

Innovation prizes for environmental R&D - supplementary material

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Abstract

This note provides supplementary material to the paper entitled Innovation prizes for environmental R&D. In the paper two types of R&D were analyzed; market goods R&D (M) and environmental R&D (E). Under both types of R&D, the monopoly innovator sets a licence fee. In contrast, the government does not have any policy instrument under the main case of market goods R&D, whereas the government uses an environmental tax under environmental R&D to correct for environmental externalities. Therefore, under environmental R&D three decision sequences are possible; government moves first, simultaneous moves and innovator moves first. In the paper only government moves first was examined. In the present note we discuss the two other possible sequences: simultaneous moves and the innovator moves first.

Further, the case of the government uses emissions quotas (not an emission tax) is analyzed in this note. This case was not covered in the paper, and we therefore analyze all three sequences (government moves first, simultaneous moves and innovator moves first).

Finally, the case of the government uses a product subsidy to stimulate demand for an ordinary market good under market goods R&D was also analyzed in the paper. When the government uses a product subsidy, all three decision sequences are possible. Because only the case in which government moves first was analyzed in the paper, in this note we examine the two other sequences (simultaneous moves and innovator moves first).

We solve all games in the same way as in the paper and use the same notation as in the paper.

1 Emission tax

1.1 Simultaneous moves

Let x^0 be the pre-innovation competitive equilibrium. Hence, x^0 is the solution to *Old Marginal Cost* (γx) being equal to demand ($1 - \beta x$), where γ and β are parameters. Thus, x^0 is given by

$$x^0 = \frac{1}{\gamma + \beta}. \quad (1)$$

Let $\alpha\gamma$ be marginal cost after an innovation and let ℓ^E be the licence fee of the monopoly innovator. As explained in the paper, firms with index up to \hat{x} will choose the old technology, where \hat{x} is determined by $\gamma\hat{x} = \alpha\gamma\hat{x} + \ell^E$, implying

$$\hat{x} = \frac{\ell^E}{\gamma(1 - \alpha)}. \quad (2)$$

Abatement \bar{x}^E is determined by private marginal abatement costs being equal to the emission tax t , i.e., $\alpha\gamma\bar{x}^E + \ell^E = t$, giving

$$\bar{x}^E = \frac{t - \ell^E}{\alpha\gamma}. \quad (3)$$

The innovator sets the licence fee such that her profit is maximized:

$$v^E = \max_{\ell^E} \{ \ell^E [\bar{x}^E - \hat{x}] \}. \quad (4)$$

The government chooses its emission tax t to maximize social benefits minus social costs:

$$\max_t \{ B(\bar{x}^E(t)) - C(\bar{x}^E(t), \hat{x}) \}. \quad (5)$$

Here, $B(\bar{x}^E)$ is the quadratic benefit function, which corresponds to the area under the marginal benefit of abatement function $1 - \beta x$ between 0 and \bar{x}^E . Further, $C(\bar{x}^E(t), \hat{x})$ is the aggregate social abatement cost function, which covers costs for those firms using the old technology ($x \leq \hat{x}$) as well as costs for firms using the new technology:

$$C(\bar{x}^E(t), \hat{x}) = \int_0^{\hat{x}} \gamma x dx + \int_{\hat{x}}^{\bar{x}^E(t)} \alpha\gamma x dx. \quad (6)$$

With simultaneous moves (si) the actors solve (4) and (5) simultaneously, taking (2) and (3) into account. The solution is

$$t_{si} = \frac{2\alpha\gamma}{(\gamma\alpha + \beta)(1 + \alpha)}. \quad (7)$$

$$\ell_{si}^E = \frac{\alpha\gamma(1 - \alpha)}{(\gamma\alpha + \beta)(1 + \alpha)}. \quad (8)$$

Next, we use (2), (3), (7) and (8) to calculate the equilibrium values of \hat{x} and \bar{x}^E (\hat{x}_{si} and \bar{x}_{si}^E) as well as the licence income of the innovator:

$$v_{si}^E = \frac{\alpha\gamma(1 - \alpha)}{(\gamma\alpha + \beta)^2(1 + \alpha)^2}. \quad (9)$$

We then find the social value of innovation V_{si}^E . As explained in the paper, this value can in general be calculated as

$$V = \int_{\hat{x}}^{x^0} (\gamma x - \alpha\gamma x) dx + \int_{x^0}^{\bar{x}} (1 - \beta x) dx - \int_{x^0}^{\bar{x}} \alpha\gamma x dx \quad (10)$$

Finally, the innovation prize is given by

$$P_{si}^E = V_{si}^E - v_{si}^E = \frac{\gamma(1 - \alpha)(\alpha^3\gamma + \alpha^2\gamma - \alpha\gamma + \beta)}{2(\gamma\alpha + \beta)^2(1 + \alpha)^2(\beta + \gamma)}. \quad (11)$$

We now compare the optimal prize under environmental R&D with simultaneous moves with the optimal prize under market goods R&D. In the paper it is shown that the latter is given by:

$$P^M = V^M - v^M = \frac{(1 - \alpha)}{8\gamma(\alpha + \frac{\beta}{\gamma})(1 + \frac{\beta}{\gamma})}. \quad (12)$$

In order to compare (11) to (12) we first define $\theta = \frac{\beta}{\gamma}$ and normalize by setting $\gamma = 1$, see the paper. We then solve the equation $P_{si}^E = P^M$ with respect to θ : $\theta = \frac{\alpha(3\alpha+5)}{\alpha+3} = g(\alpha)$. Because $g'(\alpha) > 0$, the curve $P_{si}^E = P^M$

is upward sloping in an (α, θ) diagram, like Figure 4 in the paper. Further, $P_{si}^E > P^M$ above the graph. Because $g(1) = 2$, for $\theta > 2$ we have $P_{si}^E > P^M$.

1.2 Innovator moves first

When the innovator moves first (i), the government solves (5) by taking the (predetermined) licence fee for given. The resulting tax is thus a function of the licence fee:

$$t_i(\ell^E) = \frac{\alpha\gamma}{\alpha\gamma + \beta} + \ell^E. \quad (13)$$

The innovator solves (4) taking (13) into account. This gives the optimal licence:

$$\ell_i^E = \frac{\gamma(1 - \alpha)}{2(\alpha\gamma + \beta)}. \quad (14)$$

From (13) and (14) we find the optimal environmental tax:

$$t_i = \frac{\gamma(1 + \alpha)}{2(\alpha\gamma + \beta)}. \quad (15)$$

We then use (2), (3), (14) and (15) to find the equilibrium values \hat{x}_i^E and \bar{x}_i^E , and then use these to calculate the income of the innovator (v_i^E) and the social value of innovation V_i^E , using (10). Finally, we find the optimal innovation prize:

$$P_i^E = V_i^E - v_i^E = \frac{\gamma(1 - \alpha)(4\alpha\gamma - 3\gamma + \beta)}{8(\gamma\alpha + \beta)^2(\beta + \gamma)}. \quad (16)$$

Comparing the innovation prize P_i^E to the innovation prize under market goods R&D, see (12), we find

$$P_i^E - P^M = -\frac{3\gamma^2(\alpha - 1)^2}{8(\gamma\alpha + \beta)^2(\beta + \gamma)} < 0. \quad (17)$$

Hence, $P^M > P_i^E$.

To sum up: In the reference case of the government moving first, we found that if $\theta > 0.75$, then the innovation prize should be greatest under

environmental R&D, see Proposition 2 in the main paper. For simultaneous moves we find a similar result: if $\theta > 2$, then the innovation prize should be greatest under environmental R&D. However, when the innovator moves first we obtain a very different result: the innovation prize should always be lowest under environmental R&D.

2 Emission quotas

Let N be the number of firms. In our model each firm emits one unit if it does not abate. Hence, without abatement N is the amount of emissions. Assume now that in order to emit a firm needs a quota. Let Q be the number of quotas, which we assume is decided by the government. Then abatement is

$$\bar{x}_Q^E = N - Q. \quad (18)$$

Like in the previous sections, the firm being indifferent between using the old and the new technology, \hat{x} , is given by (2). We now derive the innovation prize for environmental R&D under alternative assumptions about the sequence of moves.

2.1 Government moves first

The innovator solves (4), taking the predetermined amount of quotas Q as given. This gives us the licence fee as a function of number of quotas:

$$\ell(Q) = \frac{\gamma(1 - \alpha)(N - Q)}{2}. \quad (19)$$

The government solves its optimization problem

$$\max_Q \{B(\bar{x}_Q^E) - C(\bar{x}_Q^E, \hat{x})\} \quad (20)$$

taking (2), (18) and (19) into account. The optimal number of quotas is

$$Q = \frac{-4}{3\gamma\alpha + 4\beta + \gamma} + N. \quad (21)$$

From (19) and (21) we find the optimal licence fee:

$$\ell_Q^E = \frac{2\gamma(1-\alpha)}{3\gamma\alpha + 4\beta + \gamma}. \quad (22)$$

We then use (2), (18), (21) and (22) to find the equilibrium values of \hat{x} and \bar{x}_Q^E , and then use these to calculate the income of the innovator (v_Q^E) and the social value of innovation V_Q^E (using (10)). Finally, we find the optimal innovation prize:

$$P_Q^E = V_Q^E - v_Q^E = \frac{\gamma(1-\alpha)(9\alpha\gamma - 5\gamma + 4\beta)}{2(3\gamma\alpha + 4\beta + \gamma)^2(\beta + \gamma)}. \quad (23)$$

Comparing the innovation prize P_Q^E to the innovation prize under market goods R&D, see (12), we find

$$P_Q^E - P^M = -\frac{\gamma^2(\alpha-1)^2(27\alpha\gamma + \gamma + 28\beta)}{8(3\gamma\alpha + 4\beta + \gamma)^2(\gamma\alpha + \beta)(\beta + \gamma)} < 0. \quad (24)$$

Hence, $P^M > P_Q^E$.

2.2 Simultaneous moves

With simultaneous moves (*si*) the actors solve (4) and (20) simultaneously, taking (2) and (18) into account. The solution is

$$Q_{si} = \frac{-1}{\gamma\alpha + \beta} + N. \quad (25)$$

$$\ell_{Q,si}^E = \frac{\gamma(1-\alpha)}{2(\gamma\alpha + \beta)}. \quad (26)$$

We then use (2), (18), (25) and (26) to find the equilibrium values $\hat{x}_{Q,si}$ and $\bar{x}_{Q,si}^E$, and then use these to calculate the income of the innovator ($v_{Q,si}^E$) and the social value of innovation $V_{Q,si}^E$. Finally, we find the optimal innovation prize:

$$P_{Q,si}^E = V_{Q,si}^E - v_{Q,si}^E = P_i^E. \quad (27)$$

Here, P_i^E is the innovation prize when the innovator moves first and the government uses an emission tax, see (16). From the discussion above we know that $P^M > P_i^E$. Note that $\ell_{Q,si}^E = \ell_i^E$, $v_{Q,si}^E = v_i^E$ and $V_{Q,si}^E = V_i^E$.

2.3 Innovator moves first

When the innovator moves first (i), the government solves (20) by taking the (predetermined) licence fee for given. The resulting number of quotas is

$$Q_i = \frac{-1}{\alpha\gamma + \beta} + N. \quad (28)$$

As seen from (28), the number of quotas is independent of the licence fee, which is due to our assumptions about functional forms.

The innovator solves (4) taking (28) into account. This gives the optimal licence:

$$\ell_{Q,i}^E = \frac{\gamma(1 - \alpha)}{2(\alpha\gamma + \beta)} = \ell_i^E. \quad (29)$$

We then use (2), (18), (28) and (29) to find the equilibrium values $\hat{x}_{Q,i}^E$ and $\bar{x}_{q,i}^E$, and then use these to calculate the income of the innovator ($v_{Q,i}^E$) and the social value of innovation $V_{Q,i}^E$, using (10). Finally, we find the optimal innovation prize:

$$P_{Q,i}^E = V_{Q,i}^E - v_{Q,i}^E = P_i^E, \quad (30)$$

where again P_i^E is the innovation prize when the innovator moves first and the government uses an emission tax, see (16). From the discussion above we know that $P^M > P_i^E$. Note that the two cases with the innovator moves first - government uses an emission tax or government uses quotas - have identical income of the innovator and identical social value of innovation ($v_{Q,i}^E = v_i^E$ and $V_{Q,i}^E = V_i^E$).

To sum up: When the government uses quotas as the environmental policy instrument, then the innovation prize should always, that is, for all sequences of moves, be greatest under market goods R&D. This result differs significantly from the reference case in the main paper where the government

moves first and uses an environmental tax. Then the innovation prize should be greatest under environmental R&D provided $\theta > 0.75$.

3 Product subsidy

We now examine the case of the government offering a product subsidy s to consumers of the market good (M). Demand for the good is then $p - s = 1 - \beta x$. In the paper we discussed the case when the government decides the product subsidy before the innovator sets her licence fee. Below we therefore examine the two other decision sequences: simultaneous moves and innovator moves first. Note that the firm being indifferent between the old and the new technology, \hat{x} , is still given by (2) (with ℓ^E replaced with ℓ^M).

3.1 Simultaneous moves

Total production is determined in a competitive equilibrium such that the *New Private Marginal Cost* ($\alpha\gamma x + \ell^M$) is equal to demand ($1 + s - \beta x$). Hence, total production is given by

$$\bar{x}_s^M(s, \ell^M) = \frac{1 + s - \ell^M}{\alpha\gamma + \beta}. \quad (31)$$

The innovator sets the licence fee such that her profit is maximized:

$$v^M = \max_{\ell^M} \{ \ell^M [\bar{x}_s^M(s, \ell^M) - \hat{x}] \}. \quad (32)$$

The government chooses its product subsidy s to maximize social benefits minus social costs:

$$\max_s \{ B(\bar{x}_s^M(s, \ell^M)) - C(\bar{x}_s^M(s, \ell^M), \hat{x}) \}. \quad (33)$$

With simultaneous moves (*si*) the actors solve (32) and (33) simultaneously, taking (2) and (31) into account. The solution is

$$s_{si} = \frac{\gamma(1 - \alpha)}{\gamma(1 + \alpha) + 2\beta} = \ell_{s,si}^M. \quad (34)$$

We then use (2), (31) and (34) to find the equilibrium values $\hat{x}_{s,si}$ and $\bar{x}_{s,si}^M$, and use these to calculate the income of the innovator ($v_{s,si}^M$) and the social value of innovation $V_{s,si}^M$. The optimal innovation prize is now:

$$P_{s,si}^M = V_{s,si}^M - v_{s,si}^M = \frac{\gamma(1-\alpha)(\alpha^2\gamma^2 + 3\alpha\beta\gamma + \alpha\gamma^2 + \beta^2 - \beta\gamma - \gamma^2)}{2(\gamma\alpha + 2\beta + \gamma)^2(\beta + \gamma)(\gamma\alpha + \beta)}. \quad (35)$$

Finally, we compare $P_{s,si}^M$ to the innovation prize with environmental R&D when decisions are taken simultaneously (and the government uses an environmental tax): P_{si}^E , see (11). We find that

$$P_{s,si}^M - P_{si}^E = -\frac{\gamma\beta(\alpha-1)^2(\alpha\beta + 2\alpha\gamma + 3\beta + 2\gamma)}{2(\alpha+1)^2(\gamma\alpha + 2\beta + \gamma)^2(\gamma\alpha + \beta)^2} < 0. \quad (36)$$

Hence, $P_{si}^E > P_{s,si}^M$.

3.2 Innovator moves first

In this case the government solves its maximization problem (33) taking the (predetermined) licence fee for given. This gives us a relationship between the product subsidy and the licence fee:

$$s_i = \ell^M, \quad (37)$$

that is, the government will set its product subsidy equal to the licence fee. The innovator solves (32), taking (2), (31) and (37) into account. The resulting optimal licence fee is

$$\ell_{s,i}^M = \frac{\gamma(1-\alpha)}{2(\alpha\gamma + \beta)}. \quad (38)$$

We then use (2), (31), (37) and (38) to find the equilibrium values $\hat{x}_{s,i}$ and $\bar{x}_{s,i}^M$, and use these to calculate the income of the innovator ($v_{s,i}^M$) and the social value of innovation $V_{s,i}^M$. Finally, we find the optimal innovation prize:

$$P_{s,i}^M = V_{s,i}^M - v_{s,i}^M = P_i^E. \quad (39)$$

Hence, with a product subsidy the innovation prize under market goods R&D is equal to the innovation prize under environmental R&D when the

government uses an environmental tax and the innovator moves first. Finally, note that $v_{s,i}^M = v_i^E$ and $V_{s,i}^M = V_i^E$.

To sum up: In the main paper we studied the case of the government using a product subsidy and moving first. We found that under these assumptions the innovation prize should always be greatest under environmental R&D. With simultaneous moves we obtain the same result, whereas the two prizes should be equal if the innovator moves first.