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Energy Intensive Infrastructure Investments with Retrofits in Continuous Time: Effects of Uncertainty on Energy Use and Carbon Emissions[1]

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Energy Intensive Infrastructure Investments with Retrofits in Continuous Time: Effects of Uncertainty on Energy Use and Carbon Emissions

1.   Introduction

Large and energy-intensive infrastructure poses a serious concern for climate policy. Some of it, both supply-side (power plants) and demand-side based (urban structure and transport systems), is very long-lasting (up to 100 years or more). Other shorter-lasting infrastructure, with still persistent effects on emissions, includes motor vehicles (fossil-fuel versus electric or renewable-powered), household appliances, and home heating and cooling systems. Such capital gives rise to more than half of total greenhouse gas (GHG) emissions from fossil fuels in high-income countries.[3] Importantly also, in many emerging economies the rates of such investments, and planned rates over the next 20–30 years, are very high. A basic dilemma is that "wrong" decisions about such investments could tie up inefficiently high greenhouse gas (GHG) emissions levels for long future periods. One consequence is to make it difficult to later reach ambitious climate policy targets.

Retrofits can in some cases help to alleviate this problem. Coal-fired power plants might (perhaps soon) be retrofitted with carbon capture and sequestration technologies; or possibly be modified to instead run on renewable, non-fossil, fuels. Urban areas which depend mainly on private transport might be retrofitted by adding major public transport systems.

This paper considers an abstract or generic infrastructure investment that can, at investment time, be made more or less energy intensive, and where emissions are assumed to stay constant until it is retrofitted. Upon a retrofit, the infrastructure is purged of all of its emissions (and possibly, energy use), while its basic services remain unaltered.

The two key issues explored in this paper, are 1) the optimal energy and emissions intensity of the initial infrastructure, and 2) the optimal retrofit policy (when, if ever, should the infrastructure be retrofitted). We assume that energy (including climate and other environmental) costs are uncertain and follow a geometric Brownian motion process with constant positive drift. The retrofit cost is known, and constant. Our solution for optimal

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[3] See e.g. Davis et al (2010); Ramaswami (2013)

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timing of a retrofit for given initial infrastructure reproduces results from Pindyck (2000).[4] The main novel result in our paper is to derive the initial infrastructure investment decision, simultaneously with the future retrofit decision. We are also the first to directly derive implications for accumulated carbon emissions over the infrastructure's lifetime.

Volatility – the variance on the random component of the process for marginal GHG emission cost – here plays a major role. Our main messages are simple: Increased volatility of climate damages from given emissions, when future retrofit is an option, implies – under certain plausible parametric assumptions specified below – that (a) retrofits will be carried out when marginal climate damage has reached a higher level and thus, in expectation, later (strictly later except in the event when it never happens); and (b) the initial infrastructure will be chosen with higher energy and emissions intensity.

These results have a simple intuitive explanation. First, retrofits occur later when volatility is higher since higher volatility increases the option value of waiting to retrofit. This principle is well known from e.g. Dixit and Pindyck's (1994) analysis of investment decisions under uncertainty, and from Pindyck's (2000, 2002) analysis of similar climate-related investments. It is due to the asymmetric effect of increased volatility when retrofit is an option, which is exercised only when the climate cost is high. Since one can avoid "bad" outcomes by retrofitting, the better prospects for "good" outcomes that follow from higher volatility increases the benefit from "not yet" having carried out the retrofit. Secondly, higher volatility also increases the expected net utility of the infrastructure investment (not only the option value component), by reducing the expected value of actually realized damages, when (as here) "bad" outcomes are avoided by either retrofitting or (in a worst case, when the retrofit cost is also high) abandoning the infrastructure. For a wide, and plausible, parametric class we show that higher volatility leads to a higher energy intensity of the initially chosen investment, which, in view of above, is not surprising. In the same way as for retrofits, the effect of higher volatility on future environmental cost is asymmetric by giving "good" outcomes (with low climate damage) relatively greater weight as "bad" outcomes can be avoided by retrofitting later. This makes a more energy-intensive infrastructure optimal.

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[4] In a follow-up paper Pindyck (2002) generalizes the analysis in some other directions (to simultaneously uncertain climate impact, and uncertain damage due to given climate impact). See also the review by Pindyck (2007). Balikcioglu, Fackler and Pindyck (2011) and Framstad (2011) rectify errors in the original Pindyck (2000, 2002) presentations.

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Our solution is socially optimal given that decision makers face globally correct energy, emissions and retrofit costs.[5] More typically, however, decision makers face too low energy and GHG emissions costs, usually due to either general energy subsidies, or insufficient environmental taxes. They often also face too high retrofit costs, as least-cost technologies are not available in many countries. More climate action than that initiated by private agents and planners is then desirable. These issues are elaborated in the final section 3.

Combined infrastructure investment and retrofit decisions have been studied by Strand (2011), Strand and Miller (2010) and Strand, Miller and Siddiqui (2014), albeit in simpler, discrete (two-period), models. Lecocq and Shalizi (2014) provide a more descriptive (less technical) presentation. Anas and Timilsina (2009) simulate infrastructure investments in roads for Beijing, finding that more road investments make the chosen residential pattern more dispersed, and later investments in mass transport (retrofit in our terminology) less valuable or more expensive. Vogt-Schilb, Meunier and Hallegatte (2012) and Vogt-Schilb, Hallegatte and Gouvello (2014) discuss timing of sectoral abatement policies within similar models, and where long-lasting effects of particular abatement investments can vary between sectors. They show that marginal sectoral abatement costs should differ by sector, with more "rigid" sectors investing relatively more in early abatement; and that uncertainty can lead to option values whereby energy-saving investment is scaled down initially. Our analysis supports the basic conclusions from most of this literature; but goes farther by providing more precise arguments and analytical results. Results from other contributions, including Meunier and Finon (2013), and in Ha Duong, Grubb and Hourcade (1997), however differ from ours; here inertia and learning spillover effects (not considered in our framework) are shown to overturn the option value induced incentive to wait, so that early action is instead spurred.

Let us outline the rest of the paper. In subsection 2.1 we introduce the model and its basic assumptions; alternative assumptions are discussed in Appendix A. Subsection 2.2 considers the retrofit decision for given initial infrastructure investment, in Proposition 1; for details, see Appendix B.1. Subsection 2.3 derives the initial infrastructure choice (in Proposition 2; Proposition A in Appendix A.3 considers the same optimization problem under an alternative assumption), and effects of increased retrofit cost (in Proposition 3). Subsection 2.4 considers the effect of increasing volatility on optimal decision rules, optimized project value, and climate damage (Proposition 4). Section 3 concludes. Appendix B1 derives the main results

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[5] Certain other assumptions are required for this result, in particular, that certain convexity conditions on choice and production sets are fulfilled; and that all economic actors behave competitively (are price takers).

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under a more general stochastic process; while Appendix B.2 reviews distributional properties of our basic stochastic process. Results concerning impacts on the probability distributions of time to retrofit and stock and running damage rate at retrofit time, are found in Appendix B.3 (Proposition B). Appendix C provides additional proofs.

2. Climate Damage in a Stochastic Dynamic Model

2.1. Basics

Assumptions (A)-(C) are assumed to hold for most of the analysis:

(A) The infrastructure is established at a given point of time $t = 0$.

(B) The infrastructure is operated forever. After infrastructure establishment, the only policy choice is a later retrofit.

(C) A retrofit eliminates emissions forever from the time it is implemented. A retrofit can be carried out at any point of time after establishment of the infrastructure, but only once.

Several policies can be applied to affect ex post fossil energy consumption, and GHG emissions, related to an already established infrastructure:[6]

(I) Fossil energy use of the infrastructure is eliminated upon "retrofitting". An example of such a case is where the initial fossil energy is replaced by renewable energy sources with very low ex post marginal production cost (which might include hydro, solar or wind); or some new energy source supplied in unlimited amount (such as nuclear fusion). We then need only to be concerned with one set of prices or costs, namely the combined energy and emissions cost, from the start of the project and until a retrofit takes place.

(II) Fossil energy use is not eliminated, but GHG emissions are eliminated upon retrofitting. This might happen when CCS technology is applied to existing power plants; or fossil fuels replaced by renewable energy with no net emissions load.

(III) The infrastructure can be closed down or abandoned. All (energy and environmental) costs are then eliminated, and the infrastructure provides no further utility.

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[6] A further type of policy to deal with the impacts of GHG emissions, not discussed further here, is to lower impacts directly (through "adaptation"); see Strand (2014a) for a related analysis.

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In the following we will ignore case (III); cf. assumption (B). When invoking case (II), we simplify by assuming that future energy costs are deterministic and given (and independent of whether a retrofit occurs or not, thus not affecting the retrofit decision).

The climate damage rate is given by $q_t M_t$, where the environmental cost parameter q follows an exogenous stochastic process, and M is the GHG stock level at time t, affected by the emissions rate from the infrastructure in question here, E_t; which from assumption (C) will be constant $= E_0$ until time t of retrofit, and zero from then on. The cost of retrofit is a nonnegative, increasing function K of E_0. The infrastructure has a potentially infinite lifetime. The gross discounted utility V from the services it provides, from investment time until infinity, is assumed given. Net social value to be maximized, over initial technology and time of retrofit implementation, is then given by

$$V - f(E_0) - \mathsf{E}\left[\int_0^\infty e^{-rt} q_t M_t dt + e^{-rt} K(E_0)\right] \tag{1}$$

where $\mathsf{E}[\,\cdot\,]$ – upright, sans serif font – is the expectation operator and the discount rate r is constant and > 0. The infrastructure investment cost f is assumed to satisfy $f' < 0 < f''$: investment costs are lower for infrastructure requiring a high ex post energy consumption level, whilemarginal investment cost savings are reduced when energy consumption rises.[7]

We shall assume that the pollutant stock M evolves linearly, as

$$dM_t / dt = E_t - d \cdot M_t, \qquad \text{starting at } M_0 \geq 0 \tag{2}$$

where $d\,(> 0)$ denotes a constant rate of decay of GHGs, assuming $0 < M_t < \max_t E_t / d$, where E_t is the emission rate at time t; we shall assume that E_0 is chosen initially, while by assumption (C), $E_t = E_0$ up to retrofit time t, and zero from then on. Under the linear dynamics, M could be interpreted as this project's contribution to the GHG stock as well as the overall stock itself; we shall exploit the fact that the dependence upon initial stock splits out linearly, and refer to this as "everyone else's contribution". While linearity is a crucial feature, the assumption that there is no uncertainty in the dynamics of M apart from through the E_t process is made for simplicity and admits wide generalizations (see Framstad (2014)).

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[7] The infrastructure cost function will need to take this form: considering the set of cost-minimizing infrastructure projects that all yield the same utility, it must be the case that economically relevant projects, that are less energy intensive (and thus has lower current energy cost), must have a higher establishment cost f.

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2.2. Optimal Retrofit for Given Investment

We shall throughout the analysis assume that the process q is a geometric Brownian motion with drift $a \in (0, r)$ and volatility $s^2 > 0$:

$$q_t = q_0 \cdot e^{(a - \frac{1}{2}s^2)t + sZ_t}, \qquad (3)$$

as e.g. in Pindyck (2000, 2002). Z is a standard Brownian motion. The evolution of q can also be expressed via the stochastic differential equation $dq_t = q_t \cdot (a\, dt + s\, dZ_t)$. The marginal climate damage is independent of this project's emissions, as $q_t M_t$, is linear in M with exogenous q, and the differential equation for M is linear.[8] It turns out that the optimal retrofit rule is to wait for a sufficiently high state q^* that does not depend on M; this result is not specific to the geometric Brownian model, but follows much more generally, see Appendix B.1 or Framstad (2014).

We will need the following properties of the geometric Brownian motion (see Appendix B). Its expected value is $q_0 e^{at}$, and since $s \neq 0$, the process will with positive probability hit any given positive level $\hat{q}$. Assume $\hat{q} > q_0$ and denote the first hitting time by $\hat{t}$. The power form $E[e^{-r\hat{t}}] = (q_0 / \hat{q})^g$ is valid for any $r > 0$, where the exponent is the positive zero of $\frac{1}{2}s^2 g(g-1) + ag - r$, namely

$$g = \frac{1}{2} - \frac{a}{s^2} + \sqrt{\left(\frac{a}{s^2} - \frac{1}{2}\right)^2 + \frac{2r}{s^2}} \qquad (4)$$

Given $r > a > 0$, we have $1 < g < g_0 := r / a$; g is strictly decreasing wrt. s^2, where g_0 and 1 are the limiting values as $s^2 \to 0$, resp. $+\infty$.

A similar model has already been studied by Pindyck (2000), who shows that the optimal retrofit strategy for given investment is characterized by the threshold value

$$q^* = \frac{g}{g-1}(r - a)(r + d - a) \cdot \frac{K(E)}{E} \qquad (5)$$

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[8] Linearity of climate damage wrt. M – jointly with the assumption of exogenous q – also follows as an approximation if the project is small relative to global emissions.

which optimally trades off the damage reduction against the discounting of the retrofit cost. This form is derived in Appendix B.1 as a first-order condition for this trade-off. We find it convenient to work with the quantities r, G and Q defined by

$$r = \frac{q_0}{q^*}, \quad G(r, g) = \frac{gr - r^g}{g - 1} \quad \text{and} \quad Q(E) = \frac{r^g}{g - 1} K(E) \tag{6}$$

$$\text{so that} \quad Q(E) = \frac{(g-1)^{g-1}}{g^g} \left(\frac{q_0}{(r-a)(r+d-a)} \right)^g \cdot \left(\frac{E}{K(E)} \right)^{g-1} E \tag{7}$$

r is the ratio (the convenient metric for a geometric process) of q_0 to the threshold value. The following key result from Pindyck (2000) exhibits $Q(E_0)$ as – given $q_0 \leq q^*$ – the value of the option to retrofit and $G \cdot K(E_0) + Q(E_0)$ as the climate-related cost incurred when operating forever without retrofitting:

Proposition 1: Consider maximizing (1) with respect to a single retrofit time, t, at which E_t changes from $E_0 > 0$ to 0, with laws of motion for M and q given by (2) and (3), respectively, and $a < r$. Then the optimal retrofit-time t^* is the first time – if ever – when q_t hits q^* given by (5); if q_0 exceeds q^*, retrofit immediately. The maximal value of (1) is the sum of $-q_0 M_0 / (r + d - a)$ (which would incur even if the investment were never made), and $W(E_0)$ given by

$$W(E_0) = V - f(E_0) - G(\min\{1, r\}, g) K(E_0)$$
$$= V - f(E_0) - \begin{cases} \dfrac{q_0}{(r-a)(r+d-a)} E_0 - Q(E_0) & \text{if } q_0 \leq q^* \\ K(E_0) & \text{otherwise} \end{cases} \tag{8}$$

Furthermore, the expected discounted climate damage, D, turns out as (see Appendix B):

$$D = \frac{g}{g-1}(r - r^g)K(E_0) = \frac{q_0 E_0}{(r-a)(r+d-a)} \cdot (1 - r^{g-1}) \tag{9}$$

Given $r > a > 0$, the solution in Proposition 1 converges to the deterministic case as $s \to 0$, with $g \to g_0 = r / a$ and $g / (g - 1) \to r / (r - a)$. A higher s^2 results in a lower g (closer to unity), so that $g / (g - 1)$ increases, and q^* increases.

Greater uncertainty raises the current damage rate required for mitigation action, as the option value of waiting to retrofit (or closedown) then increases. Intuitively, greater uncertainty leads to more states of the world where damages are reduced (and/or reduced by more), which makes more advantageous to wait. However, as we shall see in section 2.4,

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greater uncertainty does not necessarily increase expected environmental damage (9) for given E_0.

2.3. The Optimal Initial Emission Intensity

We can now derive the optimal initial emission intensity of infrastructure, E_*, assuming that the retrofit decision is optimal (from subsection 2.2). Let us use subscript asterisk to denote a quantity optimized at initial time, e.g. r_* denotes r with the optimal E_* inserted. Denote by L the elasticity of K with respect to E, i.e.:

$$L(E) := \mathcal{E}l \, K(E) = \frac{E \cdot K'(E)}{K(E)} \tag{10}$$

We maximize welfare (8) with respect to E_0, which then affects the trigger q^* except in the proportional cost case. Differentiating W in (8) yields the first-order condition

$$Q'(E_*) - f'(E_*) = \frac{q_0}{(r-a)(r+d-a)} \tag{11}$$

(valid as long as $q_0 \leq q^*$). Using

$$\frac{dr}{dE} = -\frac{L(E)-1}{E} r \tag{12}$$

we can write $Q'(E)$ in terms of L as

$$Q'(E) = r^{g-1} \cdot \frac{q_0}{(r-a)(r+d-a)} - r^g \cdot K'(E) = r^g \frac{K(E)}{E}[\frac{g}{g-1} - L(E)] \tag{13}$$

Notice that the elasticity L could get so high that (K so convex that) the value of the retrofit option decreases with emissions level. The threshold $g/(g-1)$ is precisely when, at the margin, the benefit of eliminating emissions is balanced by the increasing cost. We have the following two forms of (11) exhibiting the effects of the two particular elasticity values $L = 1$ vs $L = g/(g-1)$:

$$-f'(E_*) = \frac{q_0}{(r-a)(r+d-a)} - [\frac{g}{g-1} - L(E_*)] \cdot \frac{K(E_*)}{E_*} r_*^g = \left(G(r_*, g) + [L(E_*) - 1] \cdot r_*^g\right) \cdot \frac{K(E_*)}{E_*} \tag{14}$$

where G is given by (6). The second-order condition is more complicated in the general case of non-linear K function. We find for $q_0 < q^*$ that:

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$$Q''(E_*) = r_*^g \cdot \left(\frac{gK(E_*)}{E_*^2}[L(E_*) - 1]^2 - K''(E_*) \right) \tag{15}$$

which should be negative to ensure a concave relationship between E and W (without having to impose stronger conditions on f than convexity). We summarize:

Proposition 2: Consider the optimal stopping problem from Proposition 1, with continuously differentiable cost functions K (strictly increasing) and f (strictly decreasing, strictly convex and with $f(\infty) \geq 0$). Given that an optimal initial E_* exists, it depends only on parameters through g and $q_0 / (r - a)(r + d - a)$. Suppose furthermore that $Q''(E_*)$ (from (15)) is nonpositive. Then W is strictly concave, and E_* is either zero, or uniquely characterized by (14), or such that immediate retrofit is optimal.

For $Q'' \leq 0$ it is necessary, but not sufficient, that $K'' \geq 0$. The following example illustrates why the condition

$$1 \leq L(E_*) \leq \frac{g}{g - 1} \tag{16}$$

will show up in many of our results. Consider power functions $K(E) = kE^L$ for various constant L, so that (15) has same sign as $(L - 1)(L - g / (g - 1))$. If $L \in (1, g / (g - 1))$, then from the first-order condition, the presence of the retrofit option will increase the chosen E_*, and the second-order condition holds under the assumption of strictly convex f. If L takes one of the endpoint values of this interval, then Q' is constant in E and we can solve for E_* by inverting f'. At larger elasticity we face the following properties: from the first-order condition, the presence of the option reduces E_*, as it is then much cheaper (per unit of purged emissions) to retrofit a less polluting infrastructure[9]; however, there is no guarantee that the second-order condition holds, as Q'' is positive and tends to $+\infty$ as $E_* \searrow 0$. However, at low enough E_* we will have $q^* = q_0$, i.e. immediate retrofit, in which case the model does arguably lose validity. To find how E_* changes with the level of retrofit cost, defined by $K(E) = k \cdot J(E)$, and consider changes in the scaling k. Since K and J have the same elasticity L, the first-order condition for E_* then takes the following form, cf. (14):

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[9] Pindyck (2000) shows how convex K could lead to gradual reduction in E through successive partial retrofits, a possibility which is however here ruled out.

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$$-f'(E_*) = \frac{q_0}{(r-a)(r+d-a)} \cdot \{1 - \frac{g-1}{g} r_*^{g-1} [\frac{g}{g-1} - L(E_*)]\} \tag{17}$$

where E_* depends on k only through r_*; at the particular elasticity $L = g/(g-1)$, the dependence degenerates completely. Straightforward implicit differentiation with respect to k yields the following comparative statics, and proof is omitted:

Proposition 3 (effect of retrofit cost on the optimized E_* and D_*): Suppose that Proposition 2 applies, the retrofit cost takes the form $K(E) = k \cdot J(E)$, and $W'(E_*) = 0 < W''(E_*)$. Then, assuming sufficient differentiability,

$$\frac{dE_*}{dk} = -r_*^g \cdot \frac{(g-1) \cdot J(E_*)}{-W''(E_*) \cdot E_*} \cdot [\frac{g}{g-1} - L(E_*)] \tag{18}$$

and

$$\mathcal{E}\ell_k E_* = -r_*^g \cdot \frac{(g-1) \cdot K(E_*)}{-W''(E_*)} \cdot [\frac{g}{g-1} - L(E_*)] \tag{19}$$

both of which are negative iff $L < g/(g-1)$, in which case also (19) tends to 0 as k increases, provided that $W''(0^+) < 0$. For the effect of k on D_*, we find

$$\begin{aligned} \frac{(r-a)(r+d-a)}{q} \cdot \frac{dD_*}{dk} &= \frac{dE_*}{dk} \cdot (1 - r_*^{g-1}) + \frac{d\ln(1/r_*)}{dk} \cdot r_*^{g-1}(g-1)E_* \\ &= \{1 - r_*^{g-1} + (g-1)r_*^{g-1}[L(E_*) - 1]\} \frac{dE_*}{dk} + r_*^{g-1}(g-1)\frac{E_*}{k} \end{aligned} \tag{20}$$

and

$$\frac{(r-a)(r+d-a)}{q} \cdot \mathcal{E}\ell_k D_* = \{1 - r_*^{g-1} + (g-1)r_*^{g-1}[L(E_*) - 1]\}\mathcal{E}\ell_k E_* + (g-1)r_*^{g-1} \tag{21}$$

A higher retrofit cost reduces the value of the option to stop (cf. formula (6)) for given emissions level; proposition 3 gives a condition that the emission level is reduced with the retrofit cost level, approaching the case with no retrofit option as the retrofit cost grows. However, the effect on the damage need not follow the effect on the optimized value. To understand this result, observe that higher retrofit cost both postpones the retrofit and reduces the optimal E_* (as long as $L < g/(g-1)$). The indirect effect – through E_* – could take any magnitude, depending on $f''(E_*)$. When f'' is very large, the initial investment is insensitive to changes in a future retrofit cost, which will then only postpone the retrofit, increasing the

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environmental damages. When f'' is very small, the dominating impact is on the initial decision.

Example: Let us give an example where dD_* / dk could indeed take either sign depending on f''. Fix an initial state q_0 with optimal choices – without loss of generality by scaling units – $E_* = 1$ and $q^* = (r - a)(r + d - a)K(1)g / (g - 1)$ assumed strictly greater than q_0. Those choices are then optimal for all strictly convex decreasing positive cost functions $\tilde{f}$ for which $\tilde{f}'(1) = f'(1)$ – in the limiting cases even with $\tilde{f}''(1)$ being $+\infty$ (with inelastic E_*, (21) reduces to $(g - 1)r_*^g$) or zero. In the latter case, consider proportional K, such that $-W''(1) = \tilde{f}''(1)$ and the elasticity tends to $-\infty$. We leave to the reader to verify that power functions do yield tractable expressions also when $L \in (1, g / (g - 1))$.

2.4. Effects of Increased Volatility

Consider now effects on the retrofit decision and initial infrastructure investment due to changes in s^2, which measures per time-unit variance of the log of climate cost. These two decisions together determine the time profile for carbon emissions, and thus expected aggregate emissions resulting from the infrastructure. From Proposition 2, for given retrofit cost function K, the optimal E_* depends on the parameters only via g and $q_0 / (r - a)(r + d - a)$. Since

$$D := -\frac{dg}{d(s^2)} = \frac{1}{2} \cdot \frac{g^2(g - 1)^2}{(r - ag)g + r(g - 1)} \quad (>0) \tag{22}$$

we can instead calculate derivatives wrt. $-g$ in place of s^2 (the negative to keep signs consistent). Using D as shorthand notation, we find (proof in Appendix C.1):

Proposition 4 (Impact on decision rules, welfare and environmental damage): Suppose the conditions of Proposition 3 hold, with $q_0 / q^* = r_* < 1$. Then the value function $W_* = W(E_*)$ increases wrt. s^2, with derivative

$$\frac{d}{d(s^2)}W_* = D \cdot \frac{d}{-dg}W_* = D \cdot \frac{K(E_*)}{g - 1}r_*^g \ln\frac{1}{r_*} > 0 \tag{23}$$

The optimal E_* and q^* both increase wrt. volatility provided that $L(E_*) \in [1, g / (g - 1)]$, with derivatives, given sufficient differentiability:

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$$\frac{dE_*}{d(s^2)} = D \cdot \frac{dE_*}{-dg} = \frac{D}{-W''(E_*)} \cdot \frac{K(E_*)}{E_*} r_*^g \left\{ \frac{L(E_*) - 1}{g - 1} + [\frac{g}{g - 1} - L(E_*)]\ln\frac{1}{r_*} \right\} \tag{24}$$

$$\frac{dq^*}{d(s^2)} = D \cdot \frac{dq^*}{-dg} = q^* \cdot [\frac{1}{g(g - 1)} + \frac{L(E_*) - 1}{E_*} \cdot (-\frac{dE_*}{dg})] \cdot D \tag{25}$$

For the optimized D_* we find

$$\frac{dD_*}{d(s^2)} = \frac{D \cdot q_0 E_*}{(r - a)(r + d - a)} \cdot \left[\{\frac{1}{g} - \ln\frac{1}{r_*}\} \cdot r_*^{g-1} + \left(1 - r_*^{g-1} + (g - 1)r_*^{g-1}[L(E_*) - 1]\right) \cdot \frac{1}{E_*} \cdot \frac{dE_*}{-dg} \right] \tag{26}$$

Provided $dE_* \,/\, dg \le 0$ (as when $L \in [1, g \,/\, (g - 1)]$), (26) is nonnegative given $q_0 \ge e^{-1/g} \cdot q^*$. For lower values of q_0, the sign of (26) is ambiguous and depends on $f''(E_*)$.

From Proposition 4, E_* and q^* both increase in volatility – this provided that the elasticity L is within the given interval[10]. The impacts depend on the shape of the retrofit cost function, and with proportionality, $K(E) = kE$ (i.e. $L \equiv 1$), as a special (borderline) case that admits unique sign of (24). Note that when the initial technology E_0 is exogenously fixed, $dD \,/\, d(s^2)$ changes sign precisely at $q_0 = e^{-1/g} \cdot q^*$, cf. the curled difference in (26); with E_* optimized, its derivative contributes positively to (26), and this may or may not be enough to ensure a positive sign when $q_0 < e^{-1/g} \cdot q^*$. For examples with either sign possible, copy the argument of the example following Proposition 3: on one hand we can have $f''(E_*)$ and thus $W''(E_*)$ arbitrarily large, yielding a zero in (26) for $\ln(1/r_*)$ arbitrarily close to $1/g$; on the other hand, with proportional retrofit cost we can have $f''(E_*)$ and thus $W''(E_*)$ arbitrarily close to zero, yielding a positive $\ln(1/r_*)$ coefficient in (26) and thus a sum of positive terms.

Our final results, presented as Proposition B in Appendix B.3, concern impacts of increased volatility on retrofit time and emissions. As volatility increases, so does the expected time to retrofit, if finite (i.e. if $2a > s^2$), while if the probability that retrofit never occurs is positive, then that probability will increase. When $L(E_*) \in [1, g \,/\, (g - 1)]$, then E_*, q^* (from Proposition 4) and t^* (in expected value or point mass at ∞, from Proposition B) all increase in volatility. Moreover, expected emissions increase "faster" in volatility than either initial energy intensity,

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[10] Otherwise, (24) takes both signs; it has a zero when $\ln r_* = -[L - 1] \,/\, [(g - 1)L - g]$, and becomes negative for q_0 closer to q^* resp. closer to 0 for $L(E_*) < 1$ resp. $> g \,/\, g - 1$.

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E_*, or expected time to retrofit, q^*, since both increase. Proposition B also gives examples of cases with either sign of the relationship between volatility and expected peak stock.

3. Discussion

In this paper we have studied two combined decision problems:

1. The choice of initial fossil-fuel and carbon emissions intensity of an infrastructure object at the time of investment and until the object is retrofitted.
2. The chosen time of retrofit (if ever), at which time the carbon emissions, and possibly also the fossil-fuel consumption, are eliminated from the infrastructure forever thereafter.

Both decisions affect the aggregate fossil-fuel consumption and carbon emissions resulting from the infrastructure's lifetime operation. It is assumed that the marginal cost of carbon emissions follows a geometric Brownian motion process with positive drift.

Our solution to problem 2 above (in Proposition 1) restates (but slightly modifies) a well-known result from Pindyck (2000): Greater volatility of the stochastic process for climate costs leads to postponement of the retrofit decision, due to an increased option value of waiting when volatility increases.

Our solution to problem 1 (Proposition 2) is new, and gives conditions for the chosen optimal energy and emissions intensity of the infrastructure, also to increase in volatility. Intuitively, the value of such a project increases in volatility, as the resulting increase in benign risk (when climate costs turn out to be small) is given greater weight relative to adverse risk (when future climate costs turn out high): the latter risk, associated with this project, can and will be avoided by retrofitting. This effect is reinforced by the increased option value of waiting to retrofit when volatility increases: both effects work to make higher volatility more attractive. In consequence, the dominating effect of a more uncertain future climate cost (for given expected cost) is that benign, low-cost, outcomes become more frequent relative to high-cost ones. This increases both the project value, and the expected return to a high energy consumption level. As volatility increases, so does the expected time to retrofit, if finite (i.e. if $2a > s^2$); while with positive probability that retrofit never occurs, this probability increases in volatility. We then also find that when the initial emissions level is endogenous, increased

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volatility typically leads to increased climate damage in more cases than when initial infrastructure is exogeneously given.

In order for investment decisions involving long-lasting and potentially energy-intensive infrastructure to be socially efficient, both at the investment stage and the later retrofit stage, decision makers need to face both globally correct energy and emissions prices, and correct retrofit costs. When decision makers instead face too low energy and emissions prices, two problems can result. First, the infrastructure will be established as overly energy intensive. This is particularly problematic (e g for long-run climate policy) when the infrastructure is difficult or expensive to retrofit or replace later. Secondly, necessary retrofits might be unduly postponed, or never implemented. Decision makers in such countries would then tend to choose more emissions-intensive infrastructure, and retrofit it later, when future emissions costs become more volatile. A similar problem with retrofit postponement can arise when the retrofit cost is excessive. We argue that this is likely to often occur in less developed economies with limited access to advanced and low-cost retrofit technologies.[11] Proposition 3 gives conditions that higher retrofit cost also reduces the chosen energy intensity of infrastructure (a benign effect when this intensity is otherwise excessive), and conditions that the total effect is either a reduction or an increase in expected climate damage.

Higher energy and emissions intensities chosen by decision makers when emissions prices are more uncertain, are in our model simply a feature of the optimal policy choice for rational economic actors facing such uncertainty. In an ideal world where decision makers face (globally) correct energy and emissions prices, this should not be particularly worrisome as a global concern. It is more likely to be a concern when decision makers instead face too low energy prices, and/or too high retrofit costs. Our model points to (although does not explicitly analyze) the possibility of adverse outcomes, with corresponding social losses, when decision makers do not face the full global emissions costs, or excessive costs of retrofits or in establishing low-carbon infrastructure. In emerging economies, with large planned infrastructure investments over coming years, decision makers are (now and for the foreseeable future) likely to face emissions prices below optimal levels. Such decision makers could also conceivably take advantage of (possibly large) uncertainties about these cost variables, emphasizing benign risk effects while ignoring adverse risks, on the presumption

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[11] A similar issue is that the cost of implementing low-carbon infrastructure technologies could be excessive in many low-income countries; also leading to socially excessive carbon intensity of the established infrastructure.

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(or with the hope) that they can avoid the consequences of more adverse risks.[12] A reasonable conjecture could then be that the combined problem of excessive energy intensity and postponed retrofits will be exacerbated by increased climate cost volatility. A high perceived likelihood of benign (low-cost) risks, when such risks are not warranted, might then be particularly harmful. Such benign or favorable risks can also be interpreted as a low rate at which carbon emissions from the infrastructure will actually be charged; and not necessarily as a low (true) climate cost to society (be it local, national or global). While such issues are not part of our model or analysis, we conclude that they are particularly relevant as topics for future research; and that our analysis represents a good starting point for such research.

An unrealistic feature of our model is that the "tail risk" of very high future climate costs plays no effective role in decision makers' choices, since very high costs are assumed to always be avoided by retrofitting. But if retrofits are impossible or very costly in some cases (as could apply to alternatives such as completely altering urban structure or transport systems), and such downside risk is underrated, emissions will also tend to be excessive. Lock-in of energy-intensive infrastructure could in such cases make certain climate policy goals infeasible.[13]

A complicating factor, also not discussed formally, is that low energy prices could imply that their variance (volatility) is also low. This could affect investment and retrofit decisions in the opposite direction, and lead to retrofits being executed too early.[14] A more complete analysis of these issues must await future research.

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[12] Another related problem, focused by Strand (2014a), is that defining the "baseline" for any future climate action may be a strategic choice; and that this baseline is affected by the infrastructure policy. For a country, which today is not committed to a specific climate policy, the established baseline or norm for emissions may help to define the degree of climate action required by this country under a future agreement. It may then have incentives to commit to a high emissions baseline in order to convince other parties that reducing future emissions is expensive, thus affecting the required commitments under the future agreement. High climate cost volatility could then make it politically easier to choose an infrastructure policy that leads to high emissions.

[13] This adverse feature of lock-in of long-lasting, energy-intensive infrastructure is stressed also by Lecocq and Shalizi (2014), and Vogt-Schilb et al. (2012; 2014).

[14] See Strand (2014b) for recent analysis of retrofit decisions in such cases. It is there shown that when reduced climate damage and volatility are reduced proportionately, the retrofit decision may in some cases be executed earlier when the climate damage variable, facing decision makers, is reduced.

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Appendix A: Alternative Assumptions

This section outlines consequences of relaxing assumptions (A) through (C) in section 2. Although the expressions become less tractable – and in section A.2 we lose concavity –the qualitative properties will to some extent carry over.

A.1   The Possibility of Multiple Retrofits: Relaxing Assumption (C)

Thus far we have assumed that emissions are reduced through a retrofit at most once, and if so to zero. We shall give conditions for the latter; if it is not optimal even in a one-shot model to remove all emissions, then it is neither optimal when E_* and retrofit trigger level q^* chosen optimally, however this time under the assumption that at the hitting time t^* for the level q^*, the emission level is reduced not necessarily to zero, but to some optimized level $E^* \in [0, E_*)$, at which it is kept forever; this yields a discounted environmental damage from t^* on, of $q^* E^* / (r - a)(r + d - a)$. At t^*, the optimal E^* must therefore minimize the sum of this damage and the retrofit cost which we now write as two variables, $\tilde{K}(E_*, h)$. Sufficient for choosing $h = 0$ is then convexity, or alternatively that $\tilde{K}_2' + q^* / (r - a)(r + d - a)$ is nonnegative for all $h \in (0, E_*)$. Inserting for q^*, this is ensured if

$$-\mathcal{E}\ell_2 \tilde{K}(E_*, \cdot) \leq \frac{g}{g - 1} \tag{27}$$

Arguably, a natural generalization of the assumption that all emissions are eliminated by the retrofit at cost $K(E)$, would be to instead impose a functional form $\tilde{K}(E, h) = K(E - h)$ for reducing emissions from E to h. Then (27) is a direct generalization of the condition $L(E_*) \leq g / (g - 1)$ under which the presence of the retrofit option leads to higher level of initial emissions under conditions (A)–(C). The interpretation of this is that there will not be the same incentive to adapt for lower retrofit cost, if those lower costs may be attained nevertheless, at the expense of (linear!) cost of climate damage.

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Note also that the allowable maximum value $g / (g - 1)$ for the elasticity is due to the constraints on the retrofit action. Likely, a model admitting more general strategies would not share this property.

A.2   Abandoning the Infrastructure: Relaxing Assumption (B)

Thus far we have taken as given that the infrastructure is operated forever. If we drop that assumption, the cost K is capped at V, as one can at any time close down, abandoning the services from the infrastructure. We shall see that in this case, the model will very often be losing its validity, as the optimal choice when initial time is non-negotiable, could be to invest at level $E_* = +\infty$ and then immediately abandon the infrastructure. Assume that the cap becomes effective at some $\tilde{E}$; then Q' gets an upward jump $Q'(\tilde{E}^+) - Q'(\tilde{E}^-) = K'(\tilde{E}^-) \cdot r_*^g$. Assuming f' continuous, we have $W'(\tilde{E}^+) - W'(\tilde{E}^-) = K'(\tilde{E}^-) \min\{1, q_0 / q^*\}^g$. Obviously, we do not have concavity. There might be a local max to the left of $\tilde{E}$, but we know nothing of whether it will be optimal. Assuming that $K(E) = V$ for all $E \geq \tilde{E}$, then with f merely assumed convex, W will to the right of V / k be a difference between two convex functions, and any hope for uniqueness of any stationary point $> V / k$ would require further conditions or specification of f. For given q_0, notice that for large enough E (making q^* decrease), $W'(E) = -f'(E) > 0$. Further assumptions have to be made to ensure $E_* < \infty$. However, $E_* = \infty$ would lead to total payoff $W_* = -f(\infty) - K(\infty) < 0$, and it becomes absurd to assume non-negotiable initial time. This leads to the next subsection: what if initial time is subject to choice?

A.3   Endogenizing the Initial Investment Time: Relaxing Assumption (A)

Introducing an endogenous initial time for the investment as another choice variable will arguably add a further level of complexity to the problem, but it will resolve certain objectionable properties of the previous subsection; it guarantees a nonnegative value. A full analysis of this case is beyond the scope of the present paper, but certain properties can be found in Framstad (2014); The optimal rule is, however, to wait for the first time t_* for which $q_{t_*} \leq$ some sufficiently low value q_* – chosen subject to optimized choices of E_* and t^*, and

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not depending on M. Obviously, this will ensure $t^* > t_*$ and prevent immediate retrofit/closedown action.

A.4 Availability of Retrofit Technology: Relaxing Assumption (C)

Thus far, we have assumed that any initial technology E is freely and instantly available. A question is what happens in the model if the availability of technology changes over time.

Consider a situation where the new technology will not be available until some future time T, which then lower bounds the intervention time t. Once at T, we will stop the subsequent first time we hit q^* (immediately if $q_T \geq q^*$). Rather than the post-investment value $V - f(E_0) - G(\min\{r, 1\}, g) K(E_0)$ in (8) valid for $T = 0$, we get

$$V - f(E_0) - E[e^{-rT} G(\min\{\frac{q_T}{q^*}, 1\}, g)] K(E_0) \tag{28}$$

– the discount factor is kept inside the expectation, as the formula is valid also if T is a non-deterministic stopping time. However, the positive probability that $q_T > q^*$ together with the split definition makes this somewhat analytically intractable, but the following result can now be shown; the proof consists of copying the arguments leading to the concavity part of Proposition 2 for each possible value of q_T, and is omitted:

Proposition A: Suppose that (1) has been maximized over all $t \geq T$, where T is a nonnegative finite stopping time independent of everything else, and with distribution not depending on q_0, E_0 nor the cost function f. Then

$$E_0 \mapsto E[e^{-rT} G(\min\{\frac{q_T}{q^*}, 1\}, g)] \cdot K(E_0) \tag{29}$$

is convex if K is also convex, and is affine if K is proportional.

Remark: This result shows that the form of the value function, and of the optimization wrt. emission level, will be maintained, at least to a certain degree. However, we cannot count on the dependence upon merely k, g and $q_0 / (r - a)(r + d - a)$ from Proposition 2 to carry over, due to the split definition and the probability that $q_T > q^*$ even when $q_0 < q^*$.

A different approach could be to model the retrofit unit cost as a stochastic process (reasonably, a supermartingale after discounting). This is work in progress.

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Appendix B: Probabilistic Properties of the Model

In subsection B.1 we demonstrate that the form of the optimal retrofit rule does not depend on the specific geometric Brownian model, but is a consequence of the strong Markov property and the linear dynamics of the M process. Subsection B.2 displays some basic properties of geometric Brownian motion that we invoke in the analysis. These results can also be found in Borodin and Salminen (2002), see in particular pp. 295 and 622. Subsection B.3 gives the last result of this paper, involving certain additional effects of volatility.

B.1 The Linearity of the Pollutant Stock, the Strong Markov Property and the Retrofit Optimization

Solving for $M_t = M_0 e^{-dt} + e^{-dt} \int_0^t e^{ds} E_s \, ds$, we see that the dependence upon M_0 splits out additively in the objective (1). Also, splitting M additively into this project's contribution and everyone else's, it is easy to see that the latter splits out additively in (1), and the term we can actually affect through E. Therefore the optimization of emissions – be it initial level or timing of retrofit – does not depend upon M.

In our case, with $E_t = E_0$ up to the retrofit time t and zero from then on, solving out M_t yields

$$M_t = M_0 e^{-dt} + \frac{e^{-d\max\{0,t-t\}} - e^{-dt}}{d} E_0 = M_0 e^{-dt} + \frac{1 - e^{-dt}}{d} E_0 - \frac{1 - e^{-d\max\{0,t-t\}}}{d} E_0 \qquad (30)$$

This allows us to decompose as follows: the first term $M_0 e^{-dt}$ can be interpreted as everyone else's contribution, independent of the project; adding the second term yields the project's contribution if operated forever without retrofit (i.e. like putting $t = \infty$ in the left-hand side); the last term is the reduction of the emissions stock from retrofitting at time t. For a given strategy – i.e. a given pair (E_0, t), total discounted climate damage can thus be decomposed into

$$M_0 \cdot \int_0^\infty e^{-(r+d)t} \cdot E[q_t] \, dt + D, \qquad \text{where} \qquad (31)$$

$$D = D(q_0; E_0, t) = E[\int_0^\infty e^{-rt} \frac{1 - e^{-dt}}{d} q_t \, dt - \int_t^\infty e^{-rt} q_t \frac{1 - e^{-d\max\{0,t-t\}}}{d} dt] \cdot E_0 \qquad (32)$$

The positive contribution to D in the last line is $D(q_0; E_0, \infty)$ as the last term vanishes for $t = \infty$. In the rightmost term, we integrate only from t due to zero contribution for $t < t$. Only this

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last term – a damage reduction, hence a benefit – is affected by the retrofit decision. This needs to be traded off against the expected discounted cost $K(E_0)E[e^{-rt}]$ of performing the retrofit. Assuming that q is a time-homogeneous strong Markov process, we can make a time-change shift by t in the reduction term, which turns into

$$E[\int_0^\infty e^{-rt}\tilde{q}_t \frac{1 - e^{-dt}}{d}dt \cdot e^{-rt}] \cdot E_0 \tag{33}$$

where $\tilde{q}$ evolves as an independent copy of q except inserted q_t as initial value. Thus, the t-conditional expectation of the integral in (33) is $D(q_t; E_0, \infty)$, and by the double expectation law, (33) becomes $E[D(q_t; E_0, \infty) \cdot e^{-rt}]$. If furthermore q is continuous, then we can restrict ourselves to hitting times $\hat{t}$, being the first time q hits some interval $[\hat{q}, \infty)$ (the interval extending to the right because higher q is a "bad"). If we then start no higher than $\hat{q}$ we know what state we stop at (if ever), namely $q_{\hat{t}} = \hat{q}$. This holds for arbitrary $\hat{q}$, including for the optimal q^* that solves the following problem that yields the optimized option value:

$$Q = \max_{\hat{q} \le +\infty}\{(D(\hat{q}; E_0, \infty) - K(E_0)) \cdot E[e^{-r\hat{t}}]\} \tag{34}$$

Notice that, because D is proportional to E_0, the optimal level q^* at which a retrofit is executed will depend on E_0 only through the average $K(E_0) / E_0$ – in particular, if the retrofit cost K is proportional, the dependence vanishes. Only apparently does the maximization in (34) depend on what state q_0 we are in; the first-order condition for stopping "now", i.e. for the initial state to be precisely the optimal intervention threshold, is

$$0 = \left\{E[e^{-r\hat{t}}] \cdot \frac{d}{d\hat{q}}D(\hat{q}; E_0, \infty) + (D(\hat{q}; E_0, \infty) - K(E_0)) \cdot \frac{d}{d\hat{q}}E[e^{-r\hat{t}}]\right\}\Big|_{\hat{q}=q_0=q^*} \tag{35}$$

where $E[e^{-r\hat{t}}]$ becomes 1 when inserting, and where (as increased threshold postpones retrofit) the first term is positive and the second negative. For the particular case of geometric Brownian motion, we can now insert and evaluate, using well-known properties reviewed in the next section:

$$\begin{aligned}
D(\hat{q}; E_0, \infty) &= \int_0^\infty e^{-rt} \cdot \frac{1 - e^{-dt}}{d} \cdot e^{-at} \, dt \cdot \hat{q}E_0 \\
&= (\frac{1}{r - a} - \frac{1}{r + d - a}) \cdot \frac{\hat{q}E_0}{d} = \frac{\hat{q}E_0}{(r - a)(r + d - a)}
\end{aligned} \tag{36}$$

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while (cf. (40) below) $E[e^{-rt}] = (q_0/\hat{q})^g$ for $q_0 < \hat{q}$, with g from (4) as used throughout the paper. So the first-order condition (35) reduces to

$$0 = 1 \cdot \frac{E_0}{(r-a)(r+d-a)} - (\frac{\hat{q}E_0}{(r-a)(r+d-a)} - K(E_0)) \cdot g\frac{q_0{}^g}{\hat{q}^{g+1}}\big|_{\hat{q}=q_0=q^*}$$
$$= \frac{E_0}{(r-a)(r+d-a)} \cdot (1-g) + gK(E_0) \cdot \frac{1}{q^*} \tag{37}$$

confirming (5). Now with t^* being the optimal choice, we can evaluate $D = D(q_0; E_0, t^*)$ (retrofit-optimized, but for given E_0) as follows: In (32), the first term is $D(q_0; E_0, \infty)$ from which we subtract $r^g \cdot D(q^*; E_0, \infty)$ (cf. the argument following (33)), where – as usual – $r = q_0 / q^*$. Using, (36) the difference becomes

$$\frac{q_0 E_0}{(r-a)(r+d-a)} - r^g \cdot \frac{q^* E_0}{(r-a)(r+d-a)} = (1 - r^{g-1}) \cdot \frac{q_0 E_0}{(r-a)(r+d-a)} \tag{38}$$

confirming (9).

B.2   Distributional Properties of Geometric Brownian Motion

The parameterization $q_t = q_0 \exp\{(a - s^2/2)t + sZ_t\}$ yields expectation $E[q_t] = q_0 e^{at}$, as Brownian motion is a martingale and applying the expectation to the (Itô) stochastic differential equation $dq_t = aq_t dt + sq_t dZ_t$ yields $dE[q_t] = aE[q_t]dt + 0$. Assume in the following that $s \neq 0$ and that $0 < q_0 < \hat{q}$ where $\hat{q}$ is an arbitrary candidate for trigger level with corresponding first hitting time $t > 0$; for optimum values, put $\hat{q} = q^*$ and $t = t^*$. As the log of the gBm is some Brownian motion with drift, the hitting times are those for the latter. q has positive probability of hitting any given positive value (this in contrast to the deterministic case). We have distributional properties, which can be found in e.g. Borodin and Salminen (2002 p. 295): putting $\hat{L} = \ln(\hat{q}/q_0)$, the first hitting time t for $q = \hat{q}$ has the probability density

$$PDF_t(t) = \frac{\hat{L}}{\sqrt{2ps^2}} \cdot t^{-3/2} \exp\left\{-\frac{1}{2t} \cdot \left[\frac{(a - \frac{1}{2}s^2)t - \hat{L}}{s}\right]^2\right\} \tag{39}$$

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however with the reservation that there might be a point mass at infinity, see below. Regardless of finiteness of t, the following expression holds true for each given exponent $R > 0$ and each $q_0 / \hat{q} < 1$:

$$E[e^{-R\hat{t}}] = (\frac{q_0}{\hat{q}})^{G(R)} \quad \text{with} \quad G(R) = \frac{1}{2} - \frac{a}{s^2} + \sqrt{\left(\frac{a}{s^2} - \frac{1}{2}\right)^2 + \frac{2R}{s^2}} \quad (40)$$

In particular, for the discount rate we recover $G(R) = g$ as in (4).

For the distribution itself, we need however distinguish between the following parametric cases:

- For $2a < s^2$, the drift is too low to guarantee that the process will hit $\hat{q} > q_0$, and in fact q_t tends to zero almost surely. The density (39) does not integrate to 1, but – as it should – to

$$Pr[\hat{t} < \infty] = (\frac{q_0}{\hat{q}})^{1-2a/s^2} \quad (41)$$

- For $2a = s^2$, $\ln q$ is a Brownian motion without drift, and which hits every level; thus q hits every positive level in finite – but, in fact, infinite-mean – time; (39) corresponds to the so-called Lévy distribution (i.e. the totally skewed stable distribution with index of stability equal to one half, so that even $E[t^{1/2}]$ is infinite). This is a borderline case which we omit from the exposition.

- For $2a > s^2$, the hitting time has finite moments of all orders[15], and (39) is now the inverse-Gaussian distribution with

$$\text{mean: } E[\hat{t}] = \frac{1}{a - \frac{1}{2}s^2}\hat{L}, \quad \text{and variance: } E[\hat{t}^2] - E[\hat{t}]^2 = \frac{s^2}{(a - \frac{1}{2}s^2)^3}\hat{L} \quad (42)$$

B.3 Impact of Volatility on Some Functionals of the Probability Distribution

The following gives the impact in optimum of increased volatility on the expected value of the following quantities: time t^* to retrofit (provided finite; otherwise the probability of it being

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[15] Finite mean will hold in many plausible cases (Dixit and Pindyck (1994), page 81). To exemplify, consider "reasonable" values for a and s, say, a (= the mean rate of increase in climate damage) = 2 percent per year, and s (= the relative random change around trend, positive or negative, in impact of GHG accumulation) = 10 percent per year - this, we would claim, is a relatively high value). In this case, $s^2 = 0.01 = a$.

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infinite); total emissions $t^* E_*$, peak pollution stock M_{t^*} and on the running environmental damage rate at retrofit time, $q^* M_{t^*}$. Provided $L(E_*) \in [1, g / (g - 1)]$, they all increase with volatility except in some cases the latter, for which the relationship could take either sign.

Bearing in mind that the agent's optimized choices do not depend on M, we might still be interested in information on the following: at what state of the atmosphere – and at what current damage rate – would we expect the retrofit to be implemented? Contrary to the previous propositions, the initial M_0 would now matter, and we therefore allow it to be nonzero. Expected peak pollution stock is

$$E[M_{t^*}] = E_* / d + (M_0 - E_* / d) \cdot E\, e^{-dt^*} = E_* / d + (M_0 - E_* / d) \cdot r_*^G \quad (43)$$

by (40), and where we – here and in the rest of the paper – write G for $G(d)$; analogous to (22) we have

$$\tilde{D} := -\frac{dG}{d(s^2)} = \frac{1}{2} \cdot \frac{G^2 (G-1)^2}{(d - aG)G + d(G-1)} = \frac{G-1}{s^2 + 2d / G^2} \quad (44)$$

which has the same sign as $d - a$. This explains why the following result splits between $d > a$ and $d < a$:

Proposition B (Further effects of increased volatility): Assume that the conditions for Proposition 4 hold.

We have in optimum

$$\frac{d}{d(s^2)} E[M_{t^*}] = \Big[(\frac{E_*}{d} - M_0) r_*^G \cdot \frac{L(E_*) - 1}{E_*} + \frac{1 - r_*^G}{d} \Big] \cdot \frac{dE_*}{d(s^2)} + (\frac{E_*}{d} - M_0) r_*^G \cdot \Big\{ \frac{GD}{g(g-1)} - \tilde{D} \cdot \ln \frac{1}{r_*} \Big\} \quad (45)$$

which is nonnegative provided that both (i) $dE_* / d(s^2) \geq 0$ (which in particular holds if $L(E_*) \in [1, g / (g - 1)]$) and (ii) the curled difference term in (45) is ≥ 0. Sufficient for (ii) is that $a \geq d$ or if $d > a$, that

$$\frac{q_0}{q_*} \geq \exp\Big\{ -\frac{G}{\tilde{D}} \cdot \frac{D}{g(g-1)} \Big\}. \quad (46)$$

However, if $d > a$, then for each retrofit cost function K there is a f such that (45) does attain negative values for small q_0.

Furthermore, the expected current damage rate at peak stock, $q^* E[M_{t^*}]$ (in optimum), has the following derivative wrt. s^2:

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$$\frac{q_0}{d} \cdot \frac{dE_*}{d(s^2)} \Big[1 - r_*^G + \{1 + (G-1)(1 - M_0 d / E_*) r_*^G\} \cdot (L(E_*) - 1) \Big]$$
$$+ \frac{q_0 E_*}{g(g-1)d} \cdot \Big\{ D + (G-1)(1 - M_0 d / E_*) r_*^G [1 - \frac{g(g-1)}{s^2 + 2d / G^2}] \ln(1 / r_*) \Big\}$$

(47)

which is positive for all $q_0 \in (0, q^*)$ if $dE_* / d(s^2) \geq 0$, $L(E_*) \geq 1$ and $a > d > 0$ all hold. If $d > a$, then for all large enough r there exists some f and some q_0 such that (31) is negative.

Assume now that $a > s^2 / 2$. Then

$$\frac{dE[t^*]}{d(s^2)} = \frac{1}{2(a - \frac{1}{2}s^2)^2} \ln \frac{1}{r_*} + \frac{1}{(a - \frac{1}{2}s^2)} \cdot \frac{1}{q^*} \frac{dq^*}{d(s^2)}$$

(48)

which is always positive given $dq^* / d(s^2) \geq 0$. The impact on expected total emission $E[t^*] \cdot E_*$ is

$$\frac{d}{d(s^2)} \Big(E[t^*] \cdot E_* \Big) = \Big(\frac{1}{2(a - \frac{1}{2}s^2)^2} \ln \frac{1}{r_*} + \frac{1}{(a - \frac{1}{2}s^2)} \cdot \frac{1}{q^*} \frac{dq^*}{d(s^2)} \Big) E_*$$
$$+ \frac{D}{-W''(E_*)} \frac{K(E_*)}{E_*} r_*^g \Big[\frac{L(E_*) - 1}{g - 1} + [\frac{g}{g-1} - L(E_*)] \ln \frac{1}{r_*} \Big] \cdot E[t^*]$$

(49)

which is the sum of positive terms provided that $L(E_*) \in [1, g / (g-1)]$.

Assume instead $a < s^2 / 2$. Then t^* is infinite with positive probability, and from (41),

$$\frac{d \Pr[t^* = +\infty]}{d(s^2)} = r^{1 - 2a/s^2} \cdot \{ \frac{2a}{s^4} \ln \frac{1}{r_*} + (1 - \frac{2a}{s^2}) \cdot \frac{d \ln q^*}{d(s^2)} \}$$

(50)

which from Proposition 4 is positive if $L(E_*) \in [1, g / (g-1)]$.

Appendix C: Proofs

Proposition 1 is straightforward: For the first part, the problem only depends on the parameters in question. For the second part, consider the first-order condition or the derivative at zero. Propositions 2 and 3 are also straightforward calculations, Proposition A follows by copying the argument of Proposition 2. Propositions 4, and B, require several lines of calculations, which are given in the following.

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C.1   Proof of Proposition 4

Differentiating the first-order condition wrt. g, we find that $-W''(E_*)\, dE_* / dg$ equals the g-derivative of the rightmost expression of (13) for $Q'(E)$ (at E_*) – where we keep E fixed (i.e. we regard r as a function of g, not the E_*, and only after differentiation we evaluate at E_*). We shall differentiate the powers r^{g-c} for $c \in \{0,1\}$ to get

$$\frac{d}{dg}\Big(\frac{q_0 E}{(r-a)(r+d-a)K(E)} \cdot \frac{g-1}{g}\Big)^{g-c} = r^{g-c} \cdot [\ln r + \frac{g-c}{r}\frac{dr}{dg}] \tag{51}$$

and so the g-derivative of $r^{g-1} \cdot \dfrac{q_0}{(r-a)(r+d-a)} - r^g \cdot K'(E)$ becomes

$$\begin{aligned}
&r^{g-1} \cdot [\ln r + \frac{g-1}{r} \cdot \frac{dr}{dg}] \cdot \frac{q}{(r-a)(r+d-a)} - r^g \cdot [\ln r + \frac{g}{r} \cdot \frac{dr}{dg}] K'(E) \\
&= -r^g \frac{K(E)}{E} \cdot \{[\frac{g}{g-1} - L(E)] \cdot \ln \frac{1}{r} + [L(E) - 1]\frac{g}{r}\cdot\frac{dr}{dg}\}
\end{aligned} \tag{52}$$

and the result follows by inserting for the elasticity $(g/r) \cdot dr/dg = 1/(g-1)$, evaluating at E_* and then multiplying by $D/W''(E_*)$ (which is negative, by the second-order condition). Then for q^*:

$$\frac{dq^*}{d(s^2)} = q^* \cdot [-D\frac{d}{dg}\ln\frac{g}{g-1} + \frac{dE_*}{d(s^2)} \cdot \frac{d}{dE_*}\ln\frac{K(E_*)}{E_*}] \tag{53}$$

and the rest is straightforward. Now by the envelope theorem, dW_* / dg can be calculated by partially differentiating the expression for $W(E)$ from (8):

$$\begin{aligned}
\frac{dW_*}{dg} &= K(E_*)\frac{r^g}{g-1} \cdot \frac{\partial}{\partial g}[g\ln\frac{(g-1)q_0 E_*}{g(r-a)(r+d-a)K(E_*)} + \ln(g-1)] \\
&= K(E_*)\frac{r^g}{g-1} \cdot [\ln r + g(\frac{1}{g-1} - \frac{1}{g}) - \frac{1}{g-1}] = K(E_*)\frac{r^g}{g-1}\ln r
\end{aligned} \tag{54}$$

C.2   Proof of Proposition B

The differentiations (48), (49) and (50) are straightforward; note that whenever $L(E_*) \in [1, g/(g-1)]$, the expression for $E[t^*] \cdot E_*$ is a product of two nonnegative increasing functions; for the infinite-mean case, observe that from Proposition 4, $\ln q^*$ is increasing in volatility as long as $L(E_*) \in [1, g/(g-1)]$.

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Consider now $E[M_t^*] = r_*^G M_0 + (1 - r_*^G)E_* / d$; differentiation is also fairly straightforward, taking into account that volatility enters by way of E_* directly and through r_*, by way of g through r_* and by way of G. In (45), note that the derivative of E_* could take any magnitude, depending on $f''(E_*)$; in particular, we can choose it as close as we want to zero by merely modifying the second derivative at E_*. Thus by choosing a sequence of f functions for which the first derivative in optimum is fixed, we can get (45) negative if we can get the second term negative, and we can if $\tilde{D} > 0$ (i.e. iff $d > a$) by choosing r_* close enough to 0. Finally, consider the current damage rate at peak stock, namely $q^* E[M_{t^*}]$. For convenience, multiply by d / q_0 and differentiate instead $[r_*^{-1} - r_*^{G-1}]E_* + r_*^{G-1}M_0 d$ to get:

$$
\begin{aligned}
&\left[r_*^{-1} - r_*^{G-1}\right]\frac{dE_*}{d(s^2)} + \left[-r_*^{-2}E_* - (G-1)(E_* - M_0 d)r_*^{G-2}\right]\frac{dr_*}{d(s^2)} - (E_* - M_0 d)r_*^{G-1}\frac{dG}{d(s^2)}\ln r_* \\
&= \frac{1}{r_*}(1 - r_*^G)\frac{dE_*}{d(s^2)} + \frac{1}{r_*}\left[E_* + (G-1)(E_* - M_0 d)r_*^G\right]\frac{1}{q^*}\frac{dq^*}{d(s^2)} - (E_* - M_0 d)r_*^{G-1}\ln(1/r_*)\tilde{D} \\
&= \frac{1}{r_*}\frac{dE_*}{d(s^2)}\left[1 - r_*^G + \left\{1 + (G-1)(1 - M_0 d / E_*)r_*^G\right\}\cdot\left(L(E_*) - 1\right)\right] \\
&\qquad + \frac{E_* + (G-1)(E_* - M_0 d)r_*^G}{r_* g(g-1)}D - (E_* - M_0 d)r_*^{G-1}\ln(1/r_*)\tilde{D}
\end{aligned}
\tag{55}
$$

The $dE_* / d(s^2)$ coefficient is positive: $1 - r_*^g \geq 0$, if $G < 1$, then $1 - G$, $1 - M_0 d / E_*$ and r_*^G are all in the unit interval. Consider now the last line, multiplied by $g(g-1)r_* / E_* D\ \ (> 0)$:

$$
1 + (G-1)(1 - M_0 d / E_*)r_*^G\left[1 + x G \ln r_*\right]
\tag{56}
$$

where for short we have put $x = g(g-1)G / (s^2 G^2 + 2d)D$ (positive). Formula (56) could attain negative values for certain parameters, but never if $d \leq a$ (which $\Leftrightarrow G \leq 1$), in which case the minimum wrt. r_* is attained for $r_* = 1$. Suppose therefore that $d > a$. Then $\text{argmin}_{r \in [0,1]}\ r^G(1 + x G \ln r) = e^{-1-1/x}$ so there is some q_0 for which (56) equals

$$
1 - (G-1)\left[\frac{s^2(g-1/2) + 2a}{s^2 + 2d / G^2} + G - 1\right](1 - M_0 d / E_*)\exp\{-G - \frac{s^2 G^2 + 2d}{(g-1/2)s^2 + 2a}\}
\tag{57}
$$

where inside the bracket we have inserted for x and for $r = ag + \frac{1}{2}s^2 g(g-1)$. Now r does not enter directly, but only through g, and nothing else than E_* depends on g or r_*. Letting r grow – so that $g \to \infty$ and with it the bracketed term – the exponential converges to

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something strictly positive. We can also keep E_* constant by choosing a f corresponding to each r.

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Highlights

Higher climate damages volatility leads to increased option value from postponing mitigation action

Increased climate volatility also increases the chosen energy and emissions intensity of infrastructure investments

Greater climate volatility leads also to higher discounted climate damage, in a wide set of circumstances.

It is crucial, to avoid lock-in of excessive emissions, that policy appropriately accounts for future emissions costs upon investment

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