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Abstract in Norwegian:



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Grunner for å styre FoU mot nullutslippsteknologi

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Bør forskning på miljøvennlige teknologier støttes mer enn forskning på miljøfiendtlige teknologier, eller bør støtten til FoU være teknologinøytral? I denne artikkelen studerer vi dette spørsmålet ved å bygge videre på nyere forskning rundt dette temaet, ikke minst arbeidet til Acemoglu, Aghion, Bursztyn og Hémous (AABH) som ble publisert i *American Economic Review* i 2012. AABH konkluderer at målrettet støtte til miljøvennlig FoU er et veldig viktig supplement til optimal miljøpolitikk.

I motsetning til AABH legger vi til grunn at nye innovasjoner kan gi profitt til oppfinneren i mange år. I tillegg antar vi at det er avtakende utbytte av FoU på kort sikt. Ved første øyekast ville man tro at dette skulle gjøre målrettet støtte til miljøvennlig FoU mindre essensielt enn i studien til AABH, gitt at miljøpolitikken er optimalt og langsiktig bestemt, ettersom framtidig miljøpolitikk i større grad vil være med og påvirke dagens FoU-innsats.

Vi finner imidlertid at målrettet støtte til miljøvennlig FoU er viktig også i vår analyse, sammen med en skatt på utslipp. Alvorlige og langsiktige miljøproblemer krever at FoU-innsatsen flyttes over til miljøvennlige teknologier. Forskjellen mellom privatøkonomisk og samfunnsøkonomisk nytte av innovasjoner er større for miljøvennlige teknologier i en overgangsfase der teknologiene har stor kunnskaps- og produktivitetsvekst. Dette medfører at det er optimalt for myndighetene med målrettet støtte til slike teknologier.

On the rationale for directing R&D to zero emission technologies^{*}

Mads Greaker, Tom-Reiel Heggedal[†] and Knut Einar Rosendahl[§]

Abstract

Are there reasons to support innovation on 'clean' technologies more than on 'dirty' technologies? We inquire into this question, introducing two novelties in the modelling framework of the recent literature on this topic: We allow innovation profits to survive longer than one period, and introduce decreasing returns to R&D at any point in time.

At first glance, both aspects should make targeted R&D support less crucial. That is, innovations that not only give instantaneous profits imply that future environmental policies can redirect research today, and decreasing returns induce R&D to take place in both clean and dirty sectors.

Surprisingly, we find that governments should nonetheless support clean R&D more than dirty R&D. Dealing with a major environmental problem effectively requires R&D effort to shift to the clean technology. However, when a majority of researchers work with clean technology, both productivity spillovers and the future risk of being replaced increase. Consequently, the wedge between the private and the social value of an innovation is larger for clean technologies than for dirty technologies along the transition path.

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Keywords: environment, directed technological change, innovation policy.

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1 Introduction

Reducing the share of fossil fuels in the energy mix is a major challenge for climate change policy.¹ The costs and the qualities of technologies are major drivers in the choice of energy source. Research and development (R&D) drives down costs and improves technologies, and hence, facilitates the diffusion of new, clean technologies. On the other hand, this argument also holds for dirty technologies. For instance, improvements in "fracking-technology" recently made it possible to extract natural gas from under-ground shale layers, cutting the cost of electricity from natural gas, and thus reducing the relative attractiveness of clean power technologies.

A central analysis of the competition between clean and dirty technologies is the recent contribution by Acemoglu, Aghion, Bursztyn, and Hemous (2012) (henceforth AABH). They analyze R&D subsidies and carbon taxes in a model with clean and dirty technologies in which the clean technology starts off as less advanced. They argue that the gap between clean and dirty technologies will widen in the absence of targeted subsidies to clean R&D, and that it is optimal to shift all R&D effort from dirty R&D to clean R&D either immediately or in a few years' time. This can be accomplished by subsidizing clean R&D such that all research on fossil fuel technologies is brought to a halt.

In our opinion it is not clear whether this result rests on the special assumptions in the AABH model or if the result is robust to other modelling choices for the innovation sector. Firstly, AABH assume that a scientist only enjoys the current period monopoly profits, which implies that future climate policies are unable to redirect research today. In the literature on economic growth, innovators typically enjoy monopoly profits of an innovation for an extended time. Either they have an infinite monopoly such as in Romer (1990), or their innovation gets replaced with some probability in every period, as in Aghion and Howitt (1992) and Grossman and Helpman (1991).² In any case, the length of the monopoly period has implications for environmental policy as well as innovation policy.³

Secondly, AABH (2012) assume that there are no decreasing returns to R&D within a period. As a result they obtain a corner solution for the allocation of the R&D effort: Either all researchers do dirty R&D, or they all do clean R&D. Clearly, this aspect of the model could lead to more drastic policies than if there were an internal solution for the allocation of researchers. Jones and Williams (2000) include a "stepping-on-toes" effect, which says that the chance of coming up with the same idea as your fellow researchers increases the more researchers there are at each point in time. This effect will give decreasing static returns to R&D, and ensure an internal solution for the allocation of researchers between clean and dirty R&D.

In this paper we pose the following research questions: (I) Under what circumstances should innovation effort be directed away from dirty technologies and into clean technologies? and II) If so, what are the mechanisms leading to this result? Clearly, from a policy

¹In order to keep global warming below the 2°C target, a third of oil reserves, a half of gas reserves, and more than 80 percent of coal reserves must stay in the ground (McGlade og Ekins, 2015), while IEA (2011) predicts a 503 growth in total energy demand in the next 25 years. Hence, the production of clean energy must increase dramatically.

²Long lived patents is also the standard assumption in models of directed technological change (see Acemoglu, 2002; 2009)

³See e.g. Gerlagh, Kverndokk and Rosendahl (2014).

point of view, knowledge about these questions is crucial. Today total R&D effort on fossil technologies is greater than R&D on clean technologies.⁴ Turning this around may require drastic intervention, hence, research should inquire further into the validity of the theoretical argument.

Our point of departure is the AABH model with clean inputs and dirty inputs to final goods production. The dirty input gives emissions that deteriorate the environmental stock, and this stock enters positively into households utility. The productivity of the inputs may be improved by innovation, and R&D firms choose to innovate on either clean or dirty technologies based on expected profits.

We depart from AABH by relaxing the two aforementioned assumptions for the innovation sector. First, we let R&D firms retain profits on an innovation until it is replaced by an innovation of better quality. Second, we introduce duplication effects by having decreasing returns to the number of scientist innovating in a technology. At first glance both these relaxations should make targeted R&D support less crucial. That is, longer lived monopolies based on innovations imply that future environmental policies can redirect research today, and decreasing returns forces R&D to happen in both sectors independent of the level of accumulated productivity.

Surprisingly, both our theoretical results and our numerical simulations suggest that governments should nonetheless support clean R&D more than dirty R&D. However, the mechanism is not that the research subsidy deals with future emission externalities as stated by AABH.⁵

Dealing with the environmental problem effectively requires final good production to shift from dirty to clean inputs. Knowing this, the government will want to shift productivity growth from the production of dirty inputs to the production of clean inputs. When the growth rate is larger in a technology, the spillovers are larger in that technology due to state dependent innovations. Each quality increase is larger than the last, and this increasing standing on the shoulders effect is not taken into account by private firms. This implies that the intertemporal knowledge spillover is larger in the clean R&D sector than in the dirty sector along the optimal growth path.⁶

Furthermore, this is not the only reason to prioritize clean R&D. There is also a replacement effect, that is, when a majority of the researchers work with clean technology, the future risk of someone coming up with a better innovation increases, and so does also the probability of losing the monopoly profit. Thus, both the spillover effect and the replacement effect are larger in the sector with the higher productivity growth rate. Consequently, the wedge between the private and the social value of an innovation is larger for clean technologies than for dirty technologies, and a subsidy to clean R&D is the first-best policy for sustaining the optimal allocation of R&D.

⁴Fossil fuels are also subsidized more. The International Energy Agency (IEA, 2014) has estimated consumer subsidies to fossil fuels at US\$548 billion in 2013, while subsidies to renewable energy amounted to US\$121 billion.

⁵According to AABH 'the subsidy deals with future environmental externalities by directing innovation towards the clean sector, whereas the carbon tax deals more directly with the current environmental externality by reducing production of the dirty input'.

⁶Heggedal (2014) finds a similar result in a Romer (1990)-type growth setting, though without environmental considerations.

It is well known that when there is more than one market failure, it is socially optimal to have a set of policy instruments, each targeting one of the market failures, e.g. a tax on carbon emissions and a subsidy for R&D.⁷ This is also the case in our model, and we derive expressions that are useful for analyzing optimal policies. We simulate the model numerically using the same parameter values as AABH, and find that a subsidy to clean innovation together with a tax in emissions is optimal. Further, we find that a subsidy to clean R&D welfare dominates an emission tax as a stand-alone policy. However, the welfare differences between the stand-alone policies are smaller than in AABH. The reason is that a future emission tax influences the value of clean innovation today and thus reduces dependence on subsidies to direct technological change. Moreover, we find that subsidies to clean R&D should be distributed out over time (and in many cases at a rising rate). In AABH's model, temporary subsidies may be optimal, as all scientists would stay in the clean technology innovation when it has surpassed the quality of the dirty. In our model, corner solutions are avoided, and R&D should be subsidized in all time periods due to market failures in innovation.

Whether subsidizing clean innovation is important for welfare depends on the elasticity of substitution between clean and dirty inputs to production. When this elasticity is high, second-best emission taxes increase considerably compared to the first-best in case there is no innovation subsidy. In contrast, when elasticity is low, second best emission taxes only increase slightly. In contrast, emission policy is important for welfare regardless of the elasticity of substitution between production inputs. In case there are no emission taxes, for all scenarios, innovation subsidies increase considerably compared to first-best. The reason is that, in contrast to AABH, emission taxes are important for directing innovation towards the clean inputs. Thus, this paper reestablishes the role of the emission tax in a framework of directed technological change.

Related literature

There is recent literature on directed technological change and the environment (see Heutel and Fischer (2013) for an overview on macroeconomics and the environment). Several papers modify and simulate the AABH model, though in different directions and analyzing other problems than in the present paper: Hourcade, Pottier, and Espagne (2011) discuss parameter choices related to the climate part of the model; Mattauch, Creutzig, and Edenhofer (2012) add learning-by-doing effects to the framework ; Durmaz and Schroyen (2014) extend the model by adding abatement technology (carbon capture and storage); David Hemous (2014) and van den Bijgaart (2014) extend the model to include more than one country and analyze unilateral environmental policies in a global context. Importantly, none of these papers explore profits in the innovations that are retained until replaced by a better quality, so that future emission policies affect innovation decisions today.

A key assumption in our (and AABH's) model is that innovation is path (state) dependent. A new innovation builds on past quality and increases the productivity of future innovations. Such path dependency is found by Aghion, P., Dechezlepretre, Hemous, Martin, and Van Reenen (2014). They analyze clean and dirty technologies in the automotive industry, and find that there is path dependence in innovation following from spillovers and

⁷ See e.g. Goulder & Schneider (1999); Rosendahl (2004); Gillingham, Newell & Pizer, (2008); Fischer & Newell (2008); Popp et al (2010).

the firms' histories. Moreover, that productivity spillovers is a rationale for subsidizing clean innovation has empirical support. Further, in a recent paper Dechezlepretre, Martin, and Mohnen (2013) find that spillovers are larger in clean than dirty technologies. The driving force behind the result seems to be that clean technologies are newer technologies than dirty, and that a new technology field has larger spillovers than an old technology field.

On the theory side, Acemoglu, Akcigit, Hanley, and Kerr (2014) develop another model of endogenous growth with clean and dirty R&D where they model the R&D sector differently from AABH. In their model clean and dirty machines within a product line are perfect substitutes, and hence, in order to have a market, a new clean machine must in most cases outcompete the dirty machine within the same product line. This only happens rarely, and thus, innovators may not get any profits from clean R&D at all even if they improve the clean machine. As in AABH, they also find that carbon taxes may be expensive to rely on alone, and that targeted subsidies to clean R&D are a crucial part of climate policy.

The paper is organized as follows. Section 2 presents the model and the decentralized market allocation, while Section 3 shows the socially optimal R&D allocation and discusses efficient innovation and emission policies. The model is simulated numerically and the results for optimal policies are given in Section 4, while Section 5 provides a conclusion.

2 The model

The AABH model is an infinite-horizon discrete-time economy with households, a final goods sector, a clean and a dirty intermediate input sector, a machine sector that delivers machines of different qualities to the intermediate input sectors, and finally, an innovation sector that may improve these qualities. The major difference between AABH's model and our model is the innovation sector. We therefore emphasize the innovation sector in the presentation of the model, and cover the rest of the model more briefly.

2.1 Final goods

The final good is used for the production of machines and for consumption C_t , and it is produced by combining dirty and clean intermediates. The production function for this good is given by:

$$Y_t = Y_{ct}^{\frac{s-1}{s}} + Y_{dt}^{\frac{s-1}{s}} \quad (1)$$

where Y_{ct} and Y_{dt} is the input of clean and dirty inputs, respectively, and c is the elasticity of substitution. It is hard to know a priori what the elasticity ought to be, but it seems reasonable that the two inputs cannot substitute each other perfectly e.g. solar- and wind energy are intermittent and may require dirty back-up power.

2.2 Production of intermediates with a carbon tax

The production of dirty and clean intermediates uses labor and machines. Machines are given in different varieties i which are specific for either clean or dirty intermediate production. The production function for clean and dirty intermediates in sector $j \in \{c, d\}$ is given by:

$$Y_{jt} = L_{jt}^{1-a} \int_0^1 A_{jit}^{1-a} x_{jit}^a di \quad (2)$$

where $a \in (0, 1)$, L_{jt} is labor use in sector j , A_{jit} is the quality (productivity) of machine type i in sector j at time t , x_{jit} is the input of machine type i in sector j at time t , and the number of machine types is 1. Everytime a new innovation is made in one of the sectors, one particular machine type i is replaced by a better machine of the same type. The innovation is drastic, implying the older version of the machine type no longer can be sold with positive profits.

The intermediate firm's problem is:

$$\max_{L_{jt}, x_{jit}} (p_{jit} - T_{jt}) L_{jt}^{1-a} \int_0^1 A_{jit}^{1-a} x_{jit}^a di - w_t L_{jt} - \int_0^1 p_{jit} x_{jit} di$$

where p_{jit} is the price of the intermediate input of type j , T_{jt} is the carbon tax ($T_{ct} = 0$) and p_{jit} is the price of machine type i in sector $j \in \{c, d\}$. The demand for machine type i is found from the first order condition for the optimal use of machine i :

$$x_{jit} = \frac{(p_{jit} - T_{jt}) a}{p_{jit}} L_{jt}^{1-a} A_{jit} \quad (3)$$

Equation (3) is the demand function for clean and dirty machines. We note that demand depends positively on their productivity A_{jit} and the amount of labor L_{jt} entering either the clean or the dirty sector.

The demand for labor in sector j is given from the first order condition for the optimal use of labor in each sector:

$$(1 - a)(p_{jit} - T_{jt}) L_{jt}^{-a} \int_0^1 A_{jit}^{1-a} x_{jit}^a di - w_t = 0. \quad (4)$$

By rearranging (4) we have:

$$L_{jt} = \frac{(1 - a)(p_{jit} - T_{jt}) Y_{jit}}{w_t}. \quad (5)$$

Both (4) and (5) will be used later when we compare the decentralized market allocation with the socially optimal allocations of researchers.

2.3 Production of machines

A producer of a machine type ji is a monopolist and solves:

$$\max_{p_{jit}} [(p_{jit} - 1/(1 - s)) x_{jit}] \quad (6)$$

where demand x_{jit} is given by (3) above, $1/$ is the unit cost of a machine (measured in units of the final good), and s is a subsidy to correct for the static monopoly distortion. Costs are normalized to $1/ = a^2$, and the efficient subsidy rate that gives price equal to marginal cost is $s = 1 - a$, which we assume is implemented.⁸ Then, solving (6) gives the profit maximizing price on machines $p_{jit} = a^2$. Inserting back into (6), and using (3), we obtain for the per period profit Jr_{jit} of a machine producer:

$$Jr_{jit} = a(p_{jit} - T_{jt})^{1-a} L_{jt} A_{jit}$$

⁸See subsection 6.2 in the Appendix

where $a = (1 - a) a^{\frac{1-2}{1-}}$. Note that profits are only derived from holding a patent with the highest quality in each machine type.⁹

2.4 Innovation and allocation of scientists

In each period, a scientist engages in either clean or dirty innovations, and gains profits if she innovates. When a new innovation is made in machine type i , A_{jit} bumps up to $(1 + 1)A_{jit}$, where $(1 + 1)$ is the quality step rate. A scientist can choose sector, but not target a specific machine type; instead a scientist is randomly allocated to a machine type in the specific sector. Thus, the scientist makes her decision based on the average machine quality in sector A_{jt} which is given by:

$$A_{jt} = \int_0^1 A_{jit} di. \quad (7)$$

A scientist engaged in innovation in sector j then expects a quality $(1+1)A_{jt}$ upon successful innovation.

The mass of scientists in one sector is given by fl_{jt} , and we normalize the number of scientists such that $fl_{ct} + fl_{dt} = 1$. We assume that scientist earn profits on an innovation until their machine is replaced by a new machine of better quality. At each point in time there is a probability that someone successfully invents a better quality which we denote by z_{jt} .

Further, we assume that there may be duplication by other scientists, i.e. more than one scientist may have the same successful innovation in a given period. We let the duplication effect be represented by decreasing returns to labor input on aggregate sector innovation given by the function $fl_{jt}^{r_0}$ where $w \in (0, 1)$. The probability of a successful innovation in sector j is then given by $ry_j fl_{jt}^{r_0}$ where ry_j is a parameter.

The expected discounted profits IT_{jt} of a single scientist entering sector j at time t is then given by:

$$IT_{jt} = ry_j (fl_{jt})^{(r_0-1)} a (1 + 1) A_{jt-1} \prod_{k=0}^{\infty} \prod_{v=1}^{\infty} \frac{1 - z_{j,t+v}}{1 + r_{t+v}} \left((p_{j,t+k} - T_{j,t+k})^{1-\tau} L_{j,t+k} \right) \quad (8)$$

where r_t is the scientist's discount rate and $(fl_{jt})^{(r_0-1)}$ is the average productivity of a scientist in sector j . Since the average productivity of a scientist is declining in the number of scientists, we do not get a corner solution for the allocation of researchers as in AABH.

Furthermore, equation (8) includes the multiplicative term $IT_v (1 - z_{j,t+v})$ which denotes the probability of an innovation in technology j surviving from period t until period v . The probability of being replaced z_{jt} is given by $ry_j fl_{jt}^w$, that is, the probability that an innovation occurs divided by the number of machine lines, which is normalized to unity. Thus, the multiplicative term will be declining in the amount of researchers working with technology j .

Equation (8) also includes the discounted stream of future profits from an innovation $L_{j,t+k} (p_{j,t+k} - T_{j,t+k})^{1-\tau} L_{j,t+k}$, which among other things, depends on future tax rates. In

⁹As AABH, we assume that the quality difference between a new and old machine is sufficiently large, so that firms can charge the unconstrained monopoly price of the new machine. Further, the quality difference is large enough to avoid infringement problems related to patent breadth.

contrast, AABH only allow the scientists to retain profits in the same period as the innovation occurs. After that period the ownership of the technology is returned to the machine producers without compensation.

Introducing long-lived patents may have significant implications for policy. Let's say that the current per period profits are greater in the dirty sector and that the carbon tax rate rises over a number of future periods. The tax increases the value of clean machines relative to dirty machines over time. Scientists do not take into account the effect of future taxes if patents last for one period and they engage in dirty innovations. On the other hand, if patents are long-lived, scientists take into account that the value of clean machines improves over time. A switch to clean innovation may then be induced today without the need for innovation subsidies.

The decentralized allocation of scientist is given by that in equilibrium the expected profits must be the same for both sectors:

$$IT_{ct} = IT_{dt} \tag{9}$$

$$\frac{fl_{ct}}{1 - fl_{ct}} = \frac{\phi_0 \sum_{k=0}^{\infty} \sum_{v=1}^{\infty} \frac{ry_c A_{ct-1}^{1-\theta} (1-z)^{k+v} (p_{c,t+k})^{1-\alpha} L_{c,t+k}^{1-\alpha}}{1+r_{t+v}}}{\sum_{k=0}^{\infty} \sum_{v=1}^{\infty} \frac{ry_d A_{dt-1}^{1-\theta} (p_{d,t+k} - T_{d,t+k})^{1-\alpha} L_{d,t+k}^{1-\alpha}}{1+r_{t+v}}}$$

where $1 - fl_{ct} = fl_{dt}$. We will discuss equation 9 and how it relates to optimal policies in Section 3.2.

Note that in every period, scientist only base their choice of sector on the average past quality of machine types. Given the allocation of scientist, the average quality of the machine types develops according to:

$$A_{jt} = (1 + 1ry_c(fl_{jt})^{\theta})A_{jt-1} \tag{10}$$

This is also different from AABH as the total productivity of the scientist depends on the number of scientists through the term $(fl_{jt})^{\theta}$.

2.5 Consumers and the environment

There is a continuum of households with measure 1 that all have preferences:

$$\sum_{t=0}^{\infty} \frac{1}{(1+p)^t} u(C_t, S_t)$$

where p is the discount rate of the households, C_t is consumption, and S_t is the environmental quality. The instantaneous utility function $u(C_t, S_t)$ has positive first-order derivatives.

There is no asset for saving in the economy so all final goods are consumed or used as (converted) inputs in the production process in each period. The households hold equal shares of all the assets in the economy (labor income and R&D firms' (scientists') profits). Then the discount rate for the R&D firms follows from the households' valuation of getting

income in a future period. Hence, the firms' discount factor R_t for a payoff in period t seen from period zero is:

$$\frac{1}{(1+p)^t} \frac{\partial u(C_t, S_t)}{\partial C} = R_t \quad (11)$$

where $R_t = \prod_{v=0}^{t-1} \frac{1}{1+r_v}$ and r_t is the interest rate following from a standard Euler equation (see Appendix 5 for a derivation). Note that this derivation is not in the AABH model since they do not need to discount future profits from an innovation.

The law of motion for the quality of the environment is:

$$S_{t+1} = -\delta Y_{dt} + (1 + \delta)S_t \quad (12)$$

where δ denotes the rate of degradation stemming from emissions from the dirty input Y_{dt} , while δ is the rate of environmental regeneration. More details on the relationships for the environment can be found in Appendix 5.

3 Socially optimal policies

In this section we first calculate the first order conditions of the planner's problem. Subsequently, we compare the socially optimal allocation of scientist to clean and dirty R&D with the decentralized market allocation of scientists and then discuss optimal policies.

3.1 Socially optimal allocation

The planner's problem reads:

$$\begin{aligned} \max_{L_{jt}, f_{jt}} \quad & \sum_{t=0}^{\infty} \frac{1}{(1+p)^t} u(C_t, S_t) \\ \text{s.t.} \quad & C_t = Y_t - \frac{1}{\theta} \left[\int_0^1 x_{cit} di + \int_0^1 x_{dit} di \right] \\ & Y_{jt} = L_{jt}^{1-a} \left[\int_0^1 A_{jit}^{1-a} x_{jit}^a di \right] \\ & A_{jt} = (1 + r y_j (fl_{jt})^{\rho}) A_{jt-1} \\ & S_t = -\delta Y_{dt-1} + (1 + \delta) S_{t-1} \\ & L_{ct} + L_{dt} \leq 1 \\ & fl_{ct} + fl_{dt} \leq 1 \end{aligned} \quad (13)$$

given $A_{c0} < A_{d0}$ and S_0 , where $\frac{1}{\theta} \left[\int_0^1 x_{cit} di + \int_0^1 x_{dit} di \right]$ is the total expenditure of final goods in the production of intermediate goods.¹⁰

The full set of first order conditions following from the planner problem is given in Appendix 5. In this section we discuss aspects of the planner solution that are directly relevant for optimal policy.

First, from the first order condition with respect to consumption:

$$\frac{1}{(1+p)^t} \frac{\partial u(C_t, S_t)}{\partial C} = \dots \quad (14)$$

¹⁰The equilibrium exists and is unique as the objective function is continuous and strictly concave with a convex constraint set.

we see that the shadow value of the final good λ_t is given by the discounted marginal value of consumption in period t , i.e. the social discount factor. Notice the close connection between λ_t and the market discount factor R_t from equation (11). We are later going to utilize that $\lambda_t = R_t$ if the market solution is efficient.

The shadow value of environmental quality w_t is given by

$$w_t = \sum_{v>t} (1 + \delta)^{v-t} \frac{1}{(1 + p)^v} \frac{\partial u(C_v, S_v)}{\partial S} \mathbb{1}_{S_v < S}$$

where δ is the rate of environmental regeneration. Note that $\mathbb{1}_{S_t < S} = 1$ if $S_t < S$, and $\mathbb{1}_{S_t < S} = 0$ otherwise since $\mathbb{1}_{S_t < S} = 1$ if $S_t < S$, and $\mathbb{1}_{S_t < S} = 0$ otherwise, since $S_{t+1} = \delta Y_t - (1 + \delta)S_t$ only in the interval $(0, S)$ (S is the maximum environmental quality).

Next we have, for the optimal production and use of the two intermediates, Y_{ct} and Y_{dt} :

$$\left(Y_{ct}^{\frac{1}{\sigma}-1} + Y_{dt}^{\frac{1}{\sigma}-1} Y_{ct}^{-\frac{1}{\sigma}} - \lambda_{ct} \right) = 0 \quad (15)$$

$$\left(Y_{ct}^{\frac{1}{\sigma}-1} + Y_{dt}^{\frac{1}{\sigma}-1} Y_{dt}^{-\frac{1}{\sigma}} - \lambda_{dt} - \delta w_{t+1} \right) = 0.$$

The first term in both expressions in (15) is the marginal increase in final goods production from an additional unit of the input. The next term in both expressions e.g. λ_{ct}/λ_t , is the shadow value of the inputs measured in consumption units. In a laissez faire market economy these are equivalent to the prices of the inputs.

The last term in the equation for the dirty input is the marginal value of the external effect of this input (measured in consumption units). That is, δ is the factor that links the use of dirty inputs to the deterioration of environmental quality, and w_{t+1} is the shadow value of environmental quality. In a laissez faire market economy the environmental deterioration caused by dirty input usage is likely not taken into account, however, a Pigovian tax equal to $\delta w_{t+1}/\lambda_t$ would internalize this effect.¹¹

Lastly, the socially optimal allocation of scientists can be written:

$$\frac{f_{ct}}{1 - f_{ct}} = \frac{\sum_{k=0}^{\infty} \frac{A_{ct-1}}{A_{ct}} \lambda_{c,t+k} Y_{c,t+k} \delta^{1-w}}{\sum_{k=0}^{\infty} \frac{A_{dt-1}}{A_{dt}} \lambda_{d,t+k} Y_{d,t+k} \delta^{1-w}} \quad (16)$$

where λ_{ct} and λ_{dt} are the shadow values of the clean and dirty intermediate goods, respectively. The term A_{jt-1}/A_{jt} can be substituted by using (10) and we obtain:

$$\frac{f_{ct}}{1 - f_{ct}} = \frac{\sum_{v>t} \frac{c}{ry_d [1 + 1ry_d (1 - f_{ct})^{r\sigma}]^{-1}} Y_{dv}}{\sum_{v>t} \frac{c}{ry_c [1 + 1ry_c (1 - f_{ct})^{r\sigma}]^{-1}} Y_{cv}} \delta^{1-w} \quad (17)$$

As $\sum_{k=0}^{\infty} \lambda_{j,t+k} Y_{j,t+k}$ is the net present value (npv) of intermediate input j from period t , we have the following lemma on the relationship between the allocation of scientists and the relative npv of clean and dirty inputs.

¹¹ See Appendix 6.3 for a derivation of the Pigovian tax rate.

Lemma 1 The social planner allocates more scientists to the innovation sector in which the net present value of intermediate inputs is greater, when $ry_c = ry_d$.

Proof. If $ry_c = ry_d$, then (17) implies $fl_{ct} > 1/2$ if $\sum_{k=0}^{\infty} \dots_{c,t+k} Y_{c,t+k} > \sum_{k=0}^{\infty} \dots_{d,t+k} Y_{d,t+k}$. ■

3.2 The decentralized versus the social allocation of scientists

Now we will compare the decentralized market and the social allocations of scientists to innovation. Denote the social allocation to clean innovation fl_{ct}^S and the decentralized market allocation to clean innovation fl_{ct}^M . In the following we assume the probabilities of a successful innovation to be equal across industries, i.e. $ry_c = ry_d = ry$. Moreover, we assume that the interest rate by which the private sector discounts future profits is constant and equal to p , and that discounting is the same for the social planner, i.e. that $R_t = \dots_t$. We do these simplifying assumptions to get transparent comparisons between the decentralized and the social allocation ratios.¹²

We need to get the expression for the decentralized allocation of researchers (9) on a form that is comparable to the socially optimal allocation of researchers (16). First, by inserting for x_{jit} from (3) into (4), and using both that $p_{jit} = a^2$ and the expression for average machine quality (7), we get the following expression for the wage rate:

$$w_t = (1 - a)a^{1-\alpha} (p_{jt} - T_{jt})^{\frac{1}{1-\alpha}} A_{jt}.$$

Inserting this wage rate into the demand for labor (5), we can rewrite the demand for labor as:

$$L_{jt} = a^{1-\alpha} (p_{jt} - T_{jt})^{\frac{1}{1-\alpha}} \frac{Y_{jt}}{A_{jt}}. \quad (18)$$

Then, finally, by inserting (18) into the decentralized allocation of researchers (9) we obtain:

$$1 - fl_{ct}^M = \frac{\sum_{k=0}^{\infty} \sum_{v=1}^{\infty} \frac{A_{ct-1}}{A_{ct}} \left(\frac{1 - ry_{ct}^M}{1 - ry_{ct}^M} \right)^k \frac{w_{ct,t+k} (p_{c,t+k} - T_{c,t+k})^{\frac{1}{1-\alpha}} A_{ct,t+k}}{(1+p)^k Y_{c,t+k} A_{c,t+k}}}{\sum_{k=0}^{\infty} \sum_{v=1}^{\infty} \frac{A_{dt-1}}{A_{dt}} \left(\frac{1 - ry_{dt}^M}{1 - ry_{dt}^M} \right)^k \frac{w_{dt,t+k} (p_{d,t+k} - T_{d,t+k})^{\frac{1}{1-\alpha}} A_{dt,t+k}}{(1+p)^k Y_{d,t+k} A_{d,t+k}}}. \quad (19)$$

There are three major differences in (19) from the social optimal allocation (16):

1. First, the shadow prices $\dots_{j,t+k}$ are substituted by the discounted market prices $(p_{j,t+k} - T_{j,t+k})/(1+p)^k$. The negative external effect of using dirty intermediates may lead to a difference between these terms as already indicated above.
2. Second, the replacement probability $\prod_{v=1}^k (1 - ry_{j,t+v}^M)$ is not a part of (16). This term will reduce expected future profits. Furthermore, this reduction in profits is larger the more scientists there are working in a sector.

¹²Note that the results in this section do not hinge on a constant interest rate. However, in the numerical model we assume linear utility in consumption that gives a constant interest rate.

3. Third, the term $\frac{A_{jt}}{A_{j,t+k}}$ inside the summation term is not present in (16). This term will get exceedingly smaller, the higher the growth in A_{jt} .

Apart from the three points above, there is also a difference between the decentralized and the social allocation of scientists due to differences in current and future state variables A_{ct} and A_{dt} . Thus, to compare the allocations given by (16) and (19) we need comparable paths of the state variables. To this end, let there exit an optimal policy programme in which the planner commits to implementing the first best allocation in each period. In particular, the programme consists of three elements:

First, the optimal subsidy $s = 1 - a$ for the use of machines is implemented (as assumed from before). Second, the planner sets the Pigovian-tax on the use of dirty input. This tax internalizes the environmental externality perfectly, and thus, together with the subsidy s , this must imply that $\tau_{j,t+k} = (p_{j,t+k} - T_{j,t+k})/(1 + p)^k$ for all periods.¹³

Third, a subsidy to clean innovation is implemented in each period so that the first best allocation of scientist is achieved. Under this policy programme, all market failures are corrected for and the social allocation is achieved in the decentralized market equilibrium, i.e. the left hand side of (16) and (19) are the same for all periods.

Then, we pose the following question: Given the optimal policy programme, what innovation sector must be subsidized in a given period in order to implement that period's efficient allocation of scientists? We contribute the difference between the social and decentralized allocation ratio, along the policy programme path, to two effects:

- The replacement effect listed as number two above. In the decentralized market allocation ratio the future replacement rates matter. Firms are not able to appropriate the full social value of their research effort, and thus innovation in the sector with the larger replacement rate, ceteris paribus, is lower. This replacement effect is a market failure and the replacement rate is not taken into account in the social allocation ratio.¹⁴
- The productivity spillover effect listed as number three above. Research in a sector today benefits all future research in the sector through the standing-on-shoulder effects, i.e. every subsequent innovation involves a larger absolute step in product quality. However, researchers today do not take this into account, and the private value of an innovation falls short of the social value. Moreover, the higher the discrepancy is, the higher the productivity growth is in a sector, as the standing-on-shoulders effect increases in a sector's productivity.

Note that using equation (10) we can write the productivity growth rate in a sector as

$$\frac{A_{jt}}{A_{jt-1}} = (1 + 1ry_j(\frac{fl}{jt})^{r_0}). \quad (20)$$

Thus the sector with more scientists has the higher growth rate, as well as the largest replacement rate. We then have the following proposition:

¹³See Appendix 6.3 for a proof.

¹⁴Note that there is no standard business-stealing effect in our model since there is no free entry and the scientists are always fully utilized in one sector or the other.

Proposition 2 Along the optimal policy programme path, if the current and future productivity growth rates are large in one sector, then innovation should be subsidized in that sector.

Innovation in the sector with largest growth should be subsidized since market failures due to both the replacement effect and the spillover effect are largest there. Note that Proposition 1 does not say anything about subsidies if it is socially optimal to vary which sector should have the larger future productivity growth. We do not solve the model out to find closed-form expressions for the growth rates. However, Proposition 1 gives structure to a discussion of optimal subsidies to clean innovation which we utilize when analyzing the numerical results from Section 4.

It may be helpful to characterize the innovation subsidies in terms of values of the clean and the dirty inputs instead of by the growth rates of the technologies. The relationship between the value of the inputs and optimal subsidies to clean innovation is stated in the following corollary:

Corollary 3 Along an optimal policy programme path, clean innovation should be subsidized if the net present value is higher for the clean input than for the dirty input.

Proof. First, Lemma 1 states that $fl_{ct}^S > fl_{dt}^S$ iff $\sum_{k=0}^{\infty} \beta^k Y_{c,t+k} > \sum_{k=0}^{\infty} \beta^k Y_{d,t+k}$. Next, using (20), it follows that $\frac{A_{ct}}{A_{ct-1}} > \frac{A_{dt}}{A_{dt-1}}$ iff $fl_{ct} > fl_{dt}$. Then, combining this with Proposition 2, the result follows. ■

This result highlights the role the value of the environment and emission taxes play for optimal subsidies to innovation. In the event that emissions have a large impact on environmental quality and this quality again is important for utility, the value of clean inputs will be large relative to the value of dirty inputs and it will be optimal to direct innovation more towards clean technologies. In this case the optimal growth rate of clean technology is higher than for dirty so the market failures are larger for clean innovation in a decentralized market. In contrast, if emission impacts are small and not so important for utility, it is optimal to direct innovation more towards dirty technologies to build on their productivity advantage. In this case the optimal growth rate is higher for dirty technologies and subsidies to innovations on dirty innovations are needed to implement the efficient allocation. Analysis of the value of the inputs, the relative growth rates of the technologies, and policies are done by numerical simulations in the following section.

4 Numerical analysis

In this section we present numerical analysis that builds on the analytical model above. The utility function and other details are specified in appendix 5. This includes the links between emissions (Y_{dt}), concentration in the atmosphere, temperature increase, and environmental quality (S_t). We assume a quasi-linear utility function with separable preferences between consumption and environmental quality. This is different from the numerical model specification in AABH, but similar to their assumption when they consider exhaustible resources. Furthermore, the utility function is linear in consumption, which implies that interest rates

are constant over time. This reduces the complexity of the simulations and allows us to focus on how future carbon taxes influence innovation decisions today.¹⁵

When calibrating the model, we mostly follow AABH. In our benchmark case we assume a substitution elasticity of $c = 3$. AABH also simulate $c = 10$, which we find rather high. Instead we will also examine the effects of a low elasticity of substitution case with $c = 1.5$ below.¹⁶ Following AABH, we set machine share $a = 1/3$, probability of a successful innovation $ry_j = 0.02$ (per annum) for both sectors, the quality step $1 = 1$, and the discount rate $p = 0.015$ (AABH also consider $p = 0.001$). The initial productivities A_{d0} and A_{c0} are calibrated so that clean inputs constitute 203 of total inputs, which is in line with the current share of non-fossil energy in worldwide energy use.

The value of w is set to $w = 0.7$. This implies that the initial share of scientists in the clean sector is 183 in our BaU scenario (with $c = 3$). This is somewhat below the current share of clean energy R&D in global energy R&D, which is around 25-303; however, the current R&D investments may reflect that investors expect a future policy development that lies between a BaU scenario and an optimal climate policy scenario.¹⁷

We simulate the model over 50 five-year periods, i.e., 250 years. At the end of the time horizon, the temperature is falling in the policy scenarios as there is almost no use of dirty energy anymore (this is different with $c = 1.5$, see below). Hence, extending the time horizon has negligible effects on the real variables in the policy scenarios.¹⁸

4.1 Results: Benchmark case $c = 3$

In the Business-as-Usual (BaU) scenario, most scientists move to the dirty sector, so that after 50 years only one percent remains in the clean sector. Production mostly consists of dirty inputs, and the temperature increase passes the assumed threshold level of six degrees after 110 years.

The optimal policy consists of a tax on dirty inputs and a subsidy to either clean or dirty innovation (note that the subsidy can only affect the distribution of scientists between sectors, as the total number of scientists is fixed). Figures 1A and 1B show the optimal combination of tax and subsidies. The figures also show the optimal tax in the case without any subsidy, and the optimal subsidy in the case without any tax.

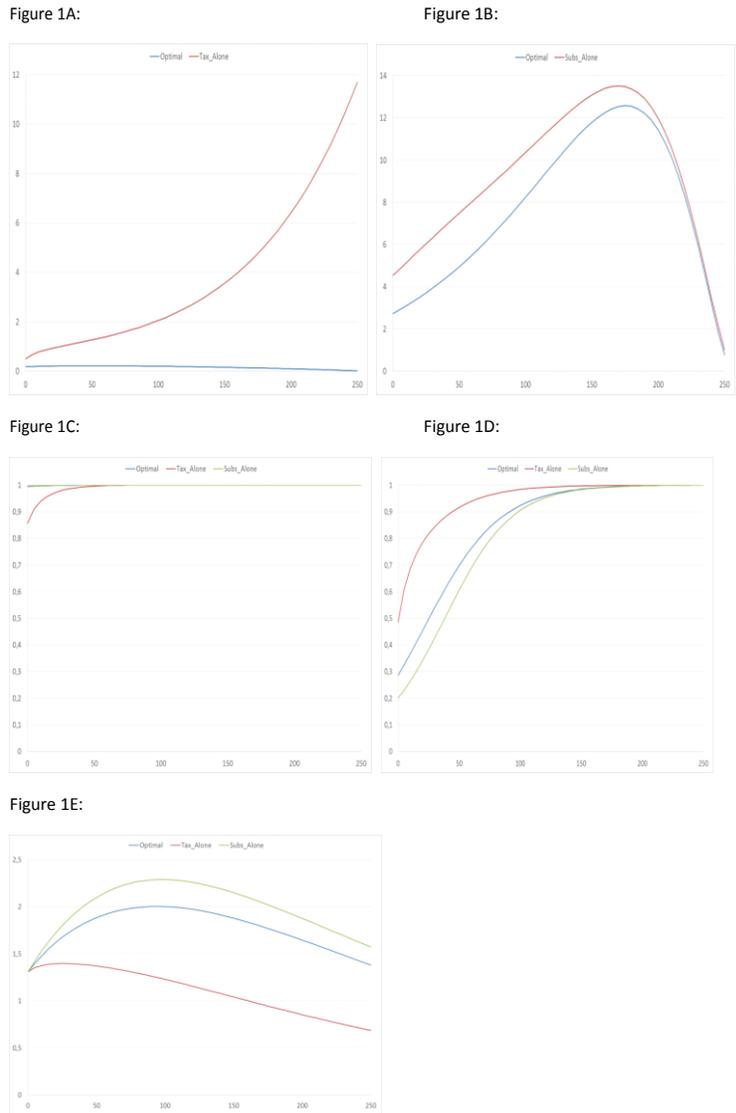
¹⁵We also use a different specification of environmental utility than AABH, as the one in AABH implies quite low damages for temperature levels close to the "disaster" level of 6 degrees. To illustrate, in their first-best solution the temperature increase passes 5 degrees after about 100 years, and then approaches the disaster level (in the case with $c = 3$ and $p = 0.015$).

¹⁶Most CGE models apply substitution elasticities around 1 or below when it comes to substitution of different energy goods at the sectoral level. For instance, Bohringer et al. (2014) apply elasticities in the range of 0.25 - 1. These elasticities may be interpreted as relevant for the intermediate term, whereas we are more interested in long-term elasticities.

¹⁷In the case with $c = 1.5$ we recalibrate the value of A_{c0} but not w . As a consequence, the initial share of clean input is not changed, but the initial share of scientists in the BaU scenario increases to 373 (since w is held fixed). If we recalibrate w to get the same initial share of scientists as with $c = 3$, w becomes approximately one, and the comparison of different elasticities would be difficult to interpret. We run a sensitivity analysis where we do not recalibrate A_{c0} .

¹⁸The optimal subsidy level, though, depends quite a lot on the time horizon.

Figure 1: First- and second best policies ($c = 3$)



First, we notice from Figure 1A that the tax is fairly constant over time in the first-best solution, which reflects a combination of a low discount rate and that environmental quality starts to improve again after 100 years, cf. Figure 1E.¹⁹

Second, Figure 1B shows that it is optimal to subsidize clean research. Thus, our results support similar findings as in AABH. However, note that there is a distinct difference in the scientist's incentives in the two models. AABH assume that the scientist can only benefit from its innovation in the first 5-year period, whereas we assume long-lived patents where

¹⁹The drop in the tax rate after 100 years is partly due to reduced temperature, but also due to the fact that the model has a finite time horizon. If we run the model over 100 more years, the tax reaches its maximum level somewhat later but tax levels differ only slightly over the first 100 years. Note that in AABH the tax is initially much lower but then increases over the first 150 years. This is probably due to their limited environmental damage costs (as long as the temperature is not too close to the disaster level of 6 degrees).

scientists do take into account future changes in the value of clean innovations due to climate policies. Rather, the reason for the subsidy in our model is the following (cf. Section 4): As practically all scientists move to the clean sector immediately (see Figure 1C), the risk of replacement is biggest in this sector. Moreover, as most scientist are in the clean sector, the productivity growth rate is highest in this sector and, thus, the spillover effects related to standing on shoulder are also highest. We see that the subsidy increases substantially over time, as the share of scientists in the dirty sector becomes smaller and smaller. Towards the end of the time horizon, however, the positive externality of clean research diminishes as the risk of replacement declines.²⁰

If taxes for some reason are not used, the second-best subsidy increases notably, especially in the beginning (see Figure 1B). Without a future tax on dirty inputs, innovators are less incentivized to do clean R&D, and hence need a higher subsidy to enter the clean research sector. From Figure 1D we notice that the share of clean inputs is lower in this scenario than in the first-best case, as there is no tax to stimulate the use of such inputs. However, the productivity growth of clean inputs is slightly higher than in the first-best scenario, and gradually it becomes profitable to switch from dirty to clean inputs. Nevertheless, the temperature increase is higher in the second-best scenario with only subsidy compared to the first-best case, cf Figure 1E.

If subsidies for some reason are not used, the second-best tax increases dramatically, especially at the end of the time horizon (see Figure 1A). The explanation is that particularly high taxes are needed to move scientists to the clean R&D sector. Nevertheless, the share of scientists in the clean sector is below the corresponding share in the optimal scenario, cf. Figure 1C. However, this tax scenario is likely to be time inconsistent, as the high future tax rates are imposed mainly to stimulate early innovation into clean inputs. Hence, when future periods arrive, the regulator would like to reduce the tax level. We notice that the share of clean inputs is much higher than the optimal share, and that the temperature increase is significantly smaller than in the first-best case.

When we compare the utility of the three policy scenarios, we find that utility is reduced by merely 0.63 in the subsidy-alone scenario compared to the optimal policy scenario, whereas utility is reduced by 6.53 in the tax-alone scenario. As the latter scenario is time inconsistent as well, our results suggest that the subsidy to clean R&D is even more important than the tax on dirty inputs in this case. This is due to the relatively high substitution elasticity between clean and dirty inputs, which implies that once clean technologies become sufficiently developed, they can take over most of the market without depending on a tax on dirty inputs.

4.2 Results: Low elasticity of substitution case $c = 1.5$

In this case, clean and dirty technologies are less substitutable than in the benchmark case and it is optimal to impose a much higher tax on dirty inputs, especially in later periods (see Figure 2A). The reason is that even if clean inputs eventually become cheaper than dirty

²⁰The corresponding subsidy path in AABH is initially zero, before it jumps suddenly after 50 years and then declines towards zero again after 100 years. This pattern is driven by the fact that AABH assume constant returns to inputs from scientists, leading to corner solutions in the innovation sector (either only dirty or only clean innovation within a period), while we assume decreasing returns due to the risk of duplication of ideas.

inputs, consumers will prefer to use a combination of inputs. Hence, a higher tax level is needed to keep emissions down.²¹ As seen in Figure 2B, the optimal subsidy level is slightly lower than in the case with $c = 3$, because the higher tax makes innovation into clean R&D more profitable. Hence, a lower subsidy is needed to direct innovation into the clean sector.²²

Figure 2 : First- and second best policies ($c = 1.5$)

Figure 2A:

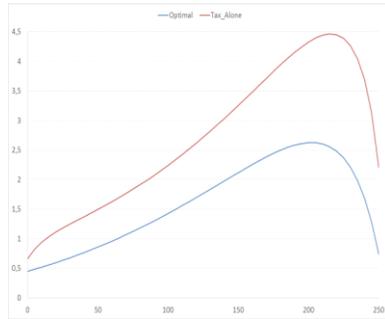


Figure 2B:

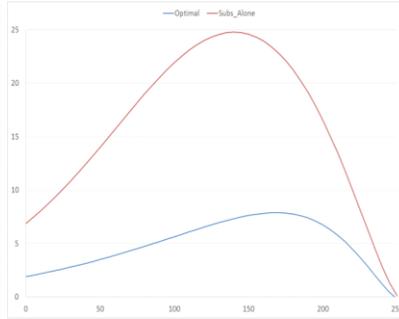


Figure 2C:

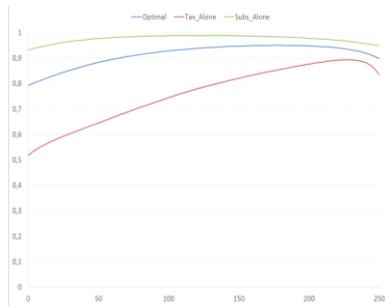


Figure 2D:

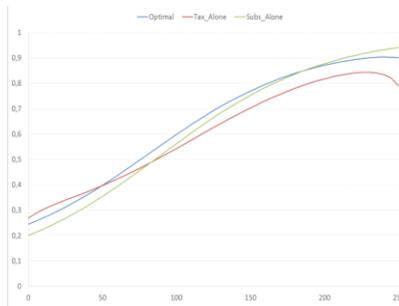
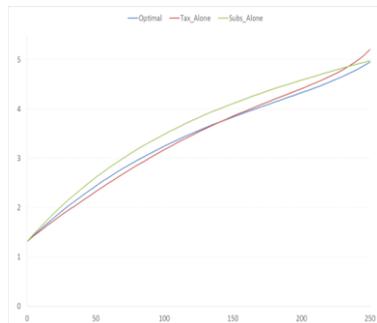


Figure 2E:



An additional reason for the lower optimal subsidy level in the case with $c = 1.5$ is that it is more valuable from a welfare perspective to use a combination of inputs instead of relying

²¹ In the figure the optimal tax starts declining after around 200 years. This is due to the simulated time horizon of 250 years. From Figure 2E we see that the temperature is increasing steadily, and would probably pass the disaster level of 6 degrees after another 100 years if the tax continued to decline after the end of our simulated time horizon. If we simulate the model for another 100 years, the tax peaks 100 years later, whereas the tax levels do not change much for the first 150 years.

²² If we rather increase the substitution elasticity to e.g. $c = 5$ we get the opposite result for both the tax and the subsidy.

mostly on either clean or dirty inputs when the substitutability is lower. This is clearly seen when comparing Figures 1D and 2D, which show the share of clean inputs in aggregate production. Thus, a social planner would prefer that both clean and dirty inputs become cheaper to use. Hence, the share of scientists in dirty innovation is much higher than with $c = 3$, see Figures 1C and 2C.

A consequence of using more dirty inputs is higher emissions and increased temperature. As seen in Figure 2E the temperature rises steadily throughout the time horizon.

If taxes are not used, the second-best subsidy increases significantly compared to the first-best policy. When the goods are less substitutable, the subsidy has to stimulate clean R&D quite heavily so that clean inputs become so cheap that consumers eventually buy only small amounts of dirty inputs.

If subsidies are not used, the second-best tax increases somewhat, but much less than with $c = 3$ as the cost difference between clean and dirty inputs is less important when the substitution elasticity is lower.

The welfare losses of second-best policies are now 4-53, irrespective of whether only taxes or only subsidies are used. With low substitution elasticity, the market is less interested in switching very much towards the clean technology even if its price become cheaper than the dirty technology. Thus, relying only on the subsidy is more costly in this case than with a high substitution between clean and dirty technology.

5 Conclusion

We introduce two novelties in the model framework of AABH: We allow innovation profits to survive longer than one period and introduce decreasing returns to R&D at any point in time.

At first glance both aspects should make targeted R&D support less crucial. That is, innovations that not only give instantaneous profits implies that future environmental policies can redirect research today, and decreasing returns force R&D to take place in all sectors independent of the level of accumulated productivity.

Surprisingly, we find that governments should nonetheless support clean R&D more than dirty R&D. Dealing with a major environmental problem effectively requires R&D effort to shift to the clean technology. However, when most researchers work with clean technology, both productivity spillovers and the future risk of being replaced increases. Consequently, the wedge between the private and the social value of an innovation is larger for clean technologies than for dirty technologies along the transition path.

Our main concern with AABH is not that they find that R&D subsidies to clean technologies are necessary, but that they downplay the role of a carbon tax. In our opinion, setting a correct price on carbon emissions now and in the future should still be an important priority of policy makers.

There are several more aspects of the AABH model that could be discussed and that will likely affect the desirability of R&D subsidies for the clean sector²³. First, there is a fixed number of scientists in the R&D sector. This assumption is not so important for the direction of the qualitative results, as the purpose of our paper is to analyze policies

²³See Hourcade, Pottier, and Espagne (2011) for a discussion of problems with key parameters and climate modelling in AABH.

related to the relative allocation of scientists between two classes of technology (although it simplifies solving the model). However, if there were more technology classes in the economy - for instance a general technology in addition to a clean and a dirty energy technology - the no free entry assumption might be less innocent, as subsidies to clean technologies then would also crowd out innovation in the general technology. This is something we plan to study in a future project. Second, there are no spillovers between the two classes of technologies. Loosening up this assumption may diminish the necessity of directing R&D to clean technologies today, as it may be better to develop the more productive technology before making the switch to clean. This is also a venue for future research.

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Appendix

A.1 The Euler equation

Let a_t be a (representative) household's asset value. The household's problem is then

$$\begin{aligned} \max_C \sum_{t=0}^{\infty} \frac{1}{(1+p)^t} u(C_t, S_t) \\ \text{s.t.} \quad a_{t+1} = (1 + r_t)a_t + w_t + J_{r_{dt}} + J_{r_{ct}} + T_{dt}Y_{dt} - C_t \end{aligned} \quad (21)$$

where the only firm profits are from selling the machines since profits will be zero in the final goods and intermediate sectors. Note that labor income is w_t as wages will be the same in the two intermediate sectors, and $L_{dt} + L_{ct} = 1$. Further, note that since there is no saving asset, all resources are spent in every period, i.e. the interest rate will be such that $C_t = w_t + J_{r_{dt}} + J_{r_{ct}} + T_{dt}Y_{dt}$. Following from ?? the Euler equation can be written

$$\frac{ou(C_t, S_t)}{oC} = \frac{(1 + r_{t+1})ou(C_{t+1}, S_{t+1})}{1 + p}.$$

A.2 Solving the Planner problem

The Lagrangian from the problem given by 13 is:

$$\begin{aligned}
 L = & \sum_{t=0}^{\infty} \frac{1}{(1+p)^t} u(Y_t - 1/\int_0^1 x_{cit} di + \int_0^1 x_{dit} di) - S_t \\
 & - \sum_{t=0}^{\infty} \lambda_t (Y_t - Y_{ct}^{\frac{s-1}{s}} + Y_{dt}^{\frac{s-1}{s}} \backslash^{\frac{s-1}{s}}) \\
 & - \sum_{t=0}^{\infty} \mu_{ct} (Y_{ct} - L_{ct}^{1-a} [\int_0^1 A_{cit}^{1-a} x_{cit}^a di]) \\
 & - \sum_{t=0}^{\infty} \mu_{dt} (Y_{dt} - L_{dt}^{1-a} [\int_0^1 A_{dit}^{1-a} x_{dit}^a di]) \\
 & - \sum_{t=0}^{\infty} \mu_{ct} (A_{ct} - (1 + 1n_c(fl_{ct})^{r_0}) A_{ct-1}) \\
 & - \sum_{t=0}^{\infty} \mu_{dt} (A_{dt} - (1 + 1n_d(1 - fl_{ct})^{r_0}) A_{dt-1}) \\
 & - \sum_{t=0}^{\infty} w_t (S_t + \diamond Y_{dt-1} - (1 + 6)S_{t-1})
 \end{aligned}$$

where λ_t is the shadow value of final goods, μ_{jt} is the shadow value of intermediate goods in $j \in \{c, d\}$, μ_{jt} is the shadow value of the average machine quality (the technology stock) in $j \in \{c, d\}$, and w_t is the shadow value of the environmental quality. Note that we have substituted in $C_t = Y_t - 1/\int_0^1 x_{cit} di + \int_0^1 x_{dit} di$ and that we have set $L_{ct} + L_{dt} = 1$ and $fl_{ct} + fl_{dt} = 1$ for simplicity.

The FOC wrt C_t is

$$\frac{1}{(1+p)^t} \frac{\partial u(C_t, S_t)}{\partial C} - \lambda_t = 0. \quad (22)$$

The FOC wrt S_t is

$$\frac{1}{(1+p)^t} \frac{\partial u(C_t, S_t)}{\partial S} - w_t + w_{t+1}(1+6)I_{S_t < S} = 0 \quad (23)$$

where $I_{S_t < S} = 1$ if $S_t < S$, and $I_{S_t < S} = 0$ otherwise, since $S_{t+1} = \diamond Y_t - (1+6)S_t$ only in the interval $(0, S)$. Equation (23) can be solved recursively to get

$$w_t = \sum_{v>t} (1+6)^{v-t} \frac{1}{(1+p)^v} \frac{\partial u(C_v, S_v)}{\partial S} I_{S_t, \dots, S_v < S}$$

The FOCs wrt Y_{ct} and Y_{dt}

$$\begin{aligned}
 & \lambda_t (Y_{ct}^{\frac{s-1}{s}} + Y_{dt}^{\frac{s-1}{s}} \backslash^{\frac{s-1}{s}} Y_{ct}^{\frac{-1}{s}} - \lambda_{ct}) = 0 \\
 & \lambda_t (Y_{ct}^{\frac{s-1}{s}} + Y_{dt}^{\frac{s-1}{s}} \backslash^{\frac{s-1}{s}} Y_{dt}^{\frac{-1}{s}} - \lambda_{dt} - \diamond w_{t+1}) = 0.
 \end{aligned} \quad (24)$$

These are discussed in the main text. Further, the FOC wrt machines x_{jit} is

$$-\frac{1}{(1+p)^t} \frac{\partial u(C_t, S_t)}{\partial C} \frac{1}{a} + \dots_{jt} L_{jt}^{1-a} A_{jit}^{1-a} a x_{jit}^{a-1} = 0$$

which using equation (22) and that the cost of a machine is given by $1/a = a^2$, can be written

$$x_{jit} = \frac{1}{a} \dots_{jt} L_{jt}^{1-a} A_{jit}^{1-a}$$

where we have used the cost $1/a = a^2$. The market solution yields the following use of machines:

$$x_{jit} = \frac{(p_{jt} - T_{jt})}{(1-s)} A_{jit} L_{jt}$$

With a Pigovian tax equal to μ_{jt+1}/\dots_{jt} , we have $p_{jt} - T_{jt} = \dots_{jt}/\dots_{jt}$ (see below). Thus, with a subsidy $s = 1 - a$, we obtain the optimal production of machines.

The relevant FOC for the allocation of scientist is

$$\mu_{ct} w_{ct} n_{ct} f_{ct}^{1-\alpha} A_{ct-1} - \mu_{dt} w_{dt} n_{dt} (1-f_{ct})^{1-\alpha} A_{dt-1} = 0 \quad (25)$$

$$\frac{f_{ct}}{1-f_{ct}} = \frac{\mu_{ct} n_{ct} A_{ct-1}^{\frac{1}{1-\alpha}}}{\mu_{dt} n_{dt} A_{dt-1}}$$

In order to get (16) we need to substitute for μ_{jt} . First, we use the FOC wrt the average quality A_{jt} which is given by

$$\dots_{jt} L_{jt}^{1-a} \int_0^1 A_{jit} x_{jit} di - \mu_{jt} + \mu_{jt+1} (1 + n_{jt} f_{jt+1}^{\alpha}) = 0 \quad (26)$$

Next we use $A_{jt} = \int_0^1 A_{jit} di$ and the definition of Y_{jt} to rewrite equation (26)

$$\dots_{jt} (1-a) \frac{Y_{jt}}{A_{jt}} - \mu_{jt} + \mu_{jt+1} (1 + n_{jt} f_{jt+1}^{\alpha}) = 0. \quad (27)$$

Then we use equation (10) to rewrite equation (27):

$$\mu_{jt} = \dots_{jt} (1-a) \frac{Y_{jt}}{A_{jt}} + \mu_{jt+1} \frac{A_{t+1}}{A_t} \quad (28)$$

Notice that equation (28) can be written as a sum of the form $\mu_{jt} = \dots_{jt} (1-a) \frac{Y_{jt}}{A_{jt}} + \dots_{jt} (1-a) \frac{Y_{jt+1}}{A_{jt+1}} + \dots_{jt} (1-a) \frac{Y_{jt+2}}{A_{jt+2}} + \dots$. We use this to obtain

$$\mu_{jt} = (1-a) \frac{1}{A_t} \sum_{v>t} \dots_{jv} Y_{jv} \quad (29)$$

Last, combining equations (25) and (29) gives the following expression for the optimal allocation of scientist

$$\frac{f_{ct}}{1-f_{ct}} = \frac{\left(\frac{n_{ct} A_{ct-1}}{n_{dt} A_{dt-1}} \right)^{\frac{1}{1-\alpha}} \sum_{v>t} \dots_{cv} Y_{cv}}{\sum_{v>t} \dots_{dv} Y_{dv}} \quad (30)$$

which we use in Subsection 4.1.

A.3 The Pigovian tax rate

The Pigovian tax rate is the tax rate that ensures that we get the optimal use of the clean and dirty intermediates. In the decentralized market solution the uses of the two intermediates by the final goods sector are given by:

$$\begin{cases} Y_{ct}^{\frac{s-1}{s}} + Y_{dt}^{\frac{s-1}{s}} \lambda^{\frac{1}{s-1}} Y_{dt}^{\frac{-1}{s}} - p_{dt} = 0 \\ Y_{ct}^{\frac{s-1}{s}} + Y_{dt}^{\frac{s-1}{s}} \lambda^{\frac{1}{s-1}} Y_{ct}^{\frac{-1}{s}} - p_{ct} = 0 \end{cases}$$

In order to obtain the optimal use of the two inputs, we see from (24) that we must have:

$$\begin{aligned} p_{ct} &= \frac{c_{ct}}{c_t} & (31) \\ p_{dt} &= \frac{c_{dt}}{c_t} + \frac{w_{t+1}}{c_t} \end{aligned}$$

In a laissez fair market equilibrium prices on the intermediates will adjust such that $p_{jt} = c_{jt}/c_t$. Hence, the Pigovian tax rate T_{dt}^* must be equal to w_{t+1}/c_t . Along an optimal growth path in which the Pigovian taxes and the subsidy to machines are both implemented, we claim in the text that:

$$c_{j,t+k} = \frac{(p_{j,t+k} - T_{j,t+k})}{(1+p)^k} \quad (32)$$

In Subsection 4.2 we use a special utility function for which we have $ou/oc = 1$. This implies that $c_{jt} = 1/(1+p)^t$, and that $T_{dt}^* = (1+p)^t w_{t+1}$. We then see that (31) and (32) are equivalent.

A.4 Specification of the numerical model

In this appendix we present how the utility function and the environmental quality function are specified in the numerical model. The rest of the model is specified before.

The instantaneous utility function is given by

$$u(C_t, S_t) = C_t + \phi(S_t) \quad (33)$$

where $\phi(S)$ is the valuation of the environmental quality. The linearity of utility with respect to consumption implies that the interest rate is exogenous and constant over time, cf. Section 3.5.

The general function $\phi(S)$ can in the context of climate change be expressed as $\phi(\Delta)$, where Δ denotes the temperature increase relative to the pre-industrial level ($\phi^1(\Delta) < 0$). In order to specify the function $\phi(\Delta)$, we first follow AABH and assume the following relationship between temperature increase and CO_2 -concentration in the atmosphere measured in parts per million (C_{CO_2}):

$$\Delta = 3 \log_2(C_{CO_2}/280) \quad (34)$$

where 280 ppm is the pre-industrial level of CO₂-concentration. Further, C_{CO₂,disaster} denotes the concentration level associated with the disaster temperature increase, which AABH sets to 6_{.disaster} = 6 degrees.

We assume a constant depreciation rate (δ) of the CO₂-concentration in the atmosphere (above the pre-industrial level):

$$C_{CO_2,t+1} - C_{CO_2,t} = \delta Y_{dt} - \delta (C_{CO_2,t} - 280)$$

where we assume δ = 0.005 (per year).²⁴ The parameter δ is calibrated so that annual concentration level increases by 2 ppm initially in the BaU-scenario.²⁵

As explained in the main text, the environmental damage costs in AABH are quite low as long as the temperature is not too close to the disaster level of 6 degrees. For this reason, and the fact that we use a separable utility function, our specification of the ϕ(6_.) function is different from theirs:

$$\phi(6_{.}) = - \frac{\dots (2_{.} - 1)^2}{6_{.disaster} - 6_{.}} \quad (35)$$

where >0 is a parameter to be calibrated. The specification in (35) implies ϕ(0) = 0, ϕ'(6_.) < 0, and that the numerator is quadratic in the concentration level of CO₂ (above the pre-industrial level). However, as the temperature increase approaches the assumed disaster level of 6 degrees, ϕ(6_.) declines towards minus infinity.

Finally, the parameter > is calibrated so that the temperature increase peaks at 2 degrees in the optimal solution when E = 3.²⁶ The motivation for this choice is the fact that the 2 degrees target has been established by the world leaders since the UNFCCC meeting in Cancun in 2010. An alternative calibration strategy could be to use an estimate of the social cost of carbon - however, these estimates vary quite substantially.

²⁴The carbon cycle in the atmosphere is much more complex. On the one hand, over the first few decades after the emissions takes place, the decay of CO₂ is more rapid than 0.53 per year, see IPCC (2013, Ch. 6). On the other hand, a not-trivial part of the CO₂ remains in the atmosphere for several millennia. Thus, our assumption can be seen as a simplification and compromise between the medium- and long-term effects. Note that this specification of the environmental regeneration differs somewhat from the general specification in (12). AABH does not specify how they model this regeneration in their numerical model.

²⁵ http://www.esrl.noaa.gov/gmd/ccgg/trends/#mlo_growth

²⁶Note that in the case with E = 1.5, the optimal temperature path does not peak at 2 degrees, cf. Figure 2E, as we do not want to change the ϕ-function when we change the value of E.