lemma establishes comparative statics results for this general model.

Lemma B22 *The rates of concession of the two players ρ_i and ρ_j, are constant over time. The rate of concession of player i, ρ_i, is increasing in W_j, L_i, λ_j, r_j, and is decreasing in W_i, L_j, λ_i, r_i. The converse is true for ρ_j.*

Proof. By Lemma B21, we have $H_i\left(t\right)=1-\exp\left(-\frac{\kappa_i t}{\lambda_i}\right)$ and $H_j\left(t\right)=1-\exp\left(-\frac{\kappa_j t}{\lambda_j}\right)$, thus the rates are constant over time and are given by $\rho_i=\frac{\kappa_i}{\lambda_i}$, $\rho_j=\frac{\kappa_j}{\lambda_j}$. This implies that these two rates may be found as the unique positive solution to the following system of equations:

$$\begin{aligned}
\rho_i &= \frac{1}{W_j-L_j}\left(\frac{1}{\lambda_j\rho_j}+r_jL_j\right); \\
\rho_j &= \frac{1}{W_i-L_i}\left(\frac{1}{\lambda_i\rho_i}+r_iL_i\right).
\end{aligned}$$

We can rewrite them as

$$\begin{aligned}
\left(\rho_i-\frac{r_jL_j}{W_j-L_j}\right)\rho_j &= \frac{1}{\lambda_j}\frac{1}{W_j-L_j}; \\
\rho_i\left(\rho_j-\frac{r_iL_i}{W_i-L_i}\right) &= \frac{1}{\lambda_i}\frac{1}{W_i-L_i}.
\end{aligned}$$

Both equations define hyperbolas on the $\left(\rho_i,\rho_j\right)$ plane that have a unique intersection in the first quadrant. The first hyperbola has asymptotes $\rho_i=\frac{r_jL_j}{W_j-L_j}<0$ and $\rho_j=0$, the second one has asymptotes $\rho_i=0$ and $\rho_j=\frac{r_iL_i}{W_i-L_i}<0$. Denoting the equilibrium values with asterisks, we get that for $0<\rho_i<\rho_i^*$, the first hyperbola lies below the second, and for $\rho_i>\rho_i^*$, the opposite is the case.

Consider the comparative statics with respect to some parameter, say W_j. If it increases, the second hyperbola does not change. For the first one, the value of ρ_i decreases for any given ρ_j, so the first hyperbola is moving left. This implies that ρ_i increases and ρ_j decreases along the first hyperbola. The comparative statics with respect to the other parameters are derived similarly. This completes the proof. ■

We conclude the analysis by calculating the expected payoffs of the players. We start by computing payoff of a player after he learns his type.

Lemma B23 *The expected payoff of player i of type θ_i from terminating at time t, if player j concedes at rate ρ_j is given by*

$$\begin{aligned} V_i(t) &= W_i \frac{\rho_j}{\rho_j + r_i}\left(1 - e^{-(\rho_j + r_i)t}\right) + L_i e^{-(\rho_j + r_i)t} \\ &\quad + \frac{1}{\theta_i (\rho_j + r_i)^2}\left(-1 + \left(1 + (\rho_j + r_i) t\right) e^{-(\rho_j + r_i)t}\right). \end{aligned} \tag{B10}$$

It is strictly increasing in ρ_j for any fixed $t > 0$.

Proof. We have $H_j(t) = 1 - \exp(-\rho_j t)$. In this case, integrating (B3) yields

$$\begin{aligned} V_i(t) &= \int_0^t \left(W_i e^{-r_i \tau} - \frac{1 - e^{-r_i\tau} - r_i \tau e^{-r_i \tau}}{\theta_i r_i^2}\right) \rho_j e^{-\rho_j \tau} d\tau + e^{-\rho_j t}\left(L_i e^{-r_i t} - \frac{1 - e^{-r_i t} - r_i t e^{-r_i t}}{\theta_i r_i^2}\right) \\ &= W_i \frac{\rho_j}{\rho_j + r_i}\left(1 - e^{-(\rho_j + r_i)t}\right) - \frac{1}{\theta_i r_i^2}\left(1 - e^{-\rho_j t}\right) + \frac{1}{\theta_i r_i^2}\frac{\rho_j}{\rho_j + r_i}\left(1 - e^{-(\rho_j + r_i)t}\right) \\ &\quad + \frac{1}{\theta_i r_i^2}\frac{r_i \rho_j}{(\rho_j + r_i)^2}\left(1 - \left(1 + r_i t + \rho_j t\right) e^{-(\rho_j + r_i)t}\right) + e^{-\rho_j t}\left(L_i e^{-r_i t} - \frac{1 - e^{-r_i t} - r_i t e^{-r_i t}}{\theta_i r_i^2}\right) \\ &= W_i \frac{\rho_j}{\rho_j + r_i}\left(1 - e^{-(\rho_j + r_i)t}\right) + L_i e^{-(\rho_j + r_i)t} - \frac{1}{\theta_i r_i^2} + \frac{1}{\theta_i r_i^2}\left(\frac{\rho_j (\rho_j + 2 r_i)}{(\rho_j + r_i)^2} + \frac{r_i^2 \left(1 + (\rho_j + r_i) t\right)}{(\rho_j + r_i)^2} e^{-(\rho_j + r_i)t}\right) \\ &= W_i \frac{\rho_j}{\rho_j + r_i}\left(1 - e^{-(\rho_j + r_i)t}\right) + L_i e^{-(\rho_j + r_i)t} + \frac{1}{\theta_i (\rho_j + r_i)^2}\left(-1 + \left(1 + (\rho_j + r_i) t\right) e^{-(\rho_j + r_i)t}\right). \end{aligned}$$

Notice that the first term is increasing in ρ_j (both factors are increasing in ρ_j and are positive). The second term is negative, but is decreasing in ρ_j in absolute terms, and thus is increasing. Finally, if we denote $x = (\rho_j + r_i) t$, the last term equals $\frac{t^2}{\theta_i x^2}\left(-1 + (1 + x) e^{-x}\right)$. We have

$$\frac{d}{dx}\frac{(1 + x) e^{-x} - 1}{x^2} = \frac{2 - \left(x^2 + 2x + 2\right) e^{-x}}{x^3},$$

which is positive for $x > 0$, because the numerator equals 0 at $x = 0$ and is increasing in x:

$$\frac{d}{dx}\left(2 - \left(x^2 + 2x + 2\right) e^{-x}\right) = x^2 e^{-x} > 0.$$

Thus, the last term is increasing in x, and since x is increasing in ρ_j, it is also increasing in ρ_j. ■

Lastly, we compute the expected payoff of a player before he learns his type.

Lemma B24 *The expected payoff of player i in equilibrium where players i and j concede at rates ρ_i and ρ_j, respectively, is given by*

$$\bar{V}_i = W_i \frac{\rho_j}{\rho_i + \rho_j + r_i} + L_i \frac{\rho_i}{\rho_i + \rho_j + r_i} + \frac{1 - \left(1 + \frac{\rho_i}{\rho_j + r_i}\right) \ln\left(1 + \frac{\rho_j + r_i}{\rho_i}\right)}{\left(\rho_j + r_i\right)\left(\rho_i + \rho_j + r_i\right)\lambda_i}. \tag{B11}$$

It is strictly increasing in ρ_j for any fixed $t > 0$.

Proof. In equilibrium, player i concedes at optimal time given by $t = \frac{1}{\rho_i \lambda_i}\theta_i$. Plugging this into (B10), we get

$$V_i(t) = W_i \frac{\rho_j}{\rho_j + r_i}\left(1 - e^{-\frac{\rho_j + r_i}{\rho_i \lambda_i}\theta_i}\right) + L_i e^{-\frac{\rho_j + r_i}{\rho_i \lambda_i}\theta_i} - \frac{1 - e^{-\frac{\rho_j + r_i}{\rho_i \lambda_i}\theta_i}}{\theta_i\left(\rho_j + r_i\right)^2} + \frac{e^{-\frac{\rho_j + r_i}{\rho_i \lambda_i}\theta_i}}{\rho_i \lambda_i \left(\rho_j + r_i\right)}.$$

To obtain $\bar{V}_i$, we need to integrate $V_i(t)$ over θ_i, distributed with density $\frac{1}{\lambda_i} e^{-\frac{\theta_i}{\lambda_i}}$. We integrate each term separately. We have

$$\int_0^\infty e^{-\frac{\rho_j + r_i}{\rho_i \lambda_i}\theta_i} \frac{1}{\lambda_i} e^{-\frac{\theta_i}{\lambda_i}} d\theta_i = \frac{1}{\lambda_i} \int_0^\infty e^{-\left(\frac{\rho_j + r_i}{\rho_i} + 1\right)\frac{\theta_i}{\lambda_i}} d\theta_i = \frac{\rho_i}{\rho_i + \rho_j + r}.$$

Thus,

$$\int_0^\infty W_i \frac{\rho_j}{\rho_j + r_i}\left(1 - e^{-\frac{\rho_j + r_i}{\rho_i \lambda_i}\theta_i}\right) \frac{1}{\lambda_i} e^{-\frac{\theta_i}{\lambda_i}} d\theta_i = W_i \left(\frac{\rho_j}{\rho_j + r_i}\left(1 - \frac{\rho_i}{\rho_i + \rho_j + r}\right)\right) = W_i \frac{\rho_j}{\rho_i + \rho_j + r}$$

and,

$$\int_0^\infty L_i e^{-\frac{\rho_j + r_i}{\rho_i \lambda_i}\theta_i} \frac{1}{\lambda_i} e^{-\frac{\theta_i}{\lambda_i}} d\theta_i = L_i \frac{\rho_i}{\rho_i + \rho_j + r}.$$

To compute the third term, consider the following calculation, which is valid whenever $\rho_j + r_i < \rho_i$, so the series is converging:

$$\begin{aligned}
& \int_0^{+\infty} \frac{1 - e^{-\left(\rho_j + r_i\right)\frac{\theta_i}{\rho_i \lambda_i}}}{\theta_i} e^{-\frac{\theta_i}{\lambda_i}} d\theta_i \\
= & \int_0^{+\infty} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\left(\left(\rho_j + r_i\right)\frac{\theta_i}{\rho_i \lambda_i}\right)^n}{n!\theta_i} e^{-\frac{\theta_i}{\lambda_i}} d\theta_i \\
= & \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{\rho_j + r_i}{\rho_i \lambda_i}\right)^{m+1}}{(m+1)!} \int_0^{+\infty} \theta_i^m e^{-\frac{\theta_i}{\lambda_i}} d\theta_i \\
= & \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{\rho_j + r_i}{\rho_i}\right)^{m+1}}{(m+1)!} \int_0^{+\infty} x^m e^{-x} dx \\
= & \sum_{m=0}^{\infty} \frac{(-1)^m \left(\frac{\rho_j + r_i}{\rho_i}\right)^{m+1}}{m+1} \\
= & \sum_{n=1}^{\infty} \frac{(-1)^{n+1}\left(\frac{\rho_j + r_i}{\rho_i}\right)^n}{n} = \ln\left(1 + \frac{\rho_j + r_i}{\rho_i}\right).
\end{aligned}$$

Since both the original expression and the last term are analytical functions of ρ_i on $(0, +\infty)$, they must coincide for all values of ρ_i (not necessarily satisfying $\rho_j + r_i < \rho_i$). Thus, the third term equals $\frac{-1}{\lambda_i(\rho_j+r_i)^2} \ln\left(1 + \frac{\rho_j+r_i}{\rho_i}\right)$. Finally, the last term equals

$$\int_0^{+\infty} \frac{e^{-\left(\frac{r_j+r_i}{\rho_i}+1\right)\frac{\theta_i}{\lambda_i}}}{\rho_i \lambda_i^2 \left(\rho_j + r_i\right)} d\theta_i = \frac{\lambda_i \frac{\rho_i}{\rho_i+\rho_j+r_i}}{\rho_i \lambda_i^2 \left(\rho_j + r_i\right)} = \frac{1}{\lambda_i \left(\rho_j + r_i\right) \left(\rho_i + \rho_j + r_i\right)}.$$

Summing all the terms yields the desired expression.

Finally, the fact that $\bar{V}_i$ is decreasing in ρ_j is easiest to see from the fact that this was true for each realization of θ_i and $t = \frac{1}{\rho_i \lambda_i} \theta_i$ (but this may be proven directly). ■