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Abstract

Growth dynamics of forests are likely to be substantially altered by climate change. Optimally, forest owners should take these changes into account when making decisions today. However, the uncertainty surrounding climate change makes these shifts hard to predict and hence this paper asks whether forecasting them is necessary for profitable management. While climate change uncertainty makes it theoretically impossible to calculate expected profit losses of not forecasting, we suggest a method utilizing Monte Carlo simulations by which to obtain a credible upper bound of these losses. We show that an owner following a rule of thumb, which completely ignores future changes and only observes changes as they come, will closely approximate optimal management. If changes are observed without too much delay, profit losses and errors in harvesting are negligible. This has implications for the effort foresters should devote to long run forecasting. It also implies the argument that boundedly rational agents may behave “as if” being fully rational has traction in forestry.

Keywords Climate change; Decision making under uncertainty; Forestry; Quantitative methods

JEL Classifications C6 D81 Q23 Q54

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1 Introduction

Is forecasting an activity that renewable resource owners should engage in? Or should they ignore future changes and base their decisions on what they observe today? These questions arise as the growth dynamics of renewable natural resources most likely will change over time. In the case of forests this may be due to technical change (e.g., genetic improvements) making trees grow both larger and faster, a spreading of disease to new environments which increases the risk of trees turning commercially worthless or due to climate change which is predicted to change the growth dynamics. These effects may be substantial. For instance, the negative effects of climate change on profitability of forestry in Europe have been predicted to be between 14 and 50 percent of net present value which corresponds to several hundred billion Euros (Hanewinkel et al., 2012). Indeed, a rather comprehensive literature (surveyed by Yousefpour et al., 2012) sets out to analyze how foresters should adapt their decision making to future climate change. Theoretically, such an exercise is very hard to perform.¹ This is possibly why the more recent literature has had a mainly computational and quantitative approach. For instance, Pukkala & Kellomäki (2012) and Schou & Meilby (2013) numerically evaluate different decision making strategies in specific climate scenarios.² Other papers (e.g., Jacobsen & Thorsen 2003) incorporate decision making with a *risk* component (i.e., a

¹The literature analyzing the optimal harvesting decisions in forestry (optimal rotation) is vast and goes back to the works of Faustmann (1849), Pressler (1860) and Ohlin (1921). In the original setup the environment is stationary making it fairly easy to characterize the optimal rotation period analytically. Since then many extensions to this basic framework have been explored (see Newman, 2002, for a survey). With technological or environmental changes, the stationarity assumption will no longer hold. McConnell et al. (1983) and Löfgren (1985) are two early attempts at analytically addressing problems where the growth characteristics of trees change over time. The former is looking at changes in prices and costs while the latter is analyzing technological change. They find that it is difficult to even characterize qualitatively how the decision rules should change over time as soon as we move away from the simplest cases. Non-stationarity has also been added through, e.g., tax and subsidy possibilities (van Kooten et al., 1995) and through evolving fire risk (Stollery, 2005).

²Pukkala & Kellomäki (2012) analyze which size and age of trees to cut and when to perform thinning. Indeed just like we do, they find that net present values are only marginally affected by the exact decision making procedure. Schou & Meilby (2013) analyze transformation from even-aged to uneven-aged forests. They find that stand development followed similar pathways during the transformation phase irrespective of the assumed climate change scenario.

probability distribution of future climate scenarios).³ Yet, one of the main problems with climate change (and other future changes) is precisely that assigning probabilities is difficult, let alone knowing which single scenario to use as a base. This is also one of the main conclusions of Yousefpour et al.'s (2012) survey – the missing piece of the puzzle is how to incorporate *uncertainty*, about the outcome and the probability distribution, into the decision making. Furthermore, we do not know whether this effort to improve our decision making makes any economic sense. That is, we do not know how much profit-making will be improved by having detailed knowledge about the future. The uncertainty surrounding these issues implies that any prognosis of the future growth of trees will be both costly to make and will most likely be inaccurate *ex post*.⁴ Bearing this in mind our paper aims at 1) answering the question whether forecasting climate change is worthwhile for forest owners or if a less demanding rule of thumb can be used, while 2) getting around the problem of uncertainty of the probability distribution.

More precisely, we compare the profits from two different decision rules. On the one hand, a “first best” rule that includes information of all future changes to the trees’ growth function and the change in the risk of fire, storms and pests. On the other hand a “naïve” rule that, at every point in time, observes the prevailing growth dynamics and risks and assumes that they will be non-changing from then onwards.⁵ Most forest owners have trees of many ages in their possession and hence it seems reasonable that they could at least assess the current growth function of the trees.

Now, the fundamental problem is which climate scenario(s) we should use to evaluate the losses of the naïve rule. Naturally, we do not ourselves know how the dynamics will change in the future. But to get around the problem of uncertainty about the probability distribution the following setup is used. We use scenarios of climate effects on trees which have a sufficiently broad range to cover changes which are, by far, more extreme than anything that can possibly happen in the future. Then, we calculate the profit losses

³Jacobsen & Thorsen (2003) analyze which composition of tree species to have under a certain probability distribution of climate change scenarios.

⁴This has also been noted by Löfgren (1985) who concludes that the assumption that forest owners can predict such changes is unlikely to hold.

⁵That is, the naïve owner makes no attempt whatsoever to predict future changes – she is adaptive. A more sophisticated owner could, for instance, be trend adaptive by inferring future changes from previous ones.

of following the naïve rule for a very large number of scenarios within this range (i.e. a Monte Carlo simulation). This setup enables getting an *upper bound* for what the losses may be if an owner is not forecasting the future. This upper bound is simply the losses of the naïve owner in the least favorable scenario. It is important to note that the upper bound of losses is not the same as the expected losses which by definition are smaller. The benefit of this approach is that *if* we find the upper bound to be of negligible size, which is exactly what we do, we can draw the conclusion that forecasting long term forest dynamics is not important. The benefit furthermore is that, to draw this conclusion, it is only necessary that the range of the scenario distribution is broad enough. That is, whether the underlying distribution of the scenarios is realistic *within* this range is of no concern. The analysis is performed for boreal forests (i.e. those covering the northern part of the northern hemisphere). For our simulations to credibly include the actual outcome, which is unknown to us today, we need to ensure that our parameter variations span over at least what can possibly happen in reality. With this in mind we include very extreme scenarios where, for example, the risk of fire changes from happening every thousand years at the onset to instead happening every third year, trees growing to become four times larger and growing to 90% of their maximum size twice as fast. These scenarios are, by an order of magnitude, more extreme than those predicted in forest research. We also vary the trajectories of climate change to include, for instance, threshold effects. Since we use so extreme scenarios our conclusions are probably on the conservative side. They are also conservative since we compare the naïve rule of thumb to an optimal rule based on perfect information. A more realistic benchmark would be a decision rule based on the best available estimate of future changes.

Our main finding is that the naïve decision rule is very close to the optimal. Using the naïve rule yields only negligible profit losses (the upper bound is 0.2% profit losses) and implies a cutting of trees which is a very close approximation of the optimal. Even when the first best rule is changing dramatically (e.g. cutting six times larger trees and halving the age at which to cut) the naïve rule closely follows it and yields only small relative losses. So, although the naïve owner will be constantly slightly wrong, since the actual changes are observed without delay, the owner will not be too far behind in updating. In a robustness check we show that losses in the worst case scenario are negligible also when using, instead of the most recent information, a moving average of

the last five years or when updating the decisions every third year. Once the delays are more than this we can no longer, strictly speaking, conclude that losses will be small as some scenarios give losses of almost 2% when the delay is ten years. Our main results are computed for when the growth function improves over time. A reversed alternative, for example due to spreading of deserts, where trees' growth falls in the next 200 years has also been simulated and is presented as a robustness check. Losses appear small here too.

One, practical, conclusion that can be drawn from this is that long run forecasting is hardly motivated within forestry as long as forest owners observe actual changes regularly. Another, more theoretical, conclusion regards whether boundedly rational agents behave "as if" being fully rational. This topic has been fiercely debated in the economics literature and there certainly is no consensus around whether full rationality is an appropriate assumption.⁶ Furthermore, as earlier work has shown that rationality assumptions sometimes matter for the outcome and sometimes not, it is hard to predict what the effects will be in an area not yet explored.⁷ Arguably, the naïve rule implies a very weak assumption on rationality since it does not require any forecasting at all. As our results show that the naïve decision maker really behaves "as if" being fully rational, so will any decision maker with rationality properties closer to the fully informed. This implies that assuming full rationality when modeling forestry most likely is without bias, even if in reality forest owners may or may not be fully able to predict the future.

The structure of the paper is as follows. We start by presenting the model and describing the decision rules in section 2. In section 3, we present the setup of the numerical simulations of the model and in section 4 how the decision rules differ in terms of profit loss and the sizes of trees to be cut. We present some robustness checks in section 5. Finally, section 6 concludes by discussing our results and how they may possibly change if extending the model. The appendix contains some technical considerations regarding the decision rules and a description of the numerical algorithm we use.

⁶See Conlisk (1996) for a discussion.

⁷See, for instance, Love (2013) and Winter et al. (2012) who evaluate welfare losses when following rules of thumb in portfolio choice and private savings decisions respectively. See also Hong et al. (2007) for an application in finance, Spiro (2012) and van Veldhuizen & Sonnemans (2011) for applications and tests in resource economics, Sims (2003) for applications in macroeconomics and Mirrlees & Stern (1972) for an early theoretical treatment.

2 The model

We set up our model in discrete time. The decision rules will be characterized by a minimum tree size that the decision maker will choose to cut at every point in time. The change of the growth function is exogenous and therefore not affected by the endogenous decisions made by the agent.

In addition to changes in the growth function, we will also assume a risk of for instance a fire, pests or a storm destroying the wood. We will treat this probability as exogenous but changing over time. For brevity we will refer to it as fire risk. One could think of several ways of complicating this by, for example, letting the risk be a function of the biomass but arguably, the way it is modeled here suffices for the purpose of comparing decision rules.⁸

The intended interpretation of the changes in the growth function and fire risk is that it is driven by changes in the surrounding environment – e.g. temperature or technology. We will, however, treat these changes as exogenous and therefore we can model the changes as depending on calendar time rather than the actual underlying driver.

2.1 The basic setup

As in earlier work, the value for the owner of the forest is due to the possibility of harvesting the trees and selling them. The price of wood is assumed to be constant over time and given by p . The cost of harvesting is a constant, c , for every harvest. It will be assumed that all trees are of the same age and that they will all be harvested at the same time.⁹ Since we assume that the agent maximizes discounted profits and that there is no incentive for income smoothing, the assumption of equal tree age makes no difference.¹⁰

In the standard formulation of the optimal rotation problem, when the environment is not changing, the problem facing the forest owner is the same after each time the trees are cut down. This means that the decision rule is independent of calendar time and usually expressed as the time that the owner should wait between each cutting. Here our decision rule will depend on calendar time and it will be described as the minimum

⁸For a more thorough treatment of the risk of fire specifically, see Stollery (2005).

⁹Mitra and Wan (1985) is an early paper modeling a forest with trees of different ages.

¹⁰An implicit assumption here is that there are no synergies between harvesting several patches of forest simultaneously.

tree size that the owner should cut down. Correspondingly, we chose the state variables in our optimization problem as calendar time t and current biomass F_t . Generally, we can then write the evolution of biomass as

$$F_{t+1} = (1 - X_t) [H_t g(0, t) + (1 - H_t) g(F_t, t)], \quad (1)$$

where $g(F, t)$ is the growth function, $H_t \in \{0, 1\}$ is the decision variable regarding harvesting and $X_t \in \{0, 1\}$ is a random variable that is equal to 1 if the forest is destroyed by fire. The growth function is assumed to be continuous and increasing in F . The distribution of X_t is given by

$$P(X_t = 1) = \pi_t \text{ and } P(X_t = 0) = (1 - \pi_t). \quad (2)$$

The assumed timing here is such that harvesting takes place at the beginning of a period and the trees then grow during the period until the next harvest opportunity. If the forest is destroyed in period t , the owner starts with biomass $F_{t+1} = 0$ in period $t + 1$. We can also define the τ times repeated application of the growth function, $g_\tau(F, t)$, iteratively as

$$g_0(F, t) = F \text{ and } g_\tau(F, t) = g(g_{\tau-1}(F, t), t + \tau - 1) \text{ for } \tau \geq 1. \quad (3)$$

The profit in period t is given by $H_t (pF_t - c)$. That is, if the trees are cut down, the profit is given by the income from selling the wood minus the harvesting cost, otherwise there is no profit in that period.

The objective of the forest owner is to maximize the expected value of the discounted profits, and the maximization problem can therefore be written as

$$\begin{aligned} & \max_{\{H_t \in \{0, 1\}\}} E \left[\sum_{t=0}^{\infty} \beta^t H_t (pF_t - c) \right] \\ & \text{s.t. } (1) \quad \forall t, \end{aligned}$$

where β is a discount factor.

Using dynamic programming, the solution of the maximization problem results in a value function $V(F, t)$ and a policy function $H(F, t)$ at each point in time. The value function satisfies the Bellman equation

$$\begin{aligned} V(F_t, t) &= \max_{H_t \in \{0, 1\}} H_t (pF_t - c) + \beta E [V(F_{t+1}, t + 1)] \\ & \text{s.t. } (1). \end{aligned} \quad (4)$$

Since g is increasing and continuous in F , V will also have these properties for all t .¹¹ The policy function $H(F, t)$ is the profit maximizing choice of cutting or leaving the trees. The harvesting rules will be of the form

$$H(F, t) = \begin{cases} 0 & \text{if } F < \bar{F}_t \\ 1 & \text{if } F \geq \bar{F}_t \end{cases} \quad (5)$$

and we will characterize the rules by \bar{F} . It is not obvious that the decision rules will always have this form. In appendix A we describe how we verify that this is in fact the case.

Since we assume that the growth function and risk of fire will change during a finite number of years and thereafter be unchanged, the optimization problem becomes autonomous in some time period T . Once the problem becomes autonomous, we can solve the problem from T onwards as an autonomous problem and this will give us the value function $V(F, T)$ that can serve as an end condition to the non-autonomous part of the solution.

We will now describe the decision rules that we analyze and how to solve for them. This will give us everything we need to numerically solve the problem. We consider three decision rules: optimal, naïve and ignorant.

2.2 The optimal decision rule

The optimal decision rule makes optimal harvesting decisions in all time periods. In order to find this rule the entire optimization problem must be solved from time zero incorporating all changes to the growth function and fire risk. We solve this problem backwards by first solving the problem facing the forest owner after period T when the problem becomes autonomous and thereafter working backwards through the non-autonomous problem.

¹¹That it is increasing follows since for any combination of future harvesting times, the first harvest will be larger the larger the initial biomass implying that you could always do better when starting from a larger initial biomass. The continuity follows since for any combination of future harvesting times the continuity of g implies that the resulting value is continuous in initial biomass. The optimal value function is then, for each F , the maximizing choice over a set of continuous functions making it continuous.

2.2.1 The autonomous problem

After period T , the problem becomes autonomous and calendar time no longer plays a role (we can therefore drop time from our notation when considering the autonomous problem). We will here describe how to find the decision rule and value function of an autonomous problem. This is the standard optimal rotation problem with a risk of fire and we can utilize that the problem is “reset” after each harvest. The value of having biomass zero, given that the owner waits for τ periods before cutting down the trees (provided that there has not been a fire in between) fulfills the equation

$$V_\tau(0) = \sum_{t=1}^{\tau} \beta^t (1 - \pi)^{t-1} \pi V_\tau(0) + \beta^\tau (1 - \pi)^\tau [pg_\tau(0) - c + V_\tau(0)]$$

where $g_\tau(F)$ is the τ times application of g . Solving this expression for $V_\tau(0)$ gives us

$$V_\tau(0) = \frac{(1 - \beta(1 - \pi)) \beta^\tau \tau (1 - \pi)^\tau}{(1 - \beta)(1 - \beta^\tau (1 - \pi)^\tau)} (pg_\tau(0) - c).$$

We can now find the maximized value $V(0)$ that is obtained when following the optimal decision rule. It must be given by $V_\tau(0)$ maximized over τ . Let

$$V(0) = \max_{\tau} \frac{(1 - \beta(1 - \pi)) \beta^\tau \tau (1 - \pi)^\tau}{(1 - \beta)(1 - \beta^\tau (1 - \pi)^\tau)} (pg_\tau(0) - c) \quad (6)$$

and let τ^* be the maximizing choice of τ . We also want to find \bar{F} in (5). From the maximization we know that

$$\bar{F} \in (g_{\tau^*-1}(0), g_{\tau^*}(0)]. \quad (7)$$

In appendix A.3 we show that \bar{F} can be found by solving the equation

$$p\bar{F} - c + V(0) = \pi\beta V(0) + (1 - \pi)\beta (pf(\bar{F}) - c + V(0)). \quad (8)$$

Given $V(0)$ and \bar{F} we can evaluate the value function for an arbitrary biomass F . If $F \geq \bar{F}$, i.e. the owner immediately cuts the trees, then $V(F) = pF - c + V(0)$. For $F < \bar{F}$, i.e. the trees are not large enough to be cut immediately, then we have that

$$V(F) = \sum_{t=1}^{\tau(F)} \beta^t \pi (1 - \pi)^{t-1} V(0) + \beta^{\tau(F)} (1 - \pi)^{\tau(F)} (pg_{\tau(F)} - c - V(0)), \quad (9)$$

where

$$\tau(F) = \min_{\tau \geq 1} g_\tau(F) \geq \bar{F}$$

is the number of time periods before the first harvest.

We now have formulas for both finding the optimal decision rule and for computing the value function for an arbitrary biomass F in an autonomous setting. Next, we move to solving a non-autonomous problem.

2.2.2 The non-autonomous problem

Above we solved the problem in the autonomous phase that begins in period T . We now move backwards into the non-autonomous phase of the problem where the growth function $g(F, t)$ and the probability of fire π_t changes over time. This also implies that the value function changes over time. We will solve the problem in this phase backwards starting with period $T - 1$. The solution of the problem in the autonomous phase thus gives us the value function in period T , $V(F, T)$, that serves as an end condition on V in the non-autonomous phase. We can therefore solve the problem in period $T - 1$ by using this end value. When considering the solution in any period $t < T$ we can thus assume that we have solved the problem in $t + 1$ and know the value function in that period. This way we can work backward until $t = 0$. We can then formulate the Bellman equation of period t as a function of the biomass F and the continuation value of leaving either $g(F)$ or $g(0)$ for the next period.

$$V(F, t) = \max_{H \in \{0,1\}} \left\{ \begin{array}{l} H [pF - c + \beta (\pi V(0, t + 1) + (1 - \pi) V(g(0, t), t + 1))] \\ + (1 - H) \beta [\pi V(0, t + 1) + (1 - \pi) V(g(F, t), t + 1)] \end{array} \right\}.$$

The solution to this equation gives us the value function and harvesting rule for any F since the right-hand side only depends on known entities when we get to period t . We want to find \bar{F}_t in (5). Since the value function is continuous in F for all t , \bar{F}_t must fulfill

$$\begin{aligned} p\bar{F}_t - c + \beta [\pi_t V(0, t + 1) + (1 - \pi_t) V(g(0, t), t + 1)] \\ = \\ \beta [\pi_t V(0, t + 1) + (1 - \pi_t) V(g(\bar{F}_t, t), t + 1)]. \end{aligned}$$

or

$$p\bar{F}_t - c = \beta(1 - \pi_t) [V(g(\bar{F}_t, t), t + 1) - V(g(0, t), t + 1)]. \quad (10)$$

This equation gives us the decision rule (as characterized by \bar{F}_t). We can also compute the value function of an arbitrary biomass F as

$$V(F, t) = \begin{cases} pF - c + \beta [\pi_t V(0, t + 1) + (1 - \pi_t) V(g(0, t), t + 1)] & \text{if } F \geq \bar{F}_t \\ \beta [\pi_t V(0, t + 1) + (1 - \pi_t) V(g(F, t), t + 1)] & \text{if } F < \bar{F}_t \end{cases}.$$

Summing up, we now have what we need to solve for the optimal decision rule and the associated value function. We first solve the autonomous problem that is relevant for $t \geq T$. We then solve backwards for the decision rule and value function associated with the optimal decision rule for all $t < T$. This gives a sequence $\{\bar{F}_t\}_{t=0}^T$. We now turn to describing the naïve and ignorant decision rules.

2.3 The naïve decision rule

The naïve decision rule observes the current growth function and probability of fire. It does not take the future changes of the environment into account. In each period the decision maker solves the optimization problem that would arise if the current growth function and risk of fire prevailed forever. Decisions are then made based on what would be optimal if the environment would not change over time. The owner is naïve in the sense that (s)he uses a rule of thumb as if noting that things have changed historically but believing that no more changes will come.

In each period t the naïve decision maker thus solves an autonomous problem as that described in section A.3 using the growth function $g(F, t)$ and risk of fire π_t . Solving this sequence of autonomous problems gives a sequence of decision rules characterized by $\{\bar{F}_t^N\}_{t=0}^\infty$.

The naïve decision maker, by construction, has incorrect expectations. This means that (s)he also has incorrect expected profits. When deriving the value of following the naïve decision rule we therefore want to compute the actual value of following this rule rather than the, incorrect, value that this person expects. To do this we need to calculate the value of following the naïve rule in the actual circumstances. That is, we have to calculate the value of an arbitrary biomass in period t when following the naïve decision rule, $V^N(F, t)$, as¹²

$$V^N(F, t) = \sum_{s=1}^{\tau^N(F, t)} \beta^s \Pi_{0, s-1} \pi_s V^N(0, t+s) + \beta^{\tau^N(F, t)} \Pi_{0, \tau^N(F, t)} \left[p g_{\tau^N(F, t)}(F, t) - c + V^N(0, \tau^N(F, t)) \right], \quad (11)$$

where

$$\tau^N(F, t) = \min_{\tau \geq 0} g_\tau(F, t) \geq \bar{F}_{t+T}^N$$

¹²We use the convention that a sum where the lower summation bound is larger than the upper is zero.

is the number of periods before the next harvest in the absence of fire and

$$\Pi_{s_1, s_2} = \begin{cases} \prod_{s'=s_1}^{s_2-1} (1 - \pi_{s'}) & \text{if } s_1 < s_2 \\ 1 & \text{if } s_1 = s_2 \end{cases} \quad (12)$$

is the probability that there is no fire in periods $s_1, \dots, s_2 - 1$.

Since the value in period t depends on the value function in future periods we must compute the naïve value function backwards from period T where the naïve and optimal value functions coincide.

2.4 The ignorant decision rule

For comparison we also include an ignorant decision rule. This rule does not observe the current growth function and risk of fire but rather makes decisions based on the initial growth function and fire risk at time zero. That is, in each period t the decision rule is characterized by the \bar{F} that results from solving the autonomous problem of section A.3 for growth function $g(F, 0)$ and risk of fire π_0 . This implies that the ignorant decision rule will be characterized by $\bar{F}_t^I = \bar{F}_0^N$ for all t .

As with the naïve decision rule we can calculate the value function of an arbitrary biomass F in period t , when following the ignorant decision rule, $V^I(F, t)$ backwards as

$$\begin{aligned} V^I(F, t) = & \sum_{s=1}^{\tau^I(F, t)} \beta^s \Pi_{0, s-1} \pi_s V^I(0, t+s) \\ & + \beta^{\tau^I(F, t)} \Pi_{0, \tau^I(F, t)} \left[pg_{\tau^I(F, t)}(F, t) - c + V^I(0, \tau^I(F, t)) \right], \end{aligned} \quad (13)$$

where

$$\tau^I(F, t) = \min_{\tau \geq 0} g_{\tau}(F, t) \geq \bar{F}_{t+T}^I$$

is the number of periods before the next harvest in the absence of fire and where Π_{s_1, s_2} is defined in (12). The value function at T , $V^I(F, T)$ is now not equal to the optimal value function and it must be computed separately.

2.5 Comparing decision rules

When evaluating the decisions rules we look at two dimensions. Firstly the relative profit losses, which are

$$L^N = \frac{V - V^N}{V} \quad (14)$$

in the naïve case and

$$L^I = \frac{V - V^I}{V} \quad (15)$$

in the ignorant case. Secondly, the relative errors in which size of tree to cut in each period, which are

$$D_t^N = \frac{\bar{F}_t - \bar{F}_t^N}{\bar{F}_t}$$

in the naïve case and

$$D_t^I = \frac{\bar{F}_t - \bar{F}_t^I}{\bar{F}_t}$$

in the ignorant case.

3 Numerical simulation

The problem defined in the previous section is very hard to solve analytically. It is even hard to characterize the optimal decision rule under any sort of general conditions. We therefore use a numerical approach that allows us to put an upper bound on the errors made by a naïve decision maker. As discussed above, it is difficult to assign probabilities to potential future climate scenarios. We address this problem by running a set of simulations (i.e. Monte Carlo) that cover a very broad range of possible future scenarios. If the errors made in all these simulations are small, which is what we find, this implies that the errors made in reality will very likely be small too. To draw such a conclusion the only requirement is that the parameters cover a wide enough range of future possibilities so that the actual realization will lie within the covered range of the parameter space. Further assumptions regarding the probability distribution within the range of scenarios covered are thus unnecessary. Note that if the resulting errors would *not* be small in all the simulations, that would still not allow us to conclude that errors would be large in reality since such a conclusion would require determining that some simulations that give large errors are in fact potentially realistic scenarios. Hence, this method allows for the following two alternative conclusions: “losses will necessarily be small” or “losses may not be small but we do not know if they will be large”.

In our main simulation we focus on a forest that changes such that it grows faster over time, where the maximum size of trees is increasing over time and where the risk of fire is increasing over time. This is meant to represent the case of climate change in boreal forests which cover the northern part of the northern hemisphere. As a robustness

check we also simulate the case where growth conditions deteriorate over time and the results are no different. If anything, our main result that profit losses from following the naïve decision rule are small is strengthened.

In all simulations, we have used $\beta = 0.95$ for discounting, which is a commonly used value for yearly discounting and real interest rate. Furthermore, we assume that it takes 200 years for the growth function to become stationary.¹³

For the numerical simulation, a particular function $g(F, t)$ must be chosen. We use a Bertalanffy growth function which has been converted to growth being a function of tree height rather than the age of the tree (for derivation, see Rammig et al., 2007). The Bertalanffy growth function in terms of tree height h is

$$h_{t+1} = h_{\max,t} \left(1 - \left(1 - \left(\frac{h_t}{h_{\max,t}} \right)^{\frac{1}{D_t}} \right) e^{-A_t} \right)^{D_t}.$$

Here $h_{\max,t}$ represents the maximum height a tree can reach under the climate prevailing at time t . We interpret A_t as the initial growth of a tree and D_t as a general growth parameter, both of which prevail at time t . In the stationary phase, these parameters are constant. We calibrate the model to boreal forests (present in the northern parts of Europe, Asia and North America) and hence use the estimation of the parameters by Rammig et al (2007) for Norwegian Spruce (*Picea Abies*) in the Swiss alps to approximately $A = 0.03$, $D = 3$ and $h_{\max} = 30$. Following Vanclay (1994, e.g. see page 70), we assume a quadratic relationship between tree height h and biomass F (implying a linear relationship between height and basal area). The growth function in terms of biomass is then

$$g(F_t, t) = F_{\max,t} \left(1 - \left(1 - \left(\frac{F_t}{F_{\max,t}} \right)^{\frac{1}{2D_t}} \right) e^{-A_t} \right)^{2D_t} \quad (16)$$

where $F_{\max,t} = h_{\max,t}^2$. The time path of the tree biomass following this growth function with the baseline parameters (from Rammig et al., 2007) is shown in Figure 1.

Another calibration that needs to be performed is that of price p and cost c . We assume their values to be fixed over time.¹⁴ While the relative proportions of these might

¹³One could think of the forest continuing to evolve also beyond this point. However, as $\beta^{200} \approx 0.00004$ it matters little for the discounted profits.

¹⁴Solving for a general equilibrium of biomass, timber prices and harvesting costs is a very complicated endeavor, especially if it is to be done for a future under climate change, involving having estimates of the price of fossil fuels, technological change, population growth etc. Although interesting, it falls outside the scope of this paper.

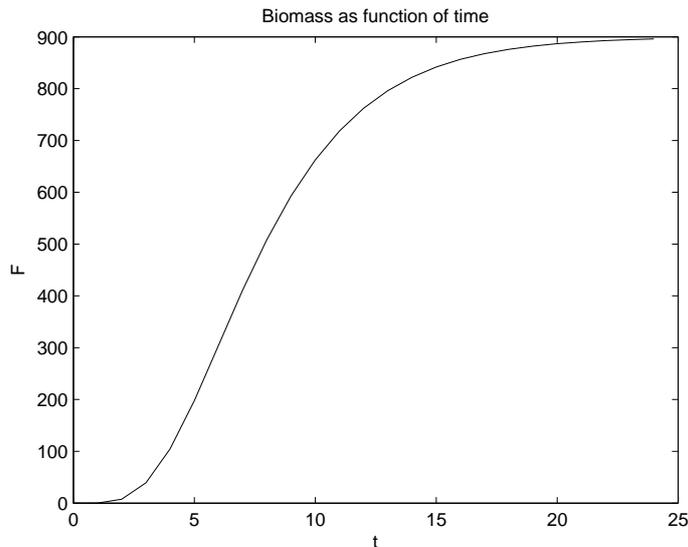


Figure 1: The typical time path of tree biomass following a Bertalanffy growth function.

be of importance for how often to harvest, their absolute values are not since they then only constitute a scaling of the absolute profits which is of no relevance here. The price is therefore normalized to one. Being interested in boreal forests we calibrate c to Swedish forestry in 2010 (see Swedish forestry authority, 2012, p. 283) so that $\frac{c}{pF_0} = 30\%$.¹⁵

We here assume that the risk of the trees being destroyed by a fire, pests or a storm, π_t , is increasing over time. This is a reasonable assumption in northern countries when the underlying driver is increased temperature. As a robustness check we also consider a case without any such risks.

In our simulations, we first set the initial parameter values (at time zero). We set $F_{\max,0} = h_{\max,0}^2 = 30^2$ and randomly draw values of the remaining parameters from uniform distributions over the intervals $A_0 \in [0.025, 0.035]$, $D_0 \in [2.5, 3.5]$ and $\pi_0 \in [0.001, 0.3]$.¹⁶ Second, we vary the parameter values when the growth function stabilizes at $t = 200$. They are randomly drawn from the uniform distributions $A_{200} \in [A_0, 3A_0]$, $D_{200} \in [0.8D_0, D_0]$, $F_{\max,200} \in [F_{\max,0}, 4F_{\max,0}]$, $\pi_{200} \in [\pi_0, 0.3]$. This encompasses very significant changes in the growth function since it allows for a quadrupling of the maximum biomass (corresponding to a doubling of the maximum

¹⁵Andersson et al. (2013) show that this ratio has been very stable for the last 70 years. In our setting it may change in later time periods as one cuts larger or smaller trees.

¹⁶We do not vary the value of $F_{\max,0}$ since it is essentially a scaling of the whole problem.

height) and trees growing to 90% of their maximum twice as fast as before, all happening within 200 years. To illustrate that these scenarios are indeed very extreme, we can compare them to some calibrated predictions in forest biology in boreal forests. For instance, Ge et al. (2011, table 2) have an increase in what corresponds to our F_{\max} of at most 20% while we allow for a 300% increase. Another example is Kellomäki et al. (2008, table 4) who estimate an increase of the tree growing to be 100% bigger within one rotation while we had several scenarios where the tree grows to be more than 500% bigger within a rotation. Furthermore, we allow for the risk of the forest being burned down to increase from once in a millennium to every third year. Third, we vary the trajectory for the changes during the 200 years to be either linearly increasing, concavely increasing or stepwise increasing. The concave and linear case represent smooth transitions with different modes of convergence. The stepwise increasing case is meant to catch climate thresholds which lead to abrupt changes. This final case is also a reduced form for abrupt technical change.

The complete Monte Carlo simulation draws 60 initial sets of parameters and 240 sets of parameter values at $T = 200$ per initial set. For each of these combinations of initial and final values it then uses all the three described trajectory shapes. In each separate simulation it calculates the optimal, naïve and ignorant decision rules and the associated discounted profits of having a forest of any possible initial biomass, that is, for any $F_0 \in [0, \max\{\bar{F}_0^N, \bar{F}_0\}]$.¹⁷ We then compute the expected values of L^N and L^I from equations (14) and (15). An algorithm for the numerical simulations can be found in appendix B. In total the Monte Carlo procedure involved 43200 simulations.

4 Numerical results¹⁸

The optimal and naïve decision rules of the simulation that yield the maximum expected losses (that is, our worst case scenario) are depicted in Figure 2. The graph on the left-hand side shows how the \bar{F} values that characterize the naïve and optimal decision rules change over time. As can be seen, the naïve policy is very similar to the optimal one throughout the transition period. On the right-hand side of Figure 2 we see how the

¹⁷We could instead have used initial values up to $F_{\max,0}$ but since both decision rules would prescribe immediate harvesting for $F > \max\{\bar{F}_0, \bar{F}_0^N\}$ that would most likely result in small profit losses.

¹⁸All material required for reproduction of the results is available from the authors upon request.

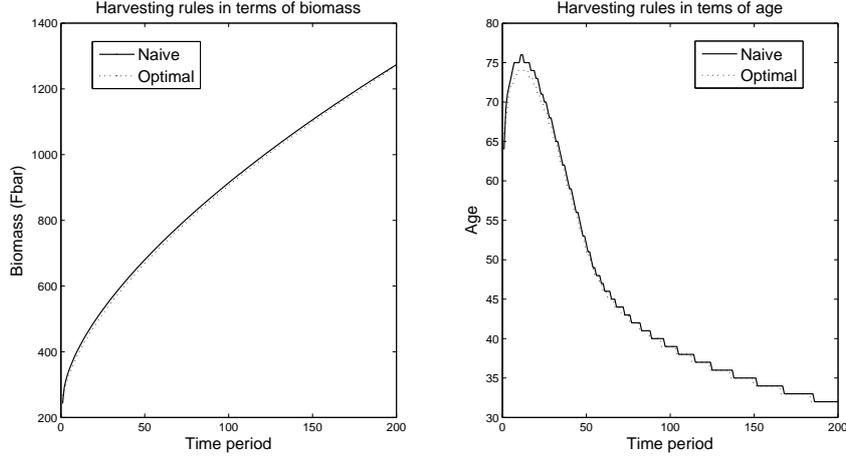


Figure 2: Decision rules of the worst case scenario. The graph on the left-hand side shows the decision rules in terms of biomass \bar{F} while the right-hand side of the graphs show them in terms of the minimum age of trees that the respective decision rules prescribe harvesting. The ages are the actual ages of trees cut in each time period. For the naïve decision rule this will not be the same as the planned rotation time at planting. The initial parameters of this scenario are $A_0 = 0.0027$, $F_{\max,0} = 900$, $D_0 = 3.4975$ and $\pi_0 = 0.0017$; the parameter values after stabilization are $A_{200} = 0.0619$, $F_{\max,200} = 3453$, $D_{200} = 3.1368$ and $\pi_{200} = 0.0055$ and the trajectory is concave.

minimum ages of trees that the naïve and optimal decision rules would prescribe cutting.

The similarity between the optimal and naïve decision rules is representative. This can be seen in Figure 3 which shows histograms over the relative errors in the naïve decision rule (in that figure we also present the harvesting errors of following the ignorant rule). Each simulated case (initial parameters, final parameters and trajectory) results in $T + 1$ values of \bar{F} for each decision rule. Each such value of \bar{F} constitutes an observation in this histogram. With the exception of a few outliers (the most extreme being 37%), the harvesting errors when following the naïve decision rule are small. While these outliers imply that we cannot discard the possibility that errors in harvesting rules can, in some cases, be large, the central question is whether they yield large profit losses.

Histograms over the relative profit losses made can be found in Figure 4. In these histograms we have one observation per simulated case for each decision rule. These are the mean profit losses made, seen from period 0, when following the respective rules. The

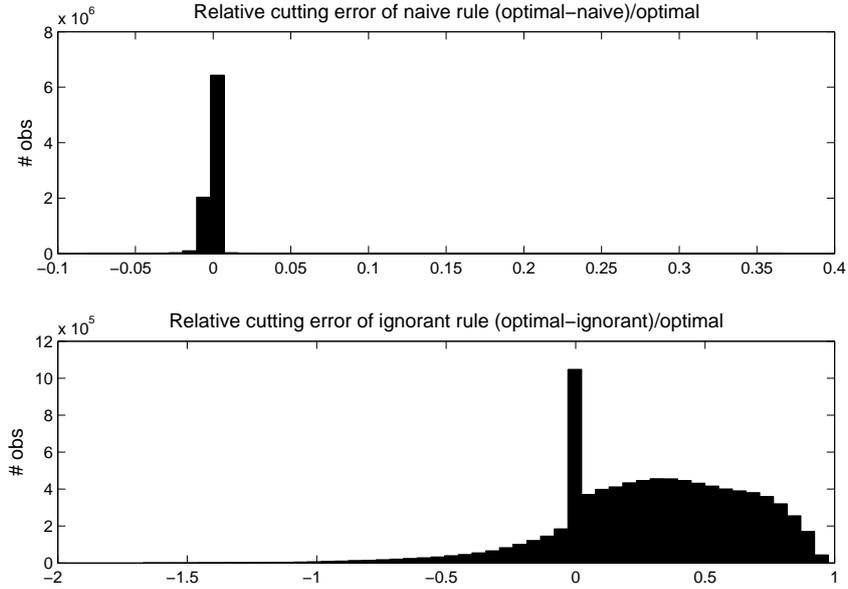


Figure 3: Histograms of the relative errors in the naïve and ignorant decision rules.

mean loss is taken across initial biomasses $F_0 \in [0, \max\{\bar{F}_0^N, \bar{F}_0\}]$. The mean loss over all possible initial biomasses is a relevant measure of expected losses within a scenario since most forest owners will have trees of different age and size within their domains. From the histogram for the naïve decision rule we can see that the expected profit losses are small for all scenarios. The worst case scenario (the harvesting decisions of which are depicted in Figure 2) gives a loss of 0.2% and hence we treat this as our upper bound for losses following the naïve rule.

A forest owner following the ignorant rule would make significant losses in many cases. We can see that many scenarios yield losses of 2-30% which, of course, makes them economically significant.

5 Robustness checks

To check the robustness of the results we have performed the simulations under some extensions to the model. Firstly, our main formulation of the naïve rule implies that the owner observes the actual changes to the forest dynamics without delay. This is motivated by most forest owners having trees of various ages from which they should be

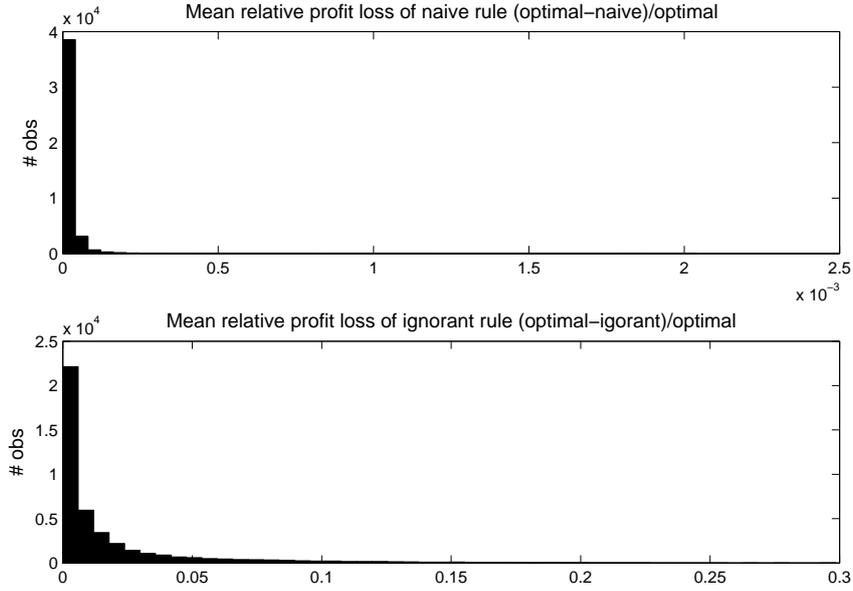


Figure 4: Histograms of the mean relative profit losses resulting from following the naïve and ignorant rather than the optimal decision rules.

able to infer the entire growth function of the trees. Now, while this may be reasonable, a practical problem facing an owner trying to perform such an exercise is that there is variability in the actual growth of trees from year to year depending on the weather and other temporary factors. This implies that the owner may need more than one year to infer what the changes are and hence that there will be a delay. A very extreme form of delay is obviously the ignorant rule, which altogether does not notice the changes. We have seen that with the ignorant rule the profit losses can be substantial so the question is what kind of delay is necessary for profits to be sizable. To address this issue we have computed the relative profit losses of the naïve rule when the owner observes the actual conditions with two forms of delay. In the first form, the lagged decision rules are computed as a moving average of the naïve decision rule without delay. That is, in a given period, the n years lagged decision rule is characterized by an \bar{F}^{Delay} that is the average of the naïve decision rules (\bar{F}^N) over the n most recent periods. The results are presented in the left part of Figure 5. There we have included the profit losses following the naïve decision rule, the decision rule using the moving average of the naïve rule of the last five years and of the last ten years. Each graph represents the cumulative

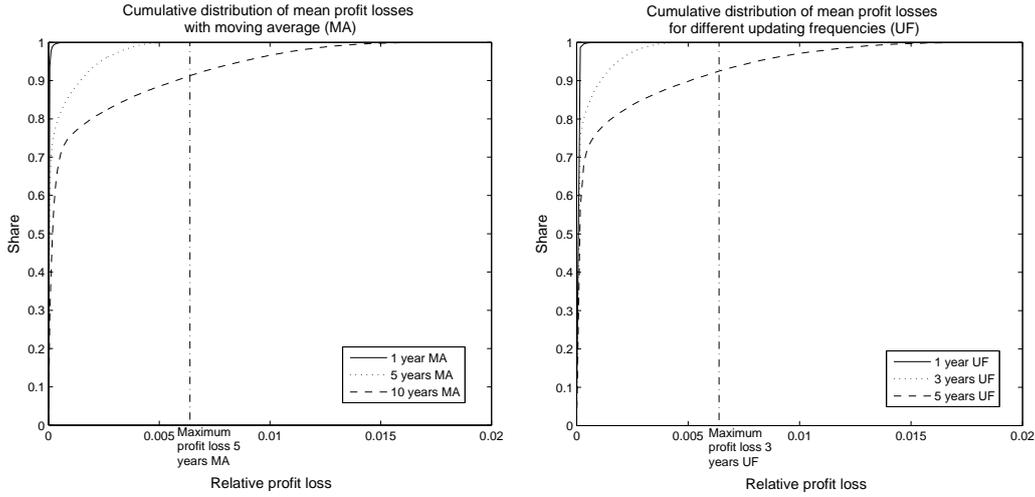


Figure 5: Cumulative distribution of mean profit losses when following lagged versions of the naïve decision rule. The vertical lines indicate the point at which the respective cumulative distributions become equal to one. That is, the largest expected profit loss in our simulations.

distribution function (CDF) of a certain number of years delay with profit losses on the x-axis. What is of interest for us is how fast each CDF converges to one. At the point where it does so we obtain the least favorable scenario and hence the upper bound for naïve losses given a certain delay. As can be seen, as long as the delay is five years or less the maximum loss is well below one percent. However, for delays of ten years the losses can possibly be close to two percent in the least favorable scenario which is large enough not to be ignored.

In the second form of delay the decision rule is updated not every year (like in the naïve rule) but more infrequently. So, for instance, with an updating every third year the decision rule follows the naïve rule of year one during years one–three, the naïve rule of year four during years four–six and so on. The results are presented in the right part of Figure 5. There we have included the CDF of relative losses following decision rules with updating frequency of one year (i.e., the naïve rule), three years and five years. As can be seen an updating frequency of three years yields losses of around 0.55 percent in the worst case scenario. But if the updating frequency is only every five years losses in the worst case scenario are above 2 percent which is no longer negligible.

Our interpretation of the results following the two forms of delay is that for short enough delays (up to five year moving average or three years updating frequency) the naïve owner will still only make marginal losses while if the delay is sufficiently long we can no longer draw the conclusion that the upper bound is small. Recall, however, that this does not imply that we can draw the conclusion that profit losses will be large – we simply cannot say anything conclusive in such a case.

Secondly, we have also performed the simulations for the case where trees are growing slower over time to represent cases where the climate becomes, for instance, drier.¹⁹ In these simulations we use the same starting values and simply use a mirror image of the end values in the basic case of trees growing faster so that trees now grow slower and to smaller sizes. The evolution of the fire risk is however not reversed – they increase over time here too. The results of such an exercise look very much the same as in our main simulation. If anything, the profit losses are now smaller.

Finally, although the case of trees growing faster and an increasing fire risk is probably the most relevant scenario for boreal forests these two effects tend to cancel each other out. While a faster growing tree may imply cutting larger trees, the increasing fire risk implies cutting smaller trees. To see whether this is what causes the naïve rule profit losses to be small we have also run the simulations while shutting down the fire risk. We simply assume that there is no risk of fire in any year. Again, the profit losses of the naïve rule remain small with an upper bound of 0.35%.

While the magnitudes of the errors vary somewhat between the main simulation and the robustness checks of trees growing slower and no fire, we can see that in all cases the profit losses associated with the naïve rule are small and the losses associated with the ignorant rule are about a factor 100 larger.

6 Conclusions and discussion of the results

Our results clearly show that a forest owner who is able to gradually observe, rather than foresee, changes in the growth dynamics and in the risk of, for instance, fire will

¹⁹For both the case where trees grow more slowly over time and the case, below, without risk of fire, the simulations draw 20 sets of initial parameters, then for each initial parameter set draws 80 sets of parameters at time T and then performs the computations for each of the three possible trajectories. This gives a total of 4800 simulations.

be very close to making optimal decisions of when to harvest. Even if these changes are observed with a delay of five years the profit losses of not being perfectly forward looking are only marginal. At a first glance, even the ignorant policy – which implies not even noticing the change in hindsight – often seems to only give small losses. But, arguably, this is a conceptual problem of discounting profits. To state this more clearly, how come making errors in cutting which are at the level of 100% yields profit losses that are only at the 30% level? This is due to the ignorant rule being fairly accurate to begin with, while the fact that it becomes increasingly inaccurate over time is of less importance since whatever profits occur later are safely discounted away. But, while mismanagement in a hundred years is less of a problem for an owner today, the owner in a hundred years will probably consider it highly worthwhile to update the policy. It is then reassuring that the naïve decision rule closely follows the optimal, which implies that the small profit losses under the naïve rule are not due to discounting. Therefore, we conclude that observing the forest’s change as it happens is important, but sufficient, for an owner. As the naïve policy only implies profit losses well below 0.5% and the cutting rule does not differ much from the optimal even though we include some rather wild scenarios, it seems to be accurate enough for most businesses. The reason is that observing the changes as they come ensures that the harvesting rule does not diverge from the optimal. For practical forest management, it is therefore questionable whether investing in high accuracy climate and technology forecasts is worthwhile. Thus, while noticing that a change has happened is essential for profitable management, trying to foresee this change is not. This, however, does not imply that foreseeing a catastrophic event next year is not worthwhile. It is of course of great value for a forest owner to know whether, for instance, the forest will burn down next year – if such knowledge is at all possible. But this is not the same as it being valuable to know how the probability of a fire changes, which is what this paper is about.

Our model is far more simple than the complex reality facing an actual forest owner. Strictly speaking, our result that forecasting is not important is therefore confined to the aspects covered in the model. Whether our simplifications are of any consequence to the results crucially depend on two things. Firstly, it is not enough that an additional piece of complexity would change the harvesting rules. Since we are only interested in how wrong the naïve owner is compared to an owner with perfect foresight, what is important

is whether the added component would affect one of them disproportionately more than the other and how this translates into expected profits. Secondly, depending on the initial size of the trees the loss may be smaller than the average loss (if, for instance, the first best and the naïve rule both prescribe waiting a few years before cutting) or greater (if, for instance, the first best prescribes cutting immediately while the naïve prescribes waiting). As we are interested in the expected loss within a scenario we do, however, believe that the average loss over all possible initial conditions is the relevant measure. Adding a degree of complexity to the model is only of importance if it will affect losses under many of the initial conditions, rather than under only one or a few.

One possible alteration of the model would be to allow for the choice of changing the kind of tree one plants instead of having one tree species whose growth pattern changes over time. This would require a different kind of modeling where, possibly, the growth function is constant for the currently standing tree but where planting a new tree type changes the growth function. In this sense our way of modeling may be more representative of climate change whereas R&D may be better modeled in the alternative way. Although we see no direct reason why this would be more of a problem for a naïve decision maker and why this would not be averaged out over the possible initial conditions, our model and simulations, do not reveal whether the alternative problem makes forecasting worthwhile. As has been shown by Yousefpour et al. (forthcoming), when making decisions about which trees to plant the losses of acting according to the wrong scenario can be severe. We can only speculate whether this difference compared to our results comes from the choice being made (when to cut compared to what species to plant) or whether it is due to that making inaccurate predictions is more problematic than not making any predictions at all.

Another constraint that is highly relevant within forestry is that the price of timber may fall if the tree diameter becomes too large. We are rather sure that, if anything, this should decrease the difference between the naïve and the first best rule since it puts a cap on which size of tree to cut for both types of decision makers.

What we have less intuition for is how the results would change if incorporating a decision of when to altogether stop growing trees on a certain land area. Our model only considers the direct costs and profits of growing and harvesting trees. But in reality there is also an opportunity cost since the land area may be used for other purposes.

Obviously, the fact that forestry is currently taking place strongly suggests that these alternative usages should not be more profitable today. When the trees are growing faster and faster (like in our main simulations) a first guess would be that this should make forestry a relatively profitable endeavor also in the future. But in the opposite case, when trees are growing slower and slower, it might be that the alternative usages become more profitable at some point - if not then only because forestry itself becomes less profitable. In that case it may incur losses if one plants trees to be harvested in, say, two decades when an alternative usage is preferable already after a few years' time. A perfectly forward looking agent may be able to foresee this while a naïve agent may not. While it is not obvious what these alternative usages would be we cannot disregard such a possibility. A similar issue regards the calculation of the present value of the forest before buying or selling it. If the effect of climate change on the growth of trees is extreme then this may affect the present value substantially. So to arrive at a correct price in such an important single decision it seems essential to at least roughly be able to foresee how climate change will affect growth. Likewise we cannot disregard that other long lasting decisions, such as thinning, may affect the results. Especially in the case of stepwise climatic changes. However, once those long lasting decisions have been made we show that the continuous management decisions can be based on the naïve decision rule.

These potential caveats aside, it is worth noting that the conclusions we draw regarding the elements we do model are probably on the conservative side. First of all our decision maker with perfect foresight is indeed a utopian agent. In practice no one is able to foresee climate change and its consequences and economic agents are left to either believing in a specific scenario or weighting different scenarios and letting expected profit maximization determine the harvesting rule. Secondly, the scenarios we have included are very extreme. Since this paper is about dealing with uncertainty rather than quantifiable risk, and since we do not ourselves know what probability distribution to use, we have chosen to include these extreme scenarios as to be sure that the future possibilities all fall within our simulations.

A Monotonicity of the harvesting rule

We will here derive sufficient conditions for the harvesting rule to have the form (5). We will start by verifying that our growth function is increasing and concave. We will then draw some implications from this and show how it can be used to verify the form of the harvesting rule.

A.1 Concavity of the growth function

Regarding the growth function g we will assume that it is continuous, increasing and concave for all $F \geq 0$ in all time periods. This will hold for our growth function as can be seen by differentiating it. Starting from (16), we rewrite the growthfunction as²⁰

$$g(F) = F_{\max} \left(1 - e^{-A} + e^{-A} \left(\frac{F}{F_{\max}} \right)^{\frac{1}{2D}} \right)^{2D}.$$

Differentiating it once we get

$$\begin{aligned} g'(F) &= F_{\max} 2D \left(1 - e^{-A} + e^{-A} \left(\frac{F}{F_{\max}} \right)^{\frac{1}{2D}} \right)^{2D-1} e^{-A} \frac{1}{2D} \frac{1}{F} \left(\frac{F}{F_{\max}} \right)^{\frac{1}{2D}} \\ &= e^{-A} \left(\frac{g(F)}{F_{\max}} \right)^{\frac{2D-1}{2D}} \left(\frac{F}{F_{\max}} \right)^{\frac{1-2D}{2D}} = e^{-A} \left(\frac{g(F)}{F} \right)^{\frac{2D-1}{2D}} > 0. \end{aligned}$$

Differentiating again we get

$$\begin{aligned} g''(F) &= \frac{2D-1}{2D} g'(F) \left(\frac{g'(F)}{g(F)} - \frac{1}{F} \right) \\ &= \frac{2D-1}{2D} \frac{g'(F)}{F} \left(\frac{F}{g(F)} e^{-A} \left(\frac{g(F)}{F} \right)^{\frac{2D-1}{2D}} - 1 \right) \\ &= \frac{2D-1}{2D} \frac{g'(F)}{F} \left(e^{-A} \left(\frac{g(F)}{F} \right)^{-\frac{1}{2D}} - 1 \right). \end{aligned}$$

²⁰For practical purposes it does not matter much what we assume about the growth function for $F > F_{\max}$ but for completeness we need to assume something. Assuming that the growth function is given by the same expression from there as well implies that the biomass can become smaller over time $g(F) < F$. While it is not obvious what the right assumption is, this assumptions implies that the growth function will always be increasing and strictly concave.

Assuming $2D > 1$ this is negative since $g'(F) > 0$ and

$$\begin{aligned} e^{-A} \left(\frac{g(F)}{F} \right)^{-\frac{1}{2D}} &= \frac{e^{-A}}{\left(\frac{F_{\max}}{F} \right)^{\frac{1}{2D}} \left(1 - e^{-A} + e^{-A} \left(\frac{F}{F_{\max}} \right)^{\frac{1}{2D}} \right)} \\ &= \frac{e^{-A}}{(1 - e^{-A}) \left(\frac{F_{\max}}{F} \right)^{\frac{1}{2D}} + e^{-A}} < 1 \end{aligned}$$

This shows that g is increasing and concave.

A.2 The value of waiting a prescribed number of periods

In order to verify the monotonicity of the harvesting rule for both autonomous and non-autonomous problems, we will utilize the shape, as a function of F , of the value of waiting a prescribed number of time periods before harvest. We assume now that we are in period t and that the value of having biomass zero is known for all future time periods including period t (the value $V(0, t)$ can easily be computed given the decision rule and the value of zero biomass in all future time periods). We can then define the value of waiting for τ time periods before harvesting as

$$V_0(F, t) = pF - c + V(0, t)$$

and, for $\tau \geq 1$

$$\begin{aligned} V_\tau(F, t) &= \sum_{s=1}^{\tau} \beta^s \Pi_{0, s-1} \pi_s V(0, t+s) \\ &\quad + \beta^\tau \Pi_{0, \tau} [pg_\tau(F, t) - c + V(0, \tau)] \end{aligned} \quad (17)$$

with Π_{s_1, s_2} defined in (12). We also define

$$w_\tau(F, t) = V_\tau(F, t) - V_0(F, t)$$

the net value of waiting for τ periods before harvesting rather than harvesting directly. From the concavity of g it follows that $w_\tau(F, t)$ is concave in F if $\tau \geq 1$. The concavity implies that it can change sign at most twice, once from negative to positive and once, for a larger F , from positive to negative. In particular we have that if $\tau \geq 1$, $F_2 > F_1 \geq 0$ then

$$w_\tau(F_1, t) > 0 \ \& \ w_\tau(F_2, t) < 0 \Rightarrow w_\tau(F, t) < 0 \ \forall F > F_2 \quad (18)$$

and

$$w_\tau(F_1, t) \geq 0 \ \& \ w_\tau(F_2, t) \geq 0 \Rightarrow w_\tau(F, t) \geq 0 \ \forall F \in [F_1, F_2]. \quad (19)$$

Suppose now that we have found a biomass \tilde{F} and waiting time $\tilde{\tau}$ such that

$$w_{\tilde{\tau}}(\tilde{F}, t) = 0 \text{ and } w'_{\tilde{\tau}}(\tilde{F}, t) < 0.$$

Assume, furthermore, that we can find sequences of biomass $\{F_i\}_{i=0}^I$ and waiting times $\{\tau_i\}_{i=1}^I$ such that

$$F_0 = \tilde{F}, F_{i+1} < F_i, \text{ \& } F_I = 0; \tau_1 = \tilde{\tau} \text{ \& } \tau_{i+1} > \tau_i \quad (20)$$

and that these sequences fulfill

$$w_{\tau_1}(F_1, t) > 0 \text{ and for } i > 1 \text{ } w_{\tau_i}(F_i, t) > 0 \text{ \& } w_{\tau_i}(F_{i-1}, t) > 0. \quad (21)$$

Then (19) implies that for each $F < \tilde{F}$ there is a τ such that $w_{\tau}(F, t) > 0$ and consequently that the harvesting rule will not prescribe harvest for any $F < \tilde{F}$.

A.3 The autonomous problem

We will start by deriving sufficient conditions for the harvesting rule to have the form (5) in an autonomous problem. We do not need to keep track of the calendar time and the growth, harvesting and value functions will be functions of only biomass.

We start by noting that in an autonomous problem we will always want to harvest immediately if $F > F_{\max}$ since otherwise the biomass would decrease. We therefore need only analyze the case where we start from a biomass $F < F_{\max}$ which will also imply that the biomass will remain below F_{\max} and we can assume that $g(F) > F$ always holds.

Assume now that we have found an \tilde{F} such that $w_1(\tilde{F}) = 0$ and $w'_1(\tilde{F}) < 0$. This is what our numerical algorithm will give us as a candidate for the \bar{F} of the decision rule (5). The concavity of w_1 implies that we then will have $w(F) < 0$ for all $F > \tilde{F}$. This, in turn, tells us that the decision rule will always prescribe harvest for $F > \tilde{F}$ (note that this includes $g(F)$ for any $F \in (\tilde{F}, F_{\max})$). What remains in order to verify that the decision rule has the form (5) with $\bar{F} = \tilde{F}$ is that the decision rule does not prescribe harvest for any $F < \tilde{F}$. We can do this by finding sequences $\{F_i\}_{i=0}^I$ and $\{\tau_i\}_{i=1}^I$ that fulfill conditions (20) and (21) with $\tilde{\tau} = 1$.

A.4 The non-autonomous problem

For the non-autonomous problem, the growth, harvest and value functions all depend on calendar time t . We assume that we are in period t and we know that the harvesting

rules in all future periods are monotone and we also know their \bar{F} . We will here derive sufficient conditions for the harvesting rule to be monotone in period t as well. Since, in our simulations, the autonomous situation follows the non-autonomous, we can use the conditions derived above to verify that the harvesting rule is monotone in the first period of the autonomous problem and we can work backwards from that.

Assume that we have found an \tilde{F} such that

$$p\tilde{F} - c = \beta(1 - \pi_t) \left[V(g(\tilde{F}, t), t + 1) - V(g(0, t), t + 1) \right]$$

and

$$p > \beta(1 - \pi_t)V'(\tilde{F}_t, t)$$

where prime denotes derivative with respect to F . This is what our numerical algorithm will give us. Let

$$\tilde{\tau} = \min_{\tau \geq 1} g_\tau(\tilde{F}, t) \geq \bar{F}_{t+\tau}$$

be the number of time periods before harvest if the trees are not cut down in the current period.

We now want to derive sufficient conditions for \tilde{F} to be the \bar{F} of the harvesting rule (5). An alternative way to characterize \tilde{F} is to say that

$$w_{\tilde{\tau}}(\tilde{F}, t) = 0 \text{ and } w'_{\tilde{\tau}}(\tilde{F}, t) < 0.$$

We now want to derive conditions under which for all $F > \tilde{F}$, $w_\tau(F, t) \leq 0$ for all $\tau \geq 1$ and under which for each $F < \tilde{F}$ there is a $\tau \geq 1$ such that $w_\tau(F, t) > 0$.

The monotonicity of the harvesting rule in all future time periods (and in particular in period $t + \tilde{\tau}$) implies that $w_\tau(F, t) < 0$ for all $\tau > \tilde{\tau}$ and $F > \tilde{F}$. A sufficient condition for $w_\tau(F, t) < 0$ for all $\tau \in [1, \tilde{\tau} - 1]$ (assuming that there are such τ -values) is that for each such τ , there is an $F < \tilde{F}$ such that $w_\tau(F, t) > 0$. This follows from (18) since $w_\tau(\tilde{F}, t) < 0$ for such τ . Numerically, we can verify this by simply testing values until we have found such an F for each τ .

Finally, we want to find conditions that guarantee that for each $F < \tilde{F}$ there is a $\tau \geq 1$ such that $w_\tau(F, t) > 0$. As for the autonomous problem, we can do this by finding sequences $\{F_i\}_{i=0}^I$ and $\{\tau_i\}_{i=1}^I$ that fulfill conditions (20) and (21).

B Numerical algorithm

We start by drawing initial values of A , F_{\max} , D and π . Based on these we calibrate the harvesting cost c . We do this by iteratively solving (6) and updating c until it is 30% of the revenues in an autonomous problem based on the initial parameters.

We then draw parameter values at time T and generate the parameter trajectories from 0 to T . Based on the parameters for each $t \in [0, T]$ we solve an autonomous problem based on these parameters. We do this by first finding $V(0)$ from the solution to (6) and then by finding the \bar{F} in the interval from (7) that fulfills the condition (8) using a search algorithm. We now have the naïve and ignorant decision rules. Using (9) we can also get $V(F, T)$, which is the value function of both the naïve and optimal decision rules, for an arbitrary F . We also check the sufficient condition for the decision rule to be monotone in each of the autonomous problems using the method described in section A.3.

We then find the optimal decision rule by moving backwards from $t = T$ to $t = 0$ and in each step finding the \bar{F}_t that fulfills (10). When computing the value function we, rather than storing the value function, use the formula

$$V(F, t) = \sum_{s=1}^{\tau(F,t)} \beta^s \Pi_{0,s-1} \pi_s V(0, t+s) + \beta^{\tau(F,t)} \Pi_{0,\tau(F,t)} [pg_{\tau(F,t)}(F, t) - c + V(0, \tau(F, t))], \quad (22)$$

where

$$\tau(F, t) = \min_{\tau \geq 0} g_{\tau}(F, t) \geq \bar{F}_{t+T}$$

is the number of periods before the next harvest and where Π is defined in (12). We also store $V(0, t)$ for each t . When we get to $t = 0$ we have the optimal decision rule and we check the monotonicity assumption as described in A.4.

The next step is to compute the value function associated with each decision rule at $t = 0$ and for biomasses $F \in [0, \max\{\bar{F}_0^N, \bar{F}_0\}]$. To do this we need the value of biomass 0, $V(0, t)$, for all decision rules. For the optimal decision rule we already have $V(0, t)$ for all t . For the naïve decision rule we have $V^N(0, T)$ (which, at $t = T$, is equal to the optimal value function) and for the ignorant decision rule we compute $V^I(0, T)$ using (13). We then move backwards from $t = T - 1$ to $t = 0$ computing the naïve and ignorant value functions $V^N(0, t)$ and $V^I(0, t)$ using (11) and (13) respectively. We can

now compute the value functions at $t = 0$ for different biomasses for the optimal, naïve and ignorant decision rules using (22), (11) and (13) respectively.

The last thing we do is to compute the value of following lagged naïve decision rules the same way as we computed the naïve decision rules but using a shifted version of the naïve decision rule.

References

- [1] Andersson, L., Hultkrantz, L., Mantalos, P., , "Stumpage Prices in Sweden 1909-2011: Testing for Non-Stationarity", Working Papers, Swedish Business School, Örebro University No 2013:1 (2013)
- [2] Conlisk, J (1996), "Why Bounded Rationality?", *Journal of Economic Literature*, Vol. 34, pp. 669-700
- [3] Faustmann, M (1849) "Berechnung Des Wertes Welchen Waldboden Sowie Noch Nicht Haubare Holzbestände Für Die Waldwirtschaft Besitzen", *Allgemeine Forst und Jagd-Zeitung*, 25, 441-455
- [4] Ge, Z-M., Kellomäki, S., Peltola, H., Zhou, X., Wang, K-Y., Väisänen, H., & Makela, A. "Impacts of changing climate on the productivity of Norway spruce dominant stands with a mixture of Scots pine and birch in relation to water availability in southern and northern Finland " (2011). *Tree Physiology*, 31 (3): 323-338.
- [5] Hanewinkel, M., Cullmann, D. A., Schelhaas, M. J., Nabuurs, G. J., & Zimmermann, N. E. (2012). "Climate change may cause severe loss in the economic value of European forest land." *Nature Climate Change*, 3(3), 203-207.
- [6] Hong, H., Stein, J.C., Yu, J. (2007), "Simple forecasting and paradigm shifts", *The Journal of Finance*, Vol LXII, No 3.
- [7] Jacobsen, J. B., & Thorsen, B. J. (2003). "A Danish example of optimal thinning strategies in mixed-species forest under changing growth conditions caused by climate change". *Forest Ecology and Management*, 180(1), 375-388.
- [8] Kellomäki, S., Peltola, H., Nuutinen, T., Korhonen, K. T., & Strandman, H. (2008). "Sensitivity of managed boreal forests in Finland to climate change, with implications

- for adaptive management". *Philosophical Transactions of the Royal Society B* 363, 2341-2351.
- [9] van Kooten, G C, C S Binkley & G Delcourt (1995) "Effect of Carbon Taxes and Subsidies on Optimal Forest Rotation Age and Supply of Carbon Services," *American Journal of Agricultural Economics*, Vol 77, No 2, 365-374.
- [10] Love, D. A. (2013). "Optimal Rules of Thumb for Consumption and Portfolio Choice". *The Economic Journal*, 123(571), 932-961.
- [11] Löfgren, K-G (1985) "Effect on the socially optimal rotation period in forestry of biotechnological improvements of the growth function", *Forest Ecology and Management*, 10, 233-249.
- [12] McConnell, K E, J N Daberkow & I W Hardie (1983) "Planning Timber Production with Evolving Prices and Costs", *Land Economics*, Vol 59, No 3, 292-299
- [13] Mirrlees, J.A., Stern, N.H. (1972), "Fairly Good Plans", *Journal of Economic Theory*, Vol 4, pp. 268-288.
- [14] Mitra, T & H Y Wan Jr (1985) "Some Theory Results on the Economics of Forestry", *Review of Economic Studies*, 52, 229-249.
- [15] Newman, D H (2002) "Forestry's golden rule and the development of the optimal forest rotation literature", *J. Forest Economics* 8, 5-27
- [16] Newman, D H, C B Gilbert & W F Hyde (1985) "The Optimal Rotation with Evolving Prices", *Land Economics*, Vol 61, No 4, 347-353
- [17] Ohlin B (1921) "Till frågan om skogarnas omloppstid", *Ekonomisk Tidskrift*, 22, 89-113
- [18] Pressler, M R (1860) "Aus der Holzzuwachlehre (Zweiter Artikel)", *Allgemeine Forst- und Jagdzeitung*, 36, 173-191
- [19] Pukkala, T., & Kellomäki, S. (2012). "Anticipatory vs adaptive optimization of stand management when tree growth and timber prices are stochastic." *Forestry*, 85(4), 463-472.

- [20] Rammig, A, P Bebi, H Bugmann & L Fahse (2007) "Adapting a growth function to model regeneration in mountain forests", *Eur J Forest Res* 126: 49–57.
- [21] Schou, E., & Meilby, H. (2013). "Transformation of even-aged European beech (*Fagus sylvatica* L.) to uneven-aged management under changing growth conditions caused by climate change". *European Journal of Forest Research*, 1-13.
- [22] Sims, C.A. (2003), "Implications of Rational Inattention", *Journal of Monetary Economics*, Vol 50, pp.665-690.
- [23] Spiro, D. (2012), "Some Aspects of Resource and Behavioral Economics", Institute for International Economic Studies Monograph series No 75, Stockholm.
- [24] Stollery, K R (2005) "Climate Change and Optimal Rotation in a Flammable Forest", *Natural Resource Modeling* 18(1), 91-112
- [25] Swedish Forest Authority, (2012), Swedish Statistical Yearbook of Forestry, 2012.
- [26] Vanclay, J K (1994) *Modeling Forest Growth and Yield - Applications to Mixed Tropical Forests* CAB International, Wallingford UK, ISBN 0 85198 913 6
- [27] van Veldhuizen, R. & Sonnemans, J. (2011), "Nonrenewable Resources, Strategic Behavior and the Hotelling Rule: An Experiment", Tinbergen Institute Discussion Paper, TI 2011-014/1.
- [28] Winter, J. K., Schlafmann, K., & Rodepeter, R. (2012). "Rules of Thumb in Life-cycle Saving Decisions". *The Economic Journal*, 122(560), 479-501.
- [29] Yousefpour, R., Temperli, C., Bugmann, H., Elkin, C., Hanewinkel, M., Meilby, H., & Thorsen, B. J. (2013). Updating beliefs and combining evidence in adaptive forest management under climate change: A case study of Norway spruce (*Picea abies* L. Karst) in the Black Forest, Germany. *Journal of environmental management*, 122, 56-64.
- [30] Yousefpour, R., Jacobsen, J. B., Thorsen, B. J., Meilby, H., Hanewinkel, M., & Oehler, K. (2012). A review of decision-making approaches to handle uncertainty and risk in adaptive forest management under climate change. *Annals of forest science*, 69(1), 1-15.

- [31] Yousefpour, R. Jacobsen, J.B., Meilby, H., Bo Jellesmark Thorsen A Bayesian Modeling and Simulation Concept for Knowledge Update in Adaptive Management of Forest Resources under Climate Change, *Annals of forest science* (forthcoming)