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A shaft of light into the black box of CGE analyses of tax reforms

Abstract:

While computable general equilibrium (CGE) analysis is a useful and widely applied method for studying tax policies, there is a challenge to present and substantiate the results. I present two analytical tools designed for this purpose. The first is a two-dimensional, diagrammatical exposition of the general equilibrium solutions of a large-scale model, reduced into a two-equation system. The second is a miniature representation of the large-scale model, which can sort out the main general equilibrium responses of the CGE model. By reducing the miniature model into the same two equations, the two devices can be combined to shed light into the black box of the model. I demonstrate how the devices can be utilised to provide economic intuition on welfare and unemployment effects of changes in taxation, exemplified by CO₂ taxes, VAT and payroll tax.

JEL classification:

C53; D58; H23; J68

Keywords: Taxation; Tax Analysis, Computable General Equilibrium; Unemployment

1. Introduction

Computable general equilibrium (CGE) models are useful and widely applied tools in studies of tax policy reforms because they are able to grasp the complicated interplays among economic variables and policy instruments. However, interpreting, substantiating, and presenting results of simulations on large-scale models are challenging. The present analysis introduces pedagogical devices that may be used to shed light into the else black box of CGE models, by sorting out the main driving forces behind the macroeconomic simulation results of policy shifts. While it requires some effort to establish these model-specific tools, once set up, they can be used in forthcoming model exercises and substantially ease the analysis. For researchers and analysts working with large-scale, generic models, this up-front investment can be worthwhile both for own understanding and for providing systematic insight for academic readers.

The development of the analytical tools is inspired by Holmøy (1992)'s analysis of capital deepening. He develops a stylised version of an applied CGE model in order to study how capital expansion and foreign trade assumptions affect industrial resource allocation.¹ In contrast, the tools developed in our analysis focus on macroeconomic effects, and in particular aim to grasp the interlinkages between welfare and unemployment impacts. The tendency in the vast empirical model literature is to study welfare and employment effects separately and with different numerical models.² One obvious benefit of studying unemployment and welfare within the same framework is that accounting for unemployment effects improves the welfare estimates, as unemployment reflects unexploited labour resources. Furthermore, a model incorporating involuntary unemployment caused by imperfections in the labour markets more correctly reflects distributional implications of economic reforms through the distribution of both income and meaningful activity. Studying tax reforms in models that include both welfare and unemployment impacts will reveal possible trade-offs and co-benefits between the two.

I introduce two independent, but complementary, analytical devices designed to support welfare and unemployment analysis in CGE models. One is a diagrammatical exposition of the general equilibrium solutions, where the CGE model is reduced into a two-equation system in two variables: the welfare level and the unemployment rate. We identify the positions and slopes of the corresponding curves by means of simulations of the CGE model. By means of this graphical reduced-form representation of the model we can perform graphical shift analyses of tax reforms. The various equilibrium solutions of the model under different tax regimes boil down to different intersection points of the two curves with the resulting equilibrium welfare and unemployment impacts of the reform. A main advantage of using a two-equation graphical exposition is that the concepts and interpretations used in the analysis are quite analogous to those of partial market diagram analyses – a device that economists are familiar with. Thus, analogous interpretations of the intersection point, of being out of equilibrium, and of being on the "market" curves can be exploited.

¹ See also Holmøy et al. (1999) for similar applications.

² The issue of possible welfare or employment dividends of CO₂ taxation by cutting other taxes have been thoroughly analysed in the previous literature; see Goulder (1995) for an introduction, Schöb (2003) for an empirical survey of welfare dividends and Mors (1995) and Bosquet (2000) for surveys of employment dividends.

The other analytical device is an aggregate, stylised representation of the main characteristics of the CGE model. Auxiliary models of large CGE models are frequently used as tools in CGE analyses; see Adams (2005) for another, thorough and explicit example. The value added of the current method is the reduction of the stylised model into two equations corresponding to the graphical representation described above. This facilitates using the two devices in concert. While the stylised model identifies channels through which tax reforms affect the economy, the shift analysis identifies net effects and, thus, helps sorting dominant effects from less significant and concentrating on the relevant explanatory forces at play.

I demonstrate the usefulness of the tools by analysing three different tax shifts in a CGE model of the Spanish economy: increased CO₂ tax, reduced payroll tax and reduced VAT rates. I focus primarily on macroeconomic unemployment and welfare impacts. The numerical impacts are not the main focus of this analysis. Rather, its value-added is in its step-wise analytical procedure and its presentation of devices used for this purpose.

The two analytical tools constructed for shedding light on the dynamics of the CGE model are introduced in Section 2, together with a brief presentation of the CGE model. (The appendix gives specifications and details of the CGE model.) Section 3 exploits the tools in concert in order to understand the benchmark graphs and the tax shift analysis in light of the attributes of the CGE model. Section 4 provides concluding remarks on the benefits and limitations of the analytical procedure.

2. The CGE model and the analytical tools

2.1. The numerical model

A CGE model gives details and quantified characteristics of real economies. This section describes how key characteristics of the Spanish economy are represented in the numerical model. It is a fairly standard, static CGE model. Its advantage compared to most similar models is that it features equilibrium unemployment. Unemployment is essential to account for in economic analysis of Spain. One obvious benefit of studying unemployment and welfare within the same framework is that accounting for unemployment effects improves the welfare estimates, as unemployment reflects unexploited labour resources. Furthermore, a model incorporating involuntary unemployment caused by imperfections in the labour markets more

correctly reflects distributional implications of economic reforms through the distribution of both income and meaningful activity. Studying tax reforms in models that include both welfare and unemployment impacts will reveal possible trade-offs and co-benefits between the two.

Equilibrium unemployment in the model arises from frictions in the labour market due to lack of information, immobility, heterogeneities across jobs etc. This implies that the jobseeker will demand a mark-up on the reservation wage, which can compensate for the resources spent on job searching. The unemployed views the mark-up as given. However, it is subject to externalities from labour market changes. First, an expansion of economic activity is supposed to ease the process of finding a job, and this reduces the mark-up necessary to compensate for search efforts. Second, a rise in the unemployment rate relative to the benchmark, will reduce the mark-up factor, as the jobseekers is frightened into demanding less compensation for participating in the labour market. The model adopts the specification of the search mark-up from Balistreri (2002), as it is easily integrated within the Walrasian CGE framework. Labour supply is endogenous. This feature of the model enables us to analyse to what extent adjustments of labour supply explain changes in the unemployment rates.

Besides imperfect labour markets, the model incorporates a comprehensive description of other efficiency wedges in the Spanish economy that affect welfare, including the existing tax structure and imperfect competition in the product markets. The degree of competition is allowed to vary among industries according to observed firm concentration. Fixed costs of production and Cournot mark-up pricing are assumed. Entry and exit of firms are endogenous and ensure that fixed costs are exactly covered by the mark-up in equilibrium, i.e. industry profit is zero.

Spain is modelled as a small, open economy. Imports is modelled with the common Armington assumption that goods are differentiated by origin (domestic and foreign), while exports follow from the constant elasticity of transformation assumption. The latter can be interpreted as if there are costs of diverting deliveries between the domestic and foreign markets. These modelling ensure the observed existence of imports and exports along with production and deliveries to the domestic markets of similar products even if prices differ.

The structures of household utility, production, and factor use are disaggregated in order to represent relevant substitution possibilities decisive for the policy responses. The model also computes CO₂ emissions on a detailed level both from firms and households. Climate effects are not specified in the utility or production functions. The aim of the public sector is to balance revenues according to a constant, exogenous restriction, i. e., all policy changes are per assumption revenue neutral. Revenues from market sales of national CO₂ permits are included in the public income. Public savings are fixed, as is the current account.

The next subsections introduce the two analytical tools, first, a graphical representation of the model's general equilibrium solution, and second, a miniature model of the model designed so as to grasp the main characteristics of the model that can explain the solutions of the graphical exposition. The miniature model, together with complementary information of specifications and disaggregation in the appendix, documents the necessary understanding of the whole CGE model. For further insight and quantified information, see Fæhn et al. (2009).³

2.2 Graphical exposition of the CGE model

By simulating the model's benchmark solution and some points in the vicinity of the general equilibrium, we will be able to construct a diagrammatical exposition of the model solution as the intercept between two curves, as illustrated in Figure 1. As we shall see, it will be useful to construct two curves representing the labour market equilibrium and the current account, respectively. The two curves represent the model in reduced form in the two macro variables we want to analyse, the welfare level, WF , and the unemployment rate, U :

$$U = l(WF; \varepsilon) \tag{1}$$

$$U = d(WF; \varepsilon) \tag{2}$$

Eq. (1), *the labour market curve*, expresses all the combinations of welfare levels and unemployment rates that ensure that the labour market is in equilibrium, when all direct and indirect effects of WF and U on the labour market are accounted for. The labour market equilibrium in presence of unemployment requires that $L^D = (1-U)L^S$, where L^D and L^S are labour demand and supply. Eq. (2), *the current account curve*, represents

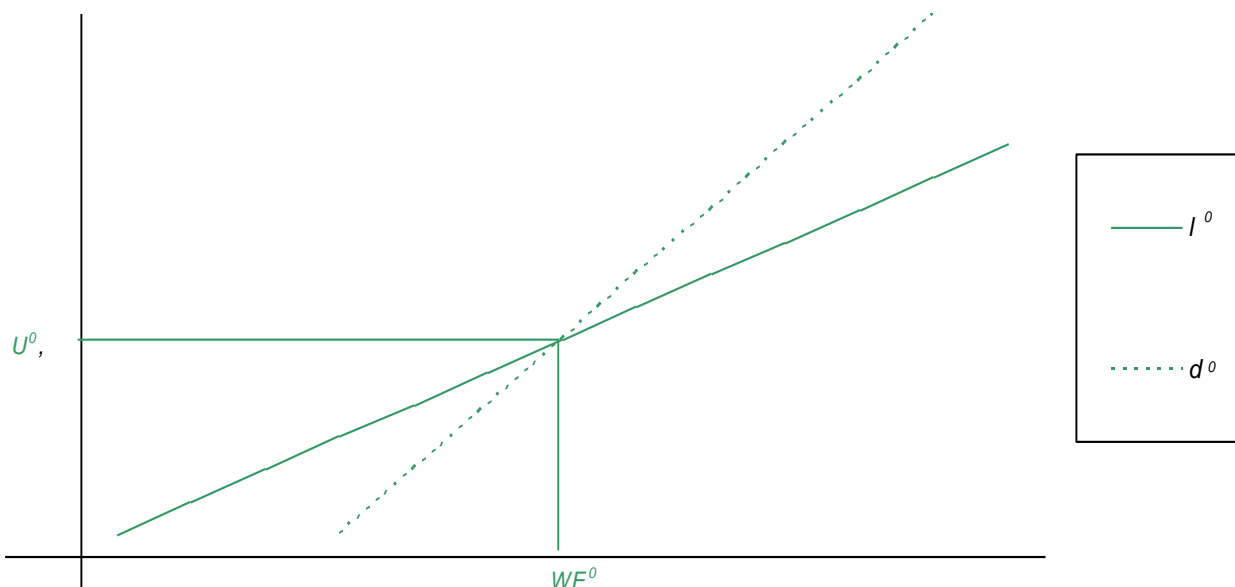
³The model in Fæhn et al. (2009) deviates from the present by having two labour markets.

combinations of WF and U that ensure unaltered current account, D . All exogenous variables, including the CO_2 tax, payroll tax and VAT rates, are represented by the exogenous vector ε – *the shift vector* – in the equations. Since the two equations represent the whole model in reduced form, there is only one (WF, U) -combination that fulfils both (1) and (2) for a given set of exogenous variables/assumptions. This (WF, U) -combination is the model's general equilibrium.

The benefit of using a general equilibrium model in this reduced form is that the 2-equation model will directly give us the welfare and unemployment results. Shift analysis on the model implies that the ε -vector changes. This will shift both the curves, and we obtain a new equilibrium solution for WF and U . It is not possible to analytically reduce the large-scale CGE model to this extent and calculate the solution. However, the solutions can be graphically illustrated.

Before we can do a shift analysis graphically we first need to establish the benchmark intersection point of the two curves and their slopes. The intersection point is easily identified by simulating the benchmark solution of the model. The resulting values of WF and U can then be marked in the diagram; see the point (WF^0, U^0) in Figure 1.

Figure 1: The general equilibrium of the benchmark



The slopes are less straightforward to identify and we need auxiliary simulations. Let's start by one of the benchmark curves, say, the l^0 -curve. As can be seen from Figure 1, in order to identify other points on the l^0 -curve than (WF^0, U^0) that can indicate its slope in the vicinity of benchmark, we need to move slightly off the d^0 -curve. We do this by swapping the roles of the naturally exogenous current account, D , and the naturally endogenous welfare, WF when running the CGE model. In other words, we change the closure of the model by choosing WF values exogenously that deviate slightly from its benchmark equilibrium values in both directions. We are then no longer on the current account curve, d^0 , but move in small steps up and down the labour market curve, l^0 . We read the corresponding U values of this stepwise procedure and plot the simulated (WF, U) -values along the l^0 -curve. The results indicate that the slope of the l^0 -curve in the vicinity of equilibrium is positive, i.e. a positive change in WF from WF^0 results in a positive change in U , and vice versa for negative changes.

Corresponding simulations for the d^0 -curve reveal that it is also upward-sloping and steeper than the l^0 -curve. Both curves are illustrated in Figure 1. The curves, intersections and shifts in this general equilibrium market diagram can be interpreted quite analogously to their partial market counterparts, a tool that economists are familiar with. A main difference between our exposition and regular partial market diagrams is that while partial demand and supply curves represent behavioural relationships, and being off them have intuitive interpretations, the general equilibrium curves do not have analogous independent definitions. Strictly, they are only defined given that we are not off the other curve. The best way of interpreting them is thus as the slopes in the (vicinity of the) general equilibrium point. In order to give intuition on these slopes, it will be useful first to introduce the other device, a miniature representation of the model.

2.3 A miniature model of the CGE model

The stylised exposition

The purpose of the stylised representation is not only to grasp the major mechanisms of the large CGE model, but at the same time analytically be able to obtain a reduced form with only two interpretable equations analogous to the simulated eqs. (1) and (2) above. For this purpose it suffices to formulate the miniature model in unspecified, deduced functional forms. Furthermore, it suppresses many details of the larger model, for instance, the product markets are merged into one and only two factors of production are

included. The result is a 16 equations' representation. The industry, goods and factor structures and functional specifications of the large-scale CGE model are given in the appendix.

Profit functions of representative, imperfectly competitive firms with fixed costs are given by eqs. (A1) in the appendix. In this miniature exposition all product markets are merged into one and only two factors of production are included. Thus, the first order conditions of the firms become:

$$P^X (1 - M) = c(W, R; \varepsilon) \quad (3)$$

expressing that there will be a mark-up wedge, M , between the price of the (merged) good, P^X , and the marginal costs, c , in order to cover fixed costs. c increases with the price of labour, W and the capital rent, R . In addition, it is affected by the exogenous shift vector, ε , which represents all exogenous variables of the model, including policy variables. This implies that all variables that can be changed in order to analyse effects of exogenous shifts by the model are represented in ε . These include the policy variables we will change in the analysis below, the CO₂, payroll and VAT tax rates.⁴ The mark-up factor

$$M = M(E; \varepsilon) \quad (4)$$

is decreasing in the number of firms, E .⁵ The number of firms is determined by a zero profit condition that regulates entry and exit; see eqs. (A1). The profit function depends on factor prices and the producer price, P^X , along with the shift vector, thus:

$$E = E(R, W, P^X; \varepsilon) \quad (5)$$

The producer price is an index of the price obtained domestically, P^H , and in the export markets. The latter is exogenous and part of the shift vector. Likewise, the price of the consumer good in the home market, P , is an index of the domestic producer price and an exogenous import price which, together with other exogenous parameters, constitutes the shift vector. (See eqs. (A6) for specifications made in the CGE model).

⁴ These policy variables are specified in the model details given in the appendix, denoted by PCO_2 , t_i^{PR} and t_i , respectively – see eqs. (A2), (A3) and (A6).

⁵ See eqs. (A4) and (A5).

$$P^X = P^X(P^H; \varepsilon) \quad (6)$$

$$P = P(P^H; \varepsilon) \quad (7)$$

In the merged, miniature model we define the welfare of the consumer, WF , as endogenously determined by the consumption of the final good (F) and of leisure, which is the (exogenous) time budget less of the labour supply (L^S):

$$WF = WF(F, L^S; \varepsilon) \quad (8)$$

Maximising welfare subject to the budget constraint yields the demand for the final good and the supply of labour (demand for leisure) as functions of the relative price of the good, P , relative to the price of leisure, W , as well as the income. Income depends on the wage rate and the capital rent, along with the unemployment rate, U , which reduces the income of supplying labour.⁶

$$F = F(W, R, U, P; \varepsilon) \quad (9)$$

$$L^S = L(W, R, U, P; \varepsilon) \quad | \quad (10)$$

Eqs. (8), (9) and (10) are also dependent on exogenous model variables in the shift vector.

In the product market, equilibrium requires that production, X , is the non-imported demand for the final good minus the exports. In general terms, where ε accounts for exogenous variables:

$$X = X(B, F, A; \varepsilon) \quad (11)$$

According to eqs. (6) and (7) exports, A , and imports, B , are endogenous shares of total consumption and production, respectively, where F and X are defined above :

$$B = b(P^H; \varepsilon)F \quad (12)$$

$$A = a(P^H; \varepsilon)X \quad (13)$$

⁶ The budget constraint is expressed in eq. (A7). In this miniature exposition, F represents the whole demand structure presented in eqs. (A8).

Capital market equilibrium is defined by the equalisation of the exogenous supply and the endogenous demand for capital. The latter is derived by Shepard's lemma and given by the right hand side of eq. (14):

$$\bar{K} = K^D = X\left(\frac{-\partial\Pi(P^x, R, W; \varepsilon)}{\partial R}\right), \quad (14)$$

where $\frac{\partial\Pi}{\partial R}$ is the derivative of the unit profit, Π , with respect to the capital rent; see eqs. (A1). For the labour market equilibrium we have:

$$L^S(1-U) = L^D = X\left(\frac{-\partial\Pi(P^x, R, W; \varepsilon)}{\partial W}\right) \quad (15)$$

The labour market equilibrium condition expresses that labour supply net of the equilibrium unemployment is equal to labour demand. The latter is determined analogously to capital demand in eq. (14).⁷

The exogenous current account restriction, \bar{D} , has net exports as the only endogenous component; see eqs. (A12). (The shift vector includes all exogenous financial transfers and export and import price components.)

$$\bar{D} = D(A, B; \varepsilon) \quad (16)$$

Finally, we have the following equations capturing the search behaviour in the labour market:

$$W = SW^0 \quad (17)$$

This arbitrage equation of the jobseeker expresses that he will participate in the labour market as long as the wage he can obtain in case of finding a job is no less than the reservation wage, W^0 , times a mark-up rate due to search costs, S . The search mark-up is subject to externalities:

$$S = S(U, L^D) \quad (18)$$

The externalities arise from both the unemployment rate and the labour demand. The higher the unemployment rate, the lesser will the compensation for searching be, in accordance with the wage curve theory (Blanchflower and Oswald, 1994; 2005). Suppliers will be “frightened” into demanding less compensation. The labour demand will affect the search costs oppositely. It can be regarded as an indicator

⁷ See eqs. (A9) and (A11) for more specified functional forms.

of economic activity, and the more activity, the easier it will be to find a job and the less compensation will be demanded in equilibrium for the search costs by suppliers. Its specification in the CGE model is given in eq. (A10).⁸

A stepwise reduction of the miniature model

By a stepwise reduction of this stylised model of 16 equations, we obtain the following four-equations general equilibrium model:⁹

$$\underbrace{L^S(W, R, U; \varepsilon)}_{L.4} \underbrace{(1-U)}_{L.5} = \underbrace{\tilde{L}^D}_{L.1} \underbrace{\{W, R, \varepsilon, X[W, R, \varepsilon, F(W, R, U; \varepsilon)]\}}_{L.2} = \underbrace{L^D(W, R, U; \varepsilon)}_{L.3} \quad (19)$$

$$\overline{D} = \underbrace{\tilde{D}}_{D.1} \underbrace{\{W, R, \varepsilon, F(W, R, U; \varepsilon)\}}_{D.2} = D(W, R, U; \varepsilon) \quad (20)$$

$$WF = \tilde{W}F \{F(W, R, U; \varepsilon), L^S(W, R, U; \varepsilon)\} = WF(W, R, U; \varepsilon) \quad (21)$$

$$\overline{K} = \tilde{K}^D \{W, R, \varepsilon, X[W, R, \varepsilon, F(W, R, U; \varepsilon)]\} = K^D(W, R, U; \varepsilon) \quad (22)$$

These four equations (19) – (22) express the reduced-form labour market equilibrium, the trade balance, the indirect utility function and the capital market equilibrium, respectively. All other equations and equilibrium conditions are implicitly defined.

Eq. (19) distinguishes between five effects on the labour market of changes in the endogenous W, R and, U , and in the exogenous variables encompassed in ε , including the tax reform components in the present analysis:

⁸ The public sector's budget constraint is also given in the appendix (eq. (A13) to complete the documentation, but it is not necessary for the subsequent analysis and omitted here.

⁹ See also Fæhn and Grünfeld (1999) for a more extensive presentation of a similar procedure.

L.1: Substitution effects: The first appearance of W , R and ε in the labour demand function, L^D , represents changes in relative labour to capital demand of altering wages (W), capital rents (R) and exogenous variables (ε).

L.2: Competitiveness effects: W , R and ε also affect labour demand through altering production costs and, thus, the international competitiveness of Spanish firms. This alters output (X) and subsequent input of labour (L^D).

L.3: Home market effects: Output, X , is, as well, dependent on the domestic demand for the final good, F , which is determined by prices and income. These are functions of factor prices, W and R , the unemployment rate, U , that affects the income of the household, as well as exogenous variables in ε .

L.4: Labour supply effects: Through the household's decisions, labour supply (L^S) is dependent on the same price and income determinants as the demand for final goods described above.

L.5: Unemployment wedge effect: The factor $(1-U)$ on the left hand side of eq. (19) captures that the unemployment rate influences the (unemployment-adjusted) labour market equilibrium directly.

The net current account restriction in eq. (20) distinguishes between two channels, through which the endogenous and exogenous variables influence the current account:

D.1: Competitiveness effects: The first appearance of W , R and ε symbolises that changes in factor prices, as well as exogenous cost factors in the shift vector, affect the competitiveness of Spanish firms, and thus the trade balance in the current account restriction.

D.2: Home market effects: The second appearance of W , R and ε in eq. (20) shows their effects through changing domestic final consumption, F . This influences the current account through import leakages.

Eq. (21) defines utility of the representative consumer. The determinations of F and L^S in eq. (21) are explained above. Capital demand in eq. (22) is influenced by *Substitution effects*, *Competitiveness effects*

and *Home market effects*, analogously to how such effects affect the labour demand; see the explanation of the labour market equilibrium in eq. (19), above.

The four equations solve for the four endogenous variables WF , W , R and U . We can reduce the model further, by solving eq. (21) for W , eq. (22) for R , and then inserting the latter into the former. For a given ε , W and R are determined by WF and U :

$$(21'): W = W(WF, U; \varepsilon)$$

$$(22'): R = \tilde{R}(W, U; \varepsilon) = R(WF, U; \varepsilon)$$

Using eqs. (21') and (22') leaves us with the labour market equilibrium and the current account expressed in eqs. (19) and (20) as functions of only two endogenous variables, WF and U , which again implicitly determine all other variables in the model, or:

$$(23) \quad U = l(WF; \varepsilon)$$

$$(24) \quad U = d(WF; \varepsilon)$$

Eq. (23) expresses the *labour market curve*, while eq. (24) is the *current account curve* of the miniature model, which correspond to eqs. (1) and (2) expressing the complete CGE model. The general equilibrium effects of the miniature model can now be described in a diagrammatical shift analysis of two curves representing the l and d curves in (23) and (24). These will correspond to the curves representing the general equilibrium of the numerical model in Figure 1. We now proceed to using the tools in concert in the reform examples simulated by the CGE model.

3. The tax shifts: Unemployment and welfare effects

3.1 Design of the CGE analysis

I demonstrate the usefulness of the tools by analysing three different tax shifts in the CGE model:

Case A: Uniform CO₂ taxation,

Case B: CO₂ taxation and reduced payroll tax rates,

Case C: CO₂ taxation and proportional reductions in VAT rates.

Case A cultivates the pure effects of introducing a uniform CO₂ tax capable of reducing economy-wide emissions by 25 per cent. For the sake of comparison, public budgets are always balanced in the model; see eq. (A13). In Case A, revenue from the CO₂ taxation is transferred to the representative consumer lump sum. In Cases B and C the recycling channels are more policy-relevant rate reductions of payroll and VAT taxes, where the dimensioning of the rates is determined so as to balance the public budget.¹⁰

Unemployment effects are measured in terms of the unemployment rate, which is affected by changes in both labour demand and supply. We report changes in the unemployment rate, employment and labour supply. Welfare changes are measured by the Hicksian Equivalent Variation index.

3.2 The benchmark: Using the tools in concert

As the miniature model captures the main mechanisms of the CGE model, exploring it can help identifying the main determinants of the slopes of the benchmark curves in Figure 1 and explaining their signs. This information gives crucial guidance as to where in the CGE model we should look for more explicit structures and parameters that are consistent with the findings from the exploitation of the auxiliary tools.

First, let us start with what can directly be learned from the simulated, positive slopes of the l^0 and d^0 -curves. From the l^0 -curve we see that a distortion in the labour market equilibrium resulting from a partial *increase* in WF will have to be neutralised by a simultaneous *increase* in U . Likewise, we see from the simulated, positive slope of the d^0 -curve that WF and U work in opposite directions on the trade balance. I will now draw on the miniature model to explain these relationships and start by addressing the slope of the l^0 -curve. Its positive slope proposes that increased WF , in isolation, creates a labour supply surplus in the Spanish economy, while an increase in U creates a deficit, which rebalances the labour market. To substantiate these relationships, we can confer the reduced version of the miniature model. Eqs. (21') and (22') show that the effects of a partial increase of WF work through factor price increases. Factor price increases have several effects on the labour market, as the explanation of eq. (19) above has made clear. The excess labour supply resulting from a partial WF increase can, thus, be followed through examining the influence of the wage

¹⁰ Since the qualitative use of the analytical tools, rather than the quantitative results, is the focus other cases or sensitivity tests are not presented. For interested readers, welfare and unemployment effects of similar carbon permit systems are addressed in Fæhn et al. (2009), where numerical sensitivity analyses of different CO₂ caps, elasticity parameters and labour markets are also discussed.

rate, W , via the labour market effects $L.1$, $L.2$, $L.3$, and $L.4$ above.¹¹ Increased wages have positive effects on the labour supply surplus via $L.1$ *Substitution effects*, $L.2$: *Competitiveness effects* and $L.4$: *Labour supply effects*. On the other hand, the $L.3$: *Home market effects* counteract. The slope of the l^0 -curve reveals that the three former, positive effects dominate the latter negative effect.

These observations lead us to look for empirical features of the CGE model that can substantiate this net result. The model's specifications indicate that though $L.4$: *Labour supply effects* contribute positively, the major explanations are the labour demand reductions resulting from $L.1$: *Substitution effects* and $L.2$: *Competitiveness effects*. In spite of relatively low Armington elasticities (between 2 and 3 for most goods), the fact that Spanish internationally competing industries (particularly metal production) are labour intensive causes significant $L.2$: *Competitiveness effects* of increased WF and wages. The $L.1$: *Substitution effects* are less easy to track, but substitution elasticities at the firm level lie between 1 and 2 for most industries, indicating rather responsive labour-to-capital rates at the firm level. Counteracting $L.3$: *Home market effects*, i.e. increased consumption, cannot be neglected. A real wage increase induces both substitution and income effects in favour of increased consumption. The subsequent labour demand increase is, however, weak due to the fact that consumer goods are relatively capital intensive – and become even more so when prices of labour intensive goods increase in relative terms. Consumption of trade services, other manufacturers, and renting are all capital intensive and constitute substantial parts of total consumption.

As the empirical slope of the l^0 -curve implies a neutralising *positive* response in U , we have to look for channels through which increased U can reduce the excessive labour supply caused by a positive partial WF -shift. Since all the channels $L.1$ to $L.4$ have *positive* partial influences on the labour supply surplus

¹¹ A partial increase in WF also affects R , see eq. (22'), and the relative impacts of R and L are not deductable from the general form miniature model. Simulations on the numerical model show that the strongest impact of WF on factor prices comes through the indirect utility expressed by eq. (21): A rise in WF will have to involve real income improvements, and for given U , factor price increases must take place. The simulations show that a partial rise in WF causes wages to increase relatively more than capital rents, and nominal wages more than the prices of consumption goods so that the real wages increase. In the following we suppress the effects on capital rents, as they only work to dampen the conclusions from a discussion focusing on nominal (and real) wages.

(increased U reduces labour demand and increases labour surplus), the slope of l^0 indicates that the *L5:Unemployment wedge effect* via the term $(1-U)$'s direct reduction of excess supply, dominates.¹²

Turning to the positive slope of the d^0 -curve, and the following opposite effects of WF and U on the trade balance, the miniature model reflects that behind lie the facts that a partial increase in WF deteriorates the trade balance, while an increase in U improves and can restore the trade balance. Partially increasing WF rises nominal and real wages and affects the trade balance adversely both through *D.1:Competitiveness effects* and *D.2:Home market effects*. An increase in U will help re-satisfying the trade balance restriction. This shows that a dominant effect of increased U in the Spanish economy is to decrease import leakage through negative *D.2:Home market effects*. A closer look at the features of the CGE model and the Spanish economy supports this finding. Metal products and other manufactured consumer goods have particularly high (input-output-adjusted) import shares and high budget weights. The slope of the d^0 -curve implies that being off and *above* the d^0 -curve represents situations with smaller deficits than required by the current account restriction, while at points *below* the curve, deficits are too large.

Having established and substantiated the positions and slopes of the labour market and current account curves in the benchmark, we can now proceed to demonstrate how graphical shift analysis and the miniature model can assist the CGE analysis of tax shifts.

3.3 Case A: Effects of increased CO₂ tax

Once having established the benchmark solution and the slopes of the curve, we are ready for simulating tax shifts. The model is now used in its usual general equilibrium manner. In particular, WF and U are endogenous variables, and the given labour market balance and current account in eqs. (A9) and (A12) apply, as usual.

The CGE simulations of the CO₂ tax increase reveal that the unemployment rate is virtually unaffected, while welfare decreases by 0.93 per cent, as reported in Table 1a. In order to wind up the main mechanisms producing these results and relevant characteristics of the model of the Spanish economy, I exploit the

¹² However, another effect of U , which is suppressed in eq. (19), also contributes to reduce excess supply somewhat: increased U reduces the search cost component of the wage rate directly (see eq. (17) due to the externalities of the matching process (see eq. (18)).

supporting analytical tools presented in Section 2. In Figure 2, the equilibrium solution of Case A is marked in the point (WF^A, U^A) , which represents the intersection between the curves l^A and d^A . The respective shifts from the l^0 and d^0 -curves reflect that the CO₂ tax, which imposes a price wedge between the consumer and producer price of fossil fuels, causes both curves to shift upwards in the relevant area. This is illustrated in Figure 2 by the shifts from the green to the pink curves.

Consider first the shift in the l -curve. As explained in Section 2, the new curve is characterised by WF s and U s that, for given shift vector $\varepsilon = \varepsilon_0$, would create a labour supply deficit. In other words, the partial effect of increasing the CO₂ taxes, which is reflected in the miniature model as moving from ε_0 to ε_A , is to create a labour supply *surplus* that has to be neutralised. This surplus is the net result of effects through the four main channels *L.1 – L.4* for ε -impacts already described in Section 2; see eq. (19). The surplus reflects dominating *L.2: Competitiveness effects* and *L.3: Home market effects*. The former are due to a labour demand fall as internalising costs of emitting deteriorates the competitiveness of domestic firms. The latter are consequences of lowered real wages when prices rise. This discourages consumers' demand for goods and, thus, firms' demand for labour.

Table 1: Effects of different schemes; 1a: CO₂ tax, 1b: Recycling, 1c: Total

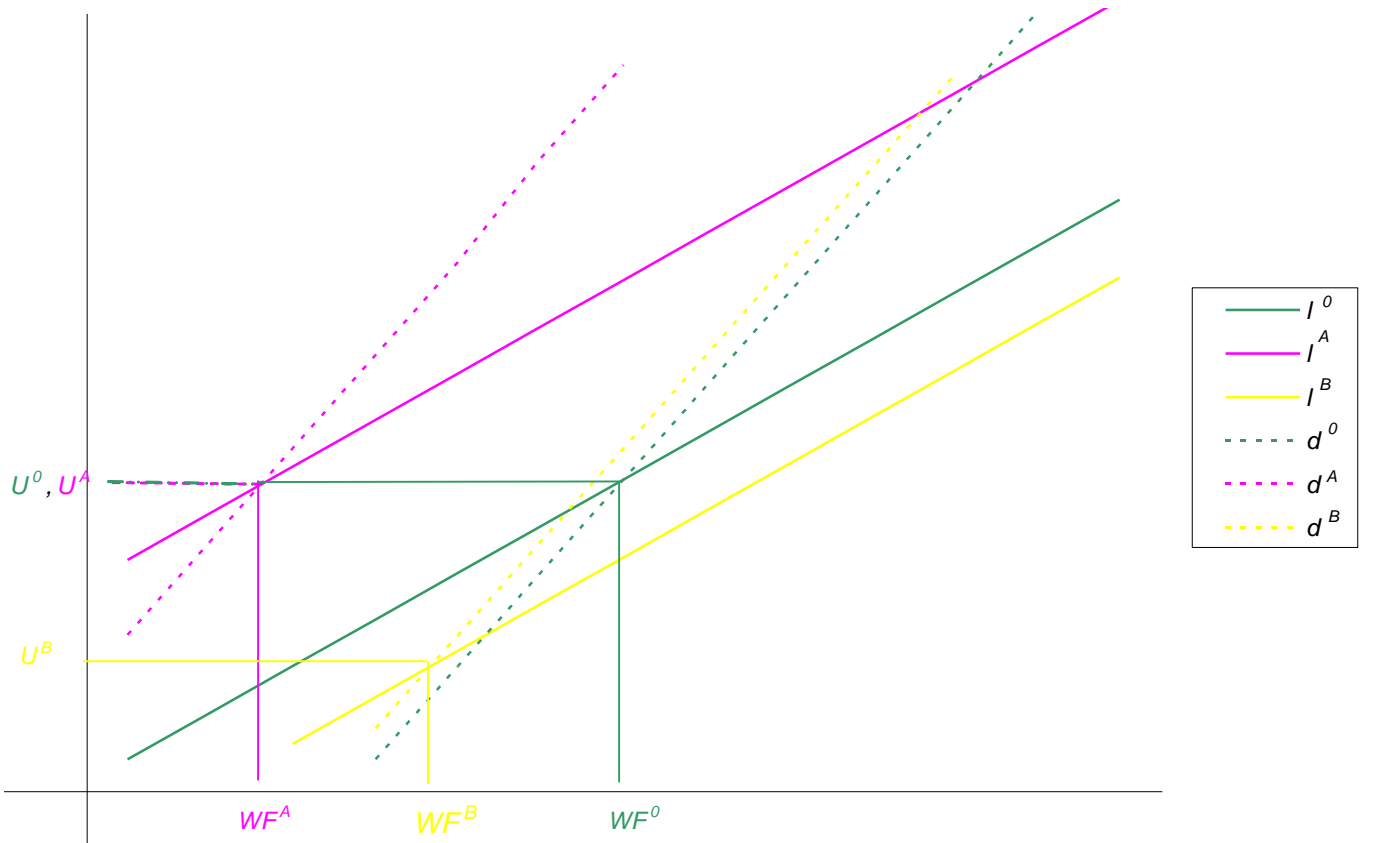
	Table 1a: CO ₂ tax effects;	Table 1b: Recycling effects;		Table 1c: Total effects;	
	% change from benchmark	% change from Case A		% change from benchmark	
	Case A	Case B	Case C	Case B	Case C
Unemployment rate	0.00	-0.24	-0.04	-0.24	-0.04
Employment	0.03	0.52	0.07	0.55	0.10
Labour supply	0.03	0.48	0.06	0.51	0.09
Welfare	-0.93	0.48	0.53	-0.46	-0.40

When we confer the CGE model, we observe that neither the internationally exposed goods, nor the consumer goods have very high *direct* fossil fuel intensities, but as prices of inputs, first of all electricity and transport services, increase, the CO₂ pricing significantly raises the costs within exposed industries and final goods industries. *L.4: Labour supply effects* and *L.1: Substitution effects* contribute to weaken, but not

offsetting, the labour supply surplus; the first through reducing labour supply, the latter through increasing labour demand. Labour supply falls as real household income drops in the wake of higher consumer prices; consumption of fossil fuels, as well as goods produced by fossil fuels, become more expensive.

L.1: Substitution effects contribute to increase labour demand, as the capital-intensive industries tend to face the highest CO₂ tax burden. This causes a substitution of relatively labour-intensive production for capital-intensive. However, as share of total capital use, the Spanish fossil fuel intensive industries are not very important, so this effect is small.

Figure 2: The Labour market curves (l) and Current account curves (d) in Case 0 (benchmark), Case A, and Case B.



Simultaneously, the d -curve shifts from d^0 to d^A in Figure 2. As the new equilibrium (WF, U) -point lies *above* the d^0 -curve, we know that the adjustments of WF and U , in isolation, would have caused a current account improvement from the benchmark. For this to be a new equilibrium, we can infer that the ε -adjustment caused by the CO₂-policy reform must have induced an isolated current account *reduction*, and the subsequent responses in WF and U were to neutralise this immediate current account deficit. The

explanation is that the reform of the CO₂ tax, represented by the shift vector ε in the miniature model, affects the current account through two main channels, *D.1* and *D.2*, as explained in Section 2. Increased emission prices imply a competitiveness loss that deteriorates the trade balance. These negative *D.1:Competitiveness effects* turn out to dominate the positive *D.2:Home market effects* caused by reduced import leakage when domestic income decreases.

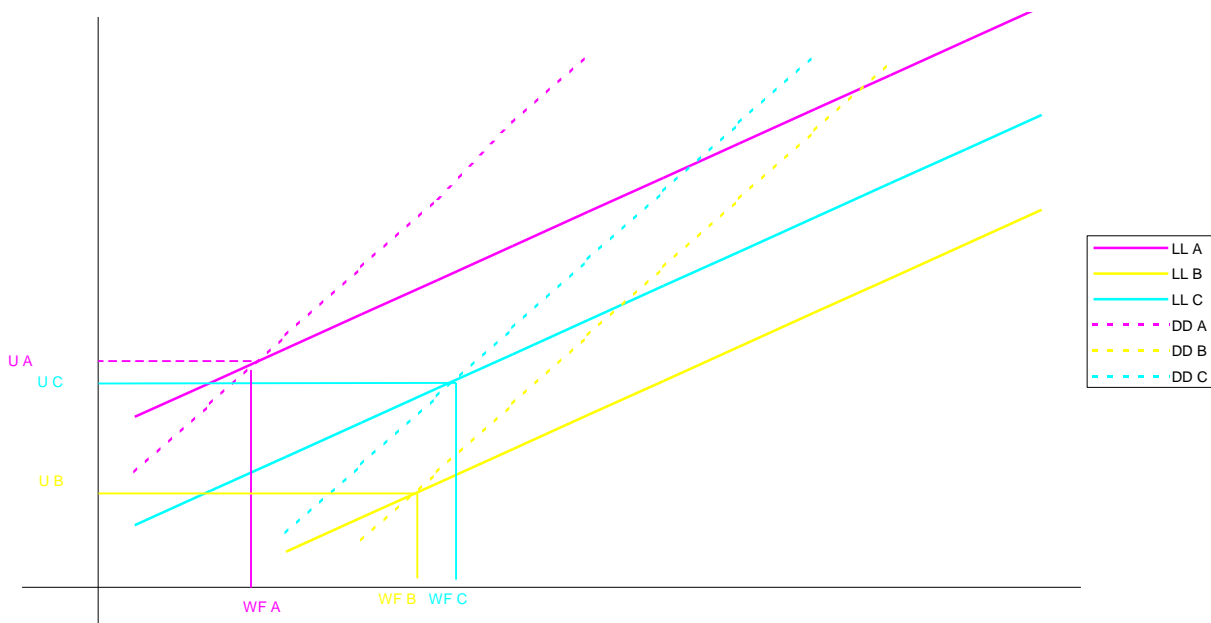
To sum up, the new intersection point reflects that introducing the CO₂ tax reform will not notably affect *U*; see also Table 1a. This mirrors that the fossil-fuel-intensive part of the Spanish economy is not very labour intensive, and only small amounts of labour is released by its contraction. In fact, employment increases slightly, reflecting that the negative shift in the labour demand caused by *Competitiveness effects* and *Home market effects* of the CO₂ permit prices is more than offset in the new equilibrium by relative wage reductions that stimulate demand. However, as labour supply simultaneously rises, the unemployment rate remains unaltered.

3.4 Case B: Effects of reduced payroll tax

Adding recycling effects of reduced payroll taxes on all labour to the pure CO₂ tax effects in Case A corresponds to the more policy-relevant Case B. Table 1b shows that the unemployment rate falls by 0.24 per cent as an effect of the recycling scheme, while welfare is positively affected, approximately bisecting the welfare costs of the CO₂ tax reform. These results are depicted in Figure 2. The move from (WF^A, U^A) to (WF^B, U^B) illustrates that the isolated recycling effects of this scheme are to reduce unemployment and to partly offset the welfare loss. In terms of the curves the change in the shift vector ε caused by the payroll tax reduction shifts the curves to l^B and d^B , both lying *below* the respective curves of Case A. Thus, *cet. par.*, the *WF* and *U* movements would cause a net supply surplus in the labour market along with an increased deficit in the current account. Accordingly, as we are in a new equilibrium, the recycling scheme has caused the opposite: A labour supply deficit and a current account improvement. These are results of counteracting effects that can be tracked by using eqs. (19) and (20) in the same manner as above. In the labour market, *L.4:Labour supply effects* of the change in ε contribute to increase net supply, as reducing payroll tax rates lowers costs and market prices. However, this effect is inferior to the other three *L.1* to *L.3*, which all

increase labour demand and cause the supply deficit: *L.1:Substitution effects* through lowered labour prices, *L.2:Competitiveness effects* through the subsequent competitiveness improvements, and *L.3:Home market effects* through higher real income and demand. The current account improvement caused by lower labour costs is explained by the favourable *D.1:Competitiveness effects*. *D.2:Home market effects* counteract somewhat through higher import leakage, but turn out to be inferior.

Figure 3: The Labour market curves (*l*) and Current account curves (*d*) in Case A, Case B and Case C.



To sum up, the fall in the unemployment rate resulting from reduced payroll tax is due to the joint strong labour demand stimulation of the *L.1:Substitution effects*, *L.2:Competitiveness effects* and *L.3:Home market effects*. The simultaneous modification of the welfare costs is explained by efficiency improvements in the economy when payroll taxes drop. Initially, a large labour tax wedge in the Spanish economy distorted labour supply, while the unemployment rate meant unexploited resources. In the new equilibrium both a smaller tax wedge and higher employment increases efficiency and welfare.

3.5 Case C: Effects of reduced VAT

The case of recycling revenue through VAT reductions is illustrated in Figure 3, together with the recycling through payroll taxes in Case B. As for Case B, both curves are shifted downwards compared to the lumpsum case, indicating that the effect of recycling is to generate a labour supply deficit, as well as a

current account improvement. However, none of the shifts are as strong as in the case of payroll tax recycling. In other words, in the labour market positive *L.4:Labour supply effects* are outperformed by *L.3:Home market effects* and *L.2:Competitiveness effects* of lower prices that stimulate labour demand. *L.1:Substitution effects* are not prominent in Case C, as opposed to Case B. The favourable *D.1:Competitiveness effects* also explain the current account improvement. The equilibrium of Case C is marked in the point (WF^C, U^C).

The modification of unemployment is markedly weaker in case of VAT recycling than of payroll tax recycling; see Table 1b. This indicates that unemployment should rather be combated through direct reductions in labour costs. Behind the weaker effects on the unemployment rate lies that the *L.2:Competitiveness effects* and *L.3:Home market effects* are counteracted, though not fully offset, by factor price increases.

The welfare impact is slightly stronger in the VAT recycling case (Case C) than in the payroll recycling case (Case B). This reflects some characteristics of the Spanish economy built into the CGE model. First, there is relatively high indirect commodity taxation through VAT in Spain. Indirect taxation contributes to distort the choice between leisure on the one hand and labour supply and consumption on the other. The increase in consumption relative to leisure due to the VAT reductions, thus, contributes to increase welfare. Second, the VAT reductions contribute to reduce the distorting tax wedge on energy goods. Third, the initial VAT taxation on domestic output tends to outperform the joint VAT and tariff wedge on imports, implying an initial distortion of resource allocation in disfavour of home-made products. In this situation, the relative price reduction of domestic goods resulting from the VAT recycling results in a welfare-improving increase of Spanish market shares at home.

4. Concluding remarks

We can obtain deeper and more exact insight from CGE model studies by exploiting two analytical devices jointly with the information contained by the CGE model. The part of the devices is first of all to sort out where to look for relevant empirical information in the model. This paper exploits two simplified representations of a complex CGE model in order to interpret results of policy reforms or other exogenous

shifts. Combined with knowledge on detailed features of the model, parameters and exogenous variables, the tools add insight into the simulated results, which in the present context have been the impacts of reforms in CO₂, payroll and VAT taxation.

The first device, a two-dimensional graphical exposition of the CGE model, has analogies to partial market equilibrium diagrams, which makes the type of reasoning familiar to economists. However, the analogies should not be stretched too far. Contrary to normal demand and supply curves that have behavioural interpretations, the points out of equilibrium in the present diagrams have no clear-cut interpretations. Interpreting points out of and far from equilibrium makes little sense. These states out-of-equilibrium are described by a mixture of equilibrium and disequilibrium reflections, and some variables are then not well-defined. Therefore, the most interpretable way of regarding, for instance, the *l*-curve (labour market curve) is as a slope in (the near vicinity of) equilibrium, where unemployment-adjusted labour supply and demand are (nearly) the same, as are e.g. income concepts based on these variables. The simulations of the out-of-equilibrium points performed to find those slopes are, thus, but approximations. They are nevertheless useful illustrations of the general equilibrium, given that the shifts are not too large. The graphical analysis will most probably be qualitative illustrations of the directions of the slopes and the shifts.

Caution must also be taken when using stylised miniature models. They abstract from many features of the main model. Whether or not the most influential driving forces are captured will only be obvious after repeated exercises with the CGE model and its simplified representations. The devices combined cannot but give advice on at what places to look for the explanations in the numerical model. They can never replace thorough explorations and sensitivity analyses on the complex model. Knowledge about the numerical model, including its parameters and exogenous variables, will always be the main source of understanding.

Despite their potential of gaining insight and organising analytical results, the labour of establishing the tools cannot be ignored. The more generic and complex the CGE model is and the more frequently it is used for new analytical problems, the more valuable will this up-front investment be. Once the devices are set up, they can be utilised in each forthcoming analysis on the generic model, irrespective of the specific reform or

exogenous shift analysis at hand. Moreover, repeated use of the tools in various studies will provide experience with the CGE model and feed back into the designing of the most adequate miniature model.

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Appendix: Supplementary specifications and explanations of the CGE model

This appendix provides necessary specifications and explanations of the CGE model for deducing the miniature model presented in Section 2.¹³

Production

The production sector of the CGE model is specified by 16 industries (see Table A1), whose production technologies are characterised by detailed, nested structures; see Figure A1.

Table A1: Classification of sectors

1	Agriculture
2	Coal
3	Oil
4	Gas
5	Electricity
6	Water and other energy sources
7	Non-energy minerals, chemicals
8	Metal and machinery
9	Other manufacturing
10	Construction
11	Commerce and hotel trade
12	Road transport
13	Other transport and communications
14	Finance and insurance
15	House renting
16	Other services

There are increasing returns to scale in each representative firm, i , due to fixed costs of labour and capital,

\overline{LF}_i and \overline{KF}_i , valued at their market prices, R and W . The unit profit, Π_i , for each sector i ($i=1, \dots, 16$) is:

$$\Pi_i = P_i^X - \frac{(R\overline{KF}_i + W\overline{LF}_i)E_i}{X_i} - k_i PEVA_i - \sum_{j=1}^{n \neq \text{energy}} k_{ij} P_j^H \quad (\text{A1})$$

P_i^X is unit income of the producers, X_i is the production and E_i is the number of firms in sector i . The k -s are cost share parameters. The upper nest of the cost structure depicted in Figure 1 is a Leontief function of an energy-value added composite and the sum of the other (non-energy) intermediates j . Their respective unit

¹³ In Fæhn et al. (2009), some additional details are available that are left out here because they are not central for understanding the analysis, including calibration and parameter details and references.

costs are $PEVA_i$ and $\sum_{j=1}^{n \neq \text{energy}} P_j^H$. The sequence of CES nests depicted in Figure A1 that defines the unit cost

$PEVA_i$, $i=1, \dots, 16$, is:

$$PEVA_i = \frac{1}{\alpha 1_i} \left(a 1_i^{\sigma_i^{ELK}} PE_i^{1-\sigma_i^{ELK}} + (1-a 1_i)^{\sigma_i^{ELK}} PVA_i^{1-\sigma_i^{ELK}} \right)^{\frac{1}{1-\sigma_i^{ELK}}} \quad (A2)$$

$$PVA_i = \frac{1}{\alpha 2_i} \left(a 2_i^{\sigma_i^{LK}} PL_i^{1-\sigma_i^{LK}} + (1-a 2_i)^{\sigma_i^{LK}} R^{1-\sigma_i^{LK}} \right)^{\frac{1}{1-\sigma_i^{LK}}}$$

$$PE_i = \frac{1}{\alpha 3_i} \left(a 3_i^{0.1} PNE L_i^{1-0.1} + (1-a 3_i)^{0.1} P_{elec}^H^{1-0.1} \right)^{\frac{1}{1-0.1}}$$

$$PNE L_i = \frac{1}{\alpha 4_i} \left(a 4_i^{0.5} PC_{coal}^{1-0.5} + (1-a 4_i)^{0.5} PLIQ_i^{1-0.5} \right)^{\frac{1}{1-0.5}}$$

$$PLIQ_i = \frac{1}{\alpha 5_i} \left(a 5_i^2 PC_{oil}^{1-2} + (1-a 5_i)^2 PC_{gas}^{1-2} \right)^{\frac{1}{1-2}}$$

$$PC_f = a 6_f P_f^H + (1-a 6_f) PCO2, \quad f = coal, oil, gas$$

PE_i , PVA_i , PL_i , $PNE L_i$, $PLIQ_i$ are unit costs of sector i 's input composites of, respectively: energy, value added, labour, non-electricity energy and liquids. PC_f , $f = coal, oil, gas$, are the unit costs of fossil fuels including carbon permits. P^H -s are the domestic market input prices and $PCO2$ the market carbon permit price. α -s and a -s are cost share parameters. This representation allows for sector-specific payroll taxation, t_i^{PR} , so that:

$$PL_i = (1+t_i^{PR})W \quad (A3)$$

According to the first order conditions, the market power of a representative firm in sector i , M_i , is defined by the price-cost margin ratio (Lerner index) that covers fixed costs ($i=1, \dots, 16$) – see (A1):

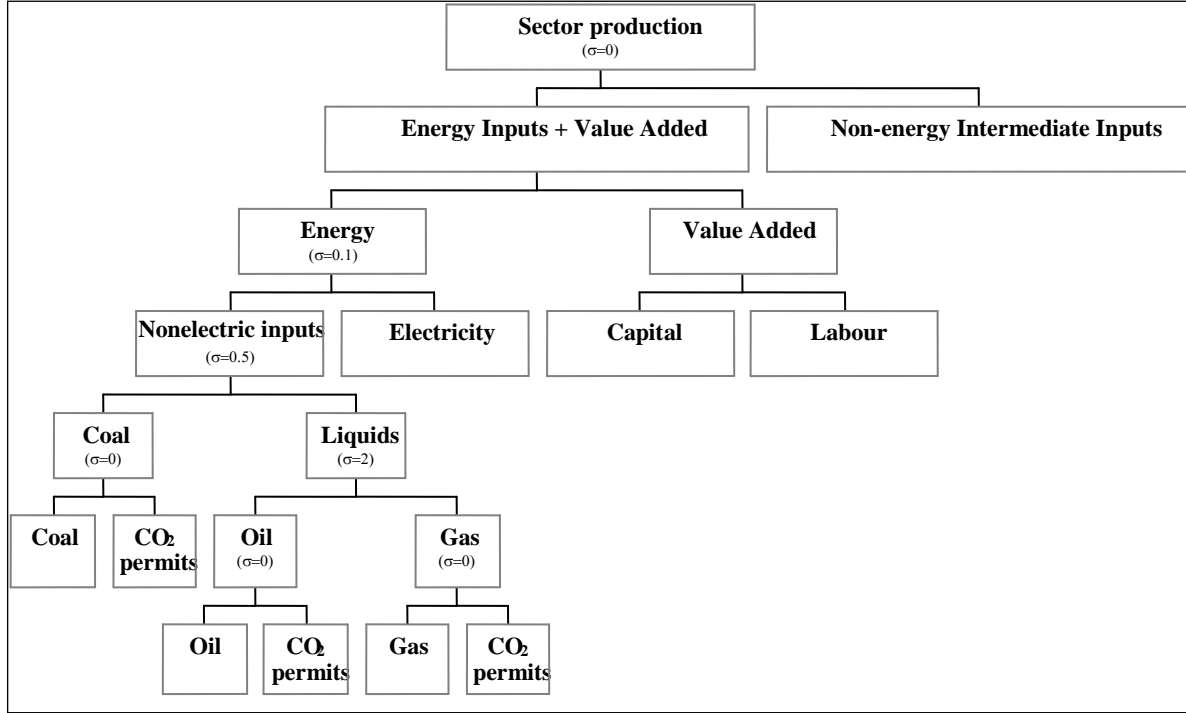
$$M_i = \frac{P_i^X - k_i PEVA_i - \sum_{j=1}^{n \neq \text{energy}} k_{ij} P_j^H}{P_i^X}, \quad (A4)$$

which can be approximated by

$$M_i = \frac{1}{E_i \kappa_i^d} \quad (\text{A5})$$

where E_i the number of firms in the sector and the parameter κ_i^d is the perceived elasticity of demand of the firms in sector i .

Figure A1: Nested cost structures of the production sectors



Unit income of the representative firm in sector i is maximised while accounting for transformation costs of redirecting sales between the domestic and export markets. This is expressed by the following constant-elasticity of transformation relation between unit income and the market prices at home and in the world markets, \overline{PFX} , the latter being exogenous.¹⁴ s_i is the transformation elasticity and ζ_i and d_i are calibrated share parameters:

$$P_i^X = \frac{1}{\zeta_i} \left(d_i^{-\varepsilon_i} P_i^H \varepsilon_i^{+1} + (1 - d_i)^{-\varepsilon_i} (\overline{PFX}_i)^{\varepsilon_i+1} \right)^{\frac{1}{\varepsilon_i+1}} \quad (\text{A6})$$

$$P_i = \left(e_i^{\sigma_i^A} \left(P_i^H (1 + t_i) \right)_i^{1-\sigma_i^A} + (1 - e_i)^{\sigma_i^A} \left(\overline{PFX}_i (1 + t_i) \right)^{1-\sigma_i^A} \right)^{\frac{1}{1-\sigma_i^A}}$$

¹⁴ The exchange rate is numeraire.

Quite analogously (A6) also yields that domestic and imported products are regarded as heterogeneous, and the optimal mixes of them are chosen according to the Armington hypothesis. This determines the consumer prices of each good, P_i , as a function of the domestic price and the world market price. σ_i^A are the Armington elasticities, e_i are share parameters and t_i are indirect taxes, including net VAT.

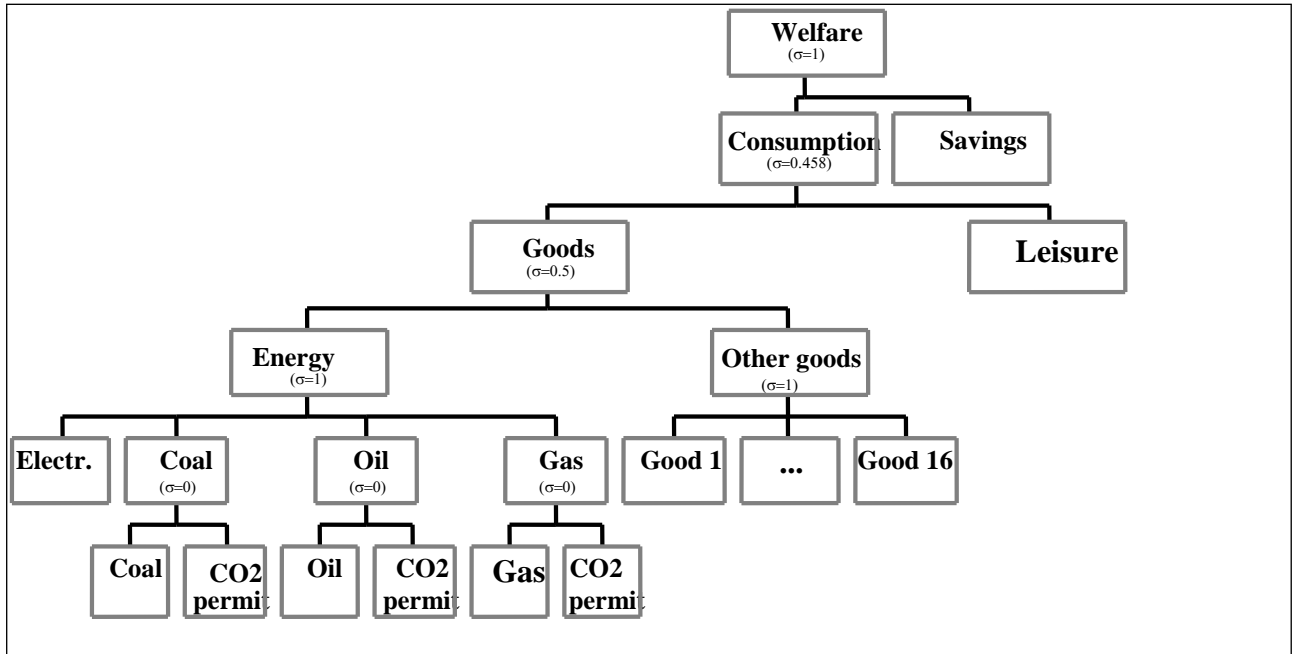
Consumption

The final demand functions are derived from the maximization of the representative consumer's nested welfare function illustrated in Figure A2, subject to the budget constraint:

$$\sum_{i=1}^m P_i F_i = WL^s (1-U) + R\overline{K}_C + TR \quad (A7)$$

where the m consumer goods, F_i , embrace all n produced goods and services, as well as CO₂ permits for the CO₂ emissions from consumption of all fuels and savings.¹⁵ P_i are consumer prices, L^s is labour supply, U is the unemployment rate, \overline{K}_C is the exogenous capital endowment of the consumer and TR are net transfers from the government.

Figure A2: The nested structure of consumption



¹⁵ Savings are given, but the price of savings is endogenous.

The nested demand structure constituting the welfare of the consumer is shown in Figure A2 and specified as:

$$WF = (F_c)^{1-\tau_{sav}} (F_{sav})^{\tau_{sav}} \quad (A8)$$

$$F_c = \left(b1^{\sigma_{cl}} F_{cg}^{1-\sigma_{cl}} + (1-b1)^{\sigma_{cl}} (1-L^s)^{1-\sigma_{cl}} \right)^{\frac{1}{1-\sigma_{cl}}}$$

$$F_{cg} = \left(b3_{en}^{0.1} F_{en}^{1-0.1} + b3_{elec}^{0.1} F_{elec}^{1-0.1} + b3_{ocg}^{0.1} F_{ocg}^{1-0.1} \right)^{\frac{1}{1-0.1}}$$

$$F_{en} = (F_{coal})^{\tau_{coal}} (F_{oil})^{\tau_{oil}} (F_{gas})^{\tau_{gas}}$$

$$F_f = h_f F_f^U + (1-h_f) F_f^{CO2}, \quad f = coal, oil, gas$$

$$F_{ocg} = \prod_{j=1}^{m \neq en} F_i^{\tau_i},$$

WF denotes welfare, which is built up of the m consumer goods in a CES composite structure. Specifically, $F_c, F_{sav}, F_{cg}, F_{en}, F_{elec}, F_{ocg}, F_f, F_f^U, F_f^{CO2}$ denote final demands for the *upper consumption composite*, for *savings*, for the *goods and services composite*, for the *energy composite*, for *electricity*, for the *fossil fuel composite*, for the *fossil fuels* ($f=coal, oil, gas$) including CO_2 emissions, for the *fossil fuels* ($f=coal, oil, gas$) exclusive of CO_2 emissions, and for the CO_2 emission permits for f , respectively. b -s and h -s are share parameters; σ -s and τ -s are elasticities. Note that the demand for leisure is defined as $1-L^s$; i.e. the total time budget is normalised to 1.

Market clearance and public sector

All prices produced and consumed goods are determined by the assumption of market clearance in all markets, i.e., demand equals supply as standard Arrow-Debreu conditions. Investments and savings also balance. The two modifications from the standard model are the modelling of fixed costs and mark-ups in production and the existence of equilibrium unemployment. These characteristics yield the following labour market equilibrium:

$$\sum_{i=1}^n E_i \overline{LF_i} + \sum_{i=1}^n X_i \left(-\frac{\partial \Pi_i}{\partial W} \right) = L^s (1-U) \quad (A9)$$

The desired labour supply net of unemployment (right hand side) balances with the demand for labour, which includes the fixed cost labour component (left hand side). This condition, together with the determination of the price wedge between the market wage rate and the reservation wage, S , below, gives the wage rate, W .

$$S = \frac{1}{(1-U)} \left(\frac{\sum_{i=1}^n X_i \left(-\frac{\partial \Pi_i}{\partial W} \right)}{\sum_{i=1}^n \overline{X_i} \left(-\frac{\partial \Pi_i}{\partial W} \right)} \right)^{\eta_0} \left(\frac{U}{\overline{U}} \right)^{\eta_1} \quad (A10)$$

Variables with bars refer to the benchmark values. The parameters η_0 and η_1 quantify the externalities arising from labour demand and unemployment, respectively.

The market equilibrium for capital that determines R is:

$$\sum_{i=1}^n E_i \overline{KF_i} + \sum_{i=1}^n X_i \left(-\frac{\partial \Pi_i}{\partial R} \right) = \overline{K_C} + \overline{K_G} \quad (A11)$$

The exogenous current account, \overline{D} , is defined as the sum of exports of each good i , A_i net of the sum of imports of each good, B_i , valued at world market prices:

$$\sum_{i=1}^n \overline{PFX_i} A_i - \sum_{i=1}^n \overline{PFX_i} B_i = \overline{D} \quad (A12)$$

The role of the public sector is to set and collect taxes and distribute transfers. Revenue neutral model shifts imply:

$$R \overline{K_G} + \sum_{i=1}^n (T_i^{PR}) + \sum_{i=1}^n (T_i) + CR = \overline{Y_G} + TR \quad (A13)$$

where income components (left hand side) are returns on the given public capital, $\overline{K_G}$, revenues from payroll taxation, T_i^{PR} , indirect taxes, T_i , and carbon tax revenue, CR. The three latter depend on the respective

exogenous tax rates; see (A2), (A3) and (A6), and their endogenous bases. Expenses are lump-sum transfers to households that (as a default modelling) balance public budgets, TR , and all other exogenous net expenses represented by $\overline{Y_G}$.