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The Impact of Uncertainty on the European Energy Market: The scenario aggregation method

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Abstract

We develop a stochastic numerical equilibrium model of the Western European energy markets. Both economic uncertainty with respect to future fossil fuel prices and GDP growth rates, and political uncertainty with respect to future climate policy, are analyzed. It is demonstrated that the equilibria under uncertainty differ significantly from the deterministic outcomes. Our approach to solve the numerical model builds on scenario aggregation, a numerical method developed to solve decision problems under uncertainty.

While carrying out this research Kjell Arne Brekke, Rolf Golombek and Sverre A.C. Kittelsen have been associated with CREE - Oslo Center for Research on Environmentally friendly Energy.

1 Introduction

Agents in the European energy market face considerable uncertainty. Recently, the region experienced a severe recession with declining energy demand. In the future, climate treaties may have substantial impact on the sector. Such abundant uncertainties can have huge consequences on investments in capacities. At the same time, if there is reluctance to invest in one technology, for other technologies the market looks more promising. Thus to fully analyze the impact of uncertainty, we need to take into account the interdependence of different technologies, energy carriers and end users – this calls for a numerical equilibrium model. But it is not trivial to solve such a model when all agents face uncertainty. Thus it is no wonder that most analyses assume full certainty, or if uncertainty is analyzed, rely on simulations in stead of examining agents optimizing under uncertainty.

In this paper we will discuss how to introduce uncertainty into computable equilibrium models. In particular, we extend LIBEMOD, a numerical multi-market equilibrium model of the Western European energy markets (Aune et al., 2008), to account for uncertainty. We then use this stochastic version of LIBEMOD to analyze the impact of uncertainty on the European energy market.

In spirit, our approach to modeling uncertainty is similar to the discussion of uncertainty in Debreu's (1959, chapter 7) classic 'Theory of Value', where uncertainty is represented by a discrete event tree. In our terminology, each branch of Debreu's event tree is called a scenario. Hence, in our model uncertainty is represented by a set of scenarios. Each scenario is one possible future realization of the uncertainty.

The models presented in this paper have two periods. In period 1, some agents make decisions under uncertainty, typically to determine their future capacities through investments. In the beginning of period 2, the uncertainty is resolved and all agents learn the true state of the economy. Then all agents make decisions; producers determine how much to produce (given the predetermined capacities) and consumers determine how much to consume. For each realization of the uncertainty, that is, for each scenario, the model determines supply of, and demand for, all goods from all agents, and the corresponding vector of prices that clear all markets.

The results indicate that uncertainty has a considerable impact on optimal investments. First, investments in electricity transmission are considerably higher under uncertainty. This is true both when the uncertainty is about GDP growth and fossil fuel prices (economic uncertainty), and with uncertainty about future climate policy (political uncertainty). Optimal investments in wind power are also much higher under uncertainty, and this result is particularly strong when there is uncertainty about future climate policy: the relative advantage of green technologies over fossil based technologies is then strong. This illustrates the importance of modeling the interdependence between different markets and technologies.

We also compare Monte Carlo simulations to the true optimal policy under uncertainty. For aggregated numbers, the results of the Monte Carlo simulations are usually closer to the optimal outcome than the deterministic solution (no uncertainty). However, in some cases, and in particular for single countries and single technologies, Monte Carlo simulations may produce numbers that are far from the optimal ones.

Our paper demonstrates that is possible to solve large computable equilibrium models with significant uncertainty in several variables. Our approach builds on scenario aggregation, a numerical method developed to solve decision problems under uncertainty (Wets, 1989, Rockafellar and Wets 1991, Kall and Wallace 1994). Scenario aggregation, and more generally stochastic programming, examines a *single* optimizing agent under uncertainty. Choosing a planner as the optimizing agent, one can find the efficient outcomes of an economy (see e.g., Kolstad, 1996). Our contribution is to use scenario aggregation in order to analyze uncertainty – within numerical multi-market equilibrium models - when *many* optimizing agents make decisions simultaneously.

The LIBEMOD model is too large and complex to be suitable for a simple representation of the scenario aggregation method. In Section 2 we therefore present a much simpler model to outline the basic approach taken.[1] Section 3 provides a short description of LIBEMOD, and in Section 4 we first describe the scenarios and then discuss the results, comparing the optimal solution under uncertainty to the equilibrium under no uncertainty and also the Monte Carlo outcomes. Finally, Section 5 concludes.

[1] The present paper builds on stochastic programming. An alternative approach is dynamic programming (Stokey et al., 1989), in particular numerical solutions to variational inequalities, see, for example, the study of Haurie, Zaccour and Smeers (1990) on ligopolistic markets under uncertainty.

We will return to a description of LIBEMOD in Section 3, but first we will present the method used. LIBEMOD is too large and complex to be suitable for a simple representation of the scenario aggregation method for modeling uncertainty in computable equilibrium models. In Section 2 we therefore present a much simpler model to outline the basic approach taken.[2]

2 Scenario aggregation

Below we first consider a model with only one source of uncertainty. This model is so simple that it can easily be solved analytically. We first solve the model in the standard way, that is, without using scenario aggregation, and then use our method to solve the simple model. We then consider the most frequently used approach to analyze uncertainty, namely Monte Carlo simulations, and compare this approach to the scenario aggregation method. Finally, we present how to use scenario aggregation within a general set up.

2.1 A simple example

Consider an economy with uncertainty in demand only. The model has one representative producer and one representative consumer. The producer has to decide on energy production capacity in period 1, that is, before observing the realization of the stochastic variable. The consumer observes the realization of the stochastic variable (in period 2) before deciding how much energy to buy. Finally, because there is no cost of production and the producer is assumed to be a price taker, the producer will, in period 2, use his entire capacity in energy production.

2.1.1 The standard solution

<u>Supply</u>

There is one representative producer who installs a capacity K at a unit cost c, and maximizes expected profits:

$$\max E\left[p(\theta)K - cK\right]$$

[2] The present paper builds on stochastic programming. An alternative approach is dynamic programming (Stokey et al., 1989), in particular numerical solutions to variational inequalities, see, for example, the study of Haurie, Zaccour and Smeers (1990) on ligopolistic markets under uncertainty.

where the price of the product, p, is assumed to depend on the uncertain weather θ. The first-order condition is

$$Ep = c. \tag{1}$$

Because the producer does not know θ before making a decision about the capacity, expected price has to equal marginal cost.

Demand

We assume a quasi-linear utility in two goods; one good, z, termed comfort, and another, y, representing all other goods:

$$u(z, y) = \sqrt{z} + y.$$

Comfort depends on energy consumption x and the weather θ:

$$z = \theta^2 x .$$

Normalizing the price of other goods to 1, the budget condition becomes $px + y = m,$ where m is the given income of the consumer. Utility can now be written as

$$\theta\sqrt{x} + (m - px) .$$

Maximizing utility yields the first-order condition

$$2p\sqrt{x} = \theta .$$

Equilibrium

Because the model is very simple, we can easily derive the equilibrium solution. In equilibrium, demand has to equal supply, which equals capacity, thus $x = K$. The first-order conditions for the producer and the consumer, along with the equilibrium condition, provide the following system of equations:

$$2p\sqrt{x} = \theta$$
$$Ep = c$$
$$x = K$$

with the solution

$$K = \left(\frac{E\theta}{2c}\right)^2.$$

2.1.2 Scnenrio aggregation

We now analyze the same model using scenarios to describe the uncertainty. Define a set of scenarios $s \in S$. In general, a scenario determines all uncertain variables, but in our simple case there is only one uncertain variable, namely the weather. The "value" of the weather depends on the scenario, which we formalize by writing θ_s. A scenario s is realized with a probability q_s, and obviously $\sum_{s\in S} q_s = 1$.

Let all decision variables depend on the scenario. Thus, we index both consumption and capacity by s; x_s and K_s. Hence, an agent now has one decision variable for each scenario. In the model above we assumed that the consumer knew θ_s before deciding on x_s (in period 2). Thus, the choice of consumption will depend on the state of the economy, that is, which scenario that has materialized. Hence, for two different scenarios s and s' we will in general have $x_s \neq x_{s'}$. For the producer, however, θ_s is *not known* when K_s is chosen (in period 1). Therefore, the producer cannot choose different capacities in different scenarios, that is, we impose the requirement $K_s = K$ for all $s \in S$. The producer maximizes expect profits subject to the latter restriction:

$$\max \sum_{s\in S} q_s \left[p_s K_s - cK_s \right]$$
$$\text{s.t. } K_s = K \text{ for all } s \in S.$$

The first-order conditions are:[3]

$$q_s p_s = q_s c + \mu_s \tag{2}$$

where μ_s is the Lagrange multiplier of the restriction $K_s = K$ for all $s \in S$. Adding all first-order conditions in (2) yields

$$Ep = c + \sum_{s \in S} \mu_s .$$

From the first-order condition (1) we know that $Ep = c$, and hence

$$\sum_{s \in S} \mu_s = 0. \tag{3}$$

Note that if we define $\tilde{\mu}_s = \frac{\mu_s}{q_s}$, the first-order condition (2) simplifies to

$$p_s = c + \tilde{\mu}_s \text{ for all } s \in S .$$

Moreover, condition (3) now becomes

$$E\tilde{\mu} = 0$$

which is the only equation where the probabilities appear.

We now collect the first-order conditions and the constraints:

$$\begin{aligned}
2 p_s \sqrt{x_s} &= \theta_s \text{ for all } s \in S \\
p_s &= c + \tilde{\mu}_s \text{ for all } s \in S \\
x_s &= K_s \text{ for all } s \in S \\
K_s &= K \text{ for all } s \in S \\
E\tilde{\mu} &= 0.
\end{aligned}$$

We see that relative to the deterministic case, uncertainty only amounts to a small modification in the second first-order condition (the term $\tilde{\mu}_s$) and the additional constraints $K_s = K$. All these

[3] In LIBEMOD, these are actually complementary slackness conditions due to the non-negativity constraint $K \geq 0$. Such conditions are essential in dynamic models, see section 6 for details.

equations apply to each scenario, so with n scenarios the number of equations equals $4n$. Finally, under uncertainty we have the additional constraint $E\tilde{\mu} = 0$. Since the first-order conditions are either (de facto) unchanged or only slightly modified relative to the deterministic case, the scenario aggregation approach requires only a modest change in computable equilibrium models like Libemod. However, the number of equations increases from 3 (under certainty) to $4n+1$ (under uncertainty).

2.1.3 Monte Carlo simulations

Consider the following system of equations:

$$
\begin{aligned}
2p_s\sqrt{x_s} &= \theta_s \text{ for all } s \in S \\
p_s &= c \text{ for all } s \in S \\
x_s &= K_s \text{ for all } s \in S.
\end{aligned}
$$

The solution of these equations corresponds to a Monte Carlo simulation of the original model, where we obtain one value for each endogenous variable p_s, x_s, K_s for each scenario *s*, that is, for each realization of θ_s. In particular, the capacity K_s will in general differ between the scenarios. Monte Carlo simulations thus simply ignore the fact that producers in the economy do not know which scenario that will materialize. Put differently: Under Monte Carlo simulations, the solution is found under the false assumption that producers consider the future as certain – which scenario that for sure will materialize differs between the simulations.

Comparing the Monte Carlo approach and the assumption that agents take the uncertainty into account when making decisions, we note some major differences. In the Monte Carlo simulations, $p_s = c$ in all scenarios. Thus there is no variation in the price, but production and capacity will be different in each scenario. On the other hand, with stochastic optimizing agents, capacity (K), which is determined before the producer knows which scenario that will materialize, and production, which is equal to capacity, do not differ between the scenarios, whereas the price differs between the scenarios: $p_s = \theta_s / (2\sqrt{K})$.

More fundamentally, according to economic theory uncertainty about future conditions change the *behavior* of agents (compared with the case of no uncertainty). This is captured by the scenario aggregation method as producers maximize expected profits, but it is not captured by Monte Carlo simulations. For each Monte Carlo simulation, a realization of the stochastic variables is drawn from a probability distribution. This realization amounts to specific parameter values that are used to find the equilibrium in a *deterministic* model. By simulating *n* times, one finds *n* equilibria, all obtained from the same deterministic model. Needless to say, the realizations (parameter values) will in general differ between each of the *n* runs, but agents neglect uncertainty simply because the model is deterministic.

2.2 The general approach of modeling uncertainty

We now turn to the general case of several sources of uncertainty and several decisions to be taken under uncertainty. In order to simplify, we neglect equilibrium aspects and consider the maximization problem of a single actor, e.g., a producer deciding on capacities to maximize expected profits. This is a stochastic optimization problem that may be written on the form

$$\max_K \left[\sum_{s \in S} f(K, \xi_s) q_s \right].$$

Here $f(K, \xi)$ is a value function, K is a vector of decision variables, ξ_s is a realization of a vector of uncertain variables, and s is a scenario index, $s \in S = \{1, 2, ..., n\}$. Finally, q_s is the probability that scenario s will materialize. Assuming that we have no information that rules out any scenario, we include all possible scenarios in the set S.

A core idea of the method of scenario aggregation is to rewrite this problem by using a vector of decision variables for each scenario:

$$\max_{K_1, K_2, \ldots} \left[\sum_{s \in S} f(K_s, \xi_s) q_s \right]$$

subject to $K_s = K_n$ for all $s \in S$, $s \neq n$, where K_s is the vector of decision variables in scenario s. With this reformulation, the Lagrange function is of the form

$$\begin{aligned} L &= \sum_{s \in S} f(K_s, \xi_s) q_s + \sum_{s=1}^{n-1} \lambda_s (K_s - K_n) \\ &= \sum_{s \in S} \left[f(K_s, \xi_s) q_s + \lambda_s K_s \right] = \sum_{s \in S} L_s \end{aligned}$$

where $\lambda_n = -\sum_{s \neq n} \lambda_s$ and where $L_s = q_s f(K_s, \xi_s) + \lambda_s K_s$ can be seen as a Lagrange function for scenario s. The first-order conditions for the maximization problem are

$$\begin{aligned} &\nabla_{x_s} L_s = 0 \text{ for all } s \in S \\ &\sum_{s \in S} \lambda_s = 0 \\ &K_s = K_n \text{ for all } s \neq n. \end{aligned}$$

Like in 2.1.2 the normalization

$$\tilde{\lambda}_s = \frac{\lambda_s}{q_s}$$

allows us to rewrite the conditions as

$$\begin{aligned} &\nabla_{x_s} f(K_s, \xi_s) + \tilde{\lambda}_s = 0 \text{ for all } s \in S \\ &E\tilde{\lambda} = 0 \\ &K_s = K_n \text{ for all } s \neq n. \end{aligned}$$

We thus find a similar modification of the first-order conditions in the general case.

3 Libemod

We now describe LIBEMOD – the numerical equilibrium model that will be used to study decisions under uncertainty. LIBEMOD allows for a detailed study of the energy markets in Western Europe, taking into account factors like inter-fuel competition, technological differences in electricity supply, transport of energy through gas pipelines/electricity lines and investment in the energy industry.

The core of LIBEMOD is a set of markets for seven energy goods: electricity, natural gas, oil, steam coal, coking coal, lignite and biomass. All energy goods are produced and consumed in each of the model countries; that is, all countries in Western Europe. Natural gas and electricity are traded in competitive Western European markets using gas pipelines and electricity transmission lines that connect the model countries. There are competitive world markets for coking coal, steam coal and oil, but only domestic (competitive) markets for lignite and biomass. While fuels are traded in annual markets, there are seasonal (summer vs. winter) and time-of-day markets for electricity.

In each model country, electricity can (with several exceptions) be produced by a number of technologies: steam coal power, lignite power, gas power, oil power, reservoir hydropower (including run-of-river and pondage), pumped storage hydropower, nuclear power, waste power, biomass power and other renewables power (primarily wind power). Each electricity producer maximizes profit. Installed and maintained electricity capacity can be used to produce electricity or is sold as reserve capacity to a domestic system operator. There are a number of costs related to production of electricity. First, there are costs directly related to combustion of fuels. These costs depend on plant efficiency, which in the model differs across countries, technologies and plants. Second, there are other inputs that are assumed to vary proportionally with production, and third, there are maintenance costs for electricity production capacity. Finally, there are start-up and ramping-up costs if the capacity used in one time period differs from the capacity used in the next time period.

In LIBEMOD there is a distinction between power plants that existed in the data year of the model (which we may refer to as old plants) and new power plants. For the first group, there is increasing marginal costs along the merit order supply curve for each type of technology in each model country. Also new reservoir hydro and new wind power have increasing marginal costs, reflecting scarcity in favorable sites. However, each type of new thermal power plants has constant returns to scale cost functions, that is, efficiency does not vary between plants using the same type of technology.

Electricity producers face some technical constraints, e.g., maintained capacity should not exceed installed (or invested) capacity, and plants need some time for technical maintenance. In addition, there are technologically specific constraints. For example, for reservoir hydro, in each season total availability of water, that is, the amount of water at the end of the previous season plus water inflow in the present season, must equal total use of water, that is, water used to produce electricity plus water saved for the next season. Moreover, water filling at the end of the season cannot exceed the reservoir capacity.

In each model country, there is demand for all types of energy from three groups of end users; the household segment (including service and the public sector), the industry segment and the transport sector. In addition, there is intermediate demand for fuels from fuel-based electricity producers. Demand from each end-user group (in each model country) is derived from a nested multi-good multi-period constant elasticity of substitution (CES) utility function. Domestic transport and distribution costs for electricity and natural gas differ across countries and user groups, but are otherwise fixed (with no capacity constraints).

There are several versions of LIBEMOD. These differ with respect to the data year (1996 vs. 2000), market structure (competitive markets vs. imperfect competition), time horizon (short run vs. long run), number of periods over the day (two vs. six), and heterogeneity in supply of, and demand for, coking coal and steam coal. In this study we use a version of LIBEMOD with competitive markets, long-run horizon (that is, there are investments in electricity production capacity, in international transmission capacity for natural gas and in international transmission capacity for electricity), two periods over the 24 hour cycle (day and night) and a simple modelling of coking coal and steam coal. The data year is 2000. The model determines all energy prices and all energy quantities invested, produced, traded and consumed in each sector in each model country. The model also determines all prices and quantities traded in the world markets, and emissions of CO_2 by country and sector.

In this paper, LIBEMOD is changed from a deterministic model into a stochastic model following the methodology outlined in Section 2, that is, (i) all decision variables depend on the scenario, and (ii) for each variable that has to be determined before the agent knows which scenario that will materialize, here investments in the energy industry, we impose the requirement that the agent has to choose the same value for all scenarios. Once the agents know the scenario, all the remaining variables are determined.

4 Simulation results

4.1 Scenarios

We focus on two types of uncertainties; economic uncertainty and political uncertainty, see Table 1. In the first case, oil and coal prices in 2030, as well as GDP in all model countries in 2030, are uncertain in period 1 (2000). Agents make investment decisions in period 1, and in the beginning of period 2 (2030) capacities have been expanded.

Table 1 Sources of uncertainty. Investment in 2000.

A	Economic uncertainty. 2030 oil prices, coal prices and GDP levels are uncertain.
B	Political uncertainty. A carbon policy may be imposed in 2030.

In the calibration year 2000 we assume that investors know the development between 2000 and 2010, which we set equal to the true development in oil and coal prices and GDP growth rates. However, the development between 2010 and 2030 is uncertain for investors. In order to calculate the probabilities of the future states in 2030, that is, the q_s in Section 2.2, we use annual data for the period 1970 to 2010 and group these into four 10-years periods. We assume that the development from 2010 to 2020 wrt. changes in coal and oil prices and GDP growth rates is characterized by one of these four 10-years periods. Similarly, the development from 2020 to 2030 is also characterized by one the four 10-years period, which may, by coincidence, be the same as the 10-years period from 2010 to 2020. This gives us 4^2 states. However, in our model the sequence of the two 10-years time periods between 2010 and 2030 is of no importance. We thus have 10 unique possible states in 2030, and each of these is termed a scenario.

In period 1, agents know that there are 10 scenarios, and they also know the probability of each scenario.[4] In addition, for each scenario they know the equilibrium prices if this scenario is materialized.

Turning to political uncertainty, the general idea is to introduce uncertainty wrt. future climate policy: agents do not know which climate policy that will be imposed in the future. To simplify, we assume that there is a probability of 50 percent that one specific carbon policy will be implemented, and there is a 50 per chance that no carbon policy will be imposed. Hence, under political uncertainty there are two scenarios, that is, two possible future states, each with a probability of 50 per cent.

The carbon policy that may be implemented in the future is characterized by a uniform tax on CO_2 emissions that is imposed on all agents in the model countries. We set this tax equal to $90 per ton CO_2, which, according to IEA (2008), will be sufficient to stabilize global GHG concentrations in the atmosphere at 550 ppm. This is by many, among others IEA (2008), considered as the most likely scenario. Thus agents know that in the future, here 2030 (period 2), there will either be no climate policy, or a uniform tax of $90 per ton CO_2 will be imposed on all emissions. Agents also know that the probability of each scenario is 50 percent. This information is taken into account in period 1 (here 2000) when investments in the Western European energy industry are determined.

In examining the importance of economic uncertainty, we compare the equilibrium of the stochastic LIBEMOD model (scenario aggregation) with the equilibrium of the deterministic LIBEMOD model. In doing so, the stochastic parameters under scenario aggregation are replaced by their expected values. Under political uncertainty this is simply $45 per ton CO_2. We also compare the equilibrium of the stochastic LIBEMOD model with the output from the Monte Carlo simulations.

[4] Note that the probability of some scenarios is 1/16, whereas the probability of the other scenarios is 2/16.

The model is solved on a state of the art Intel-based application server with the GAMS (Brooke et al., 1998) language and the Path (Ferris and Munson, 1998) solver. For efficiency, the variables are initialized with their 2000 calibration values before solving the 2030 equilibrium under no uncertainty. This solution is then used as the starting point for each of the independent Monte Carlo equilibria, which further provides initialization for the equilibrium under uncertainty.

4.2 Economic uncertainty

<u>The importance of uncertainty</u>

Below we discuss the outcome when there are three sources of uncertainty; future oil prices, future coal prices and future GDP levels. There is an important difference in the impact of uncertain fossil fuel prices versus the impact of uncertain GDPs. The impact of uncertain GDP will differ across countries because some countries will experience high growth rates whereas other countries will experience low growth rates. In countries with high growth rates, demand for energy will increase significantly, which, *cet. par.*, tends to increase domestic energy prices, thereby providing an incentive for other countries to export energy to these countries. This suggests that international transmission capacity is of great importance. In contrast, the impact of uncertain fossil fuel prices may not differ that much between countries because under perfect competition all countries will face the same producer prices of oil and coal.[5]

In the deterministic case, that is, the stochastic parameters under scenario aggregation are replaced by their expected values, 5 GW is invested in international electricity transmission capacity between the model countries, whereas under uncertainty (scenario aggregation) the level

[5] Although countries will face the same producer prices of oil and coal, the impact of uncertain producer prices may still differ somewhat between countries. First, the change in end-user prices of oil and coal will differ between countries because costs of transport and distribution, and also taxes, differ across countries. Second, the market shares of oil and coal differ between countries, both in end-user demand and in electricity generation: if coal power has a large market share and the price of coal turns out to be high, production of coal power may decrease significantly. This tends to increase the domestic price of electricity, thereby providing an incentive for increased production from other domestic electricity technologies. This effect will, however, be dampened through international trade of electricity. The effect through international trade resembles the effect of differences in GDP growth rates, but it may be much lower in magnitude because this is a derived (indirect) effect.

of investment is 15 GW, that is, three times higher, see Table 2. Hence, under uncertainty it is more profitable to invest in flexibility between countries. This may reflect differences in growth rates between countries: In the future (2030) a country *A* may experience high demand for electricity because of high growth rates. Rather than investing a lot in power plants in 2000 so that future domestic demand for sure can be met by domestic production, electricity can partly be imported from the neighboring country *B* through existing and new transmission capacities if it turns out that future demand (in 2030) is low in country *B*. These capacities can alternatively be used to transport electricity from A to B if it turns out that demand (in 2030) is high in country *B*, but low in country *A*.

Alternatively, future production of electricity may be high in country *A* but low in country *B* because of structural differences in electricity production capacities. If, for example, country *A* invests in coal power whereas country *B* invests in oil power, and the price of coal (in 2030) turns out to be low whereas the price of oil turns out to be high, then country *A* may use its entire new coal power capacity whereas oil power production in country *B* may not profitable because oil is too expensive. In such a case, it would have been profitable (in period 1) to build a new transmission line between country *A* and *B*, which can now (period 2) be used to export electricity from country *A* to country *B*. Likewise, if it turns out that the price of coal (in 2030) is high whereas the price of oil is low, a new transmission line can be used to transport electricity from *B* to *A*. As suggested in footnote 5, it seems reasonable that this effect is smaller than the effect caused by growth rate differentials.

Table 2 Uncertain oil price, coal price and GDPs in 2030. Investments in gas transmission capacity (mtoe), electricity transmission capacity (GW) and electricity production capacity (GW) in 2000.

	International gas transmission capacity (Mtoe)	International electricity transmission capacity	Total power capacity	Hydro power capacity	Gas power capacity	Coal power capacity	Bio power capacity	Wind power capacity
Deterministic	157	5	365	9	30	304	0	9
Stochastic	154	16	358	11	49	249	0	31
MC average	157	19	354	10	37	250	13	28

As seen from Table 2, the difference in investment in international gas transmission capacity between the case of uncertainty and the deterministic outcome is marginal (2 percent). Whereas electricity transmission provides flexibility because electricity can be transported one way or the other, natural gas is mainly transported one way, that is, exported from the big extractors. Which scenario that is materialized may have significant impact on the export magnitudes, but not much impact on which countries that are exporters of natural gas. Thus, for natural gas two-way flexibility is not a big issue.

In the deterministic case, total investment in electricity production capacity is 365 GW, whereas under uncertainty investment in electricity production capacity is slightly lower; 358 GW. The distribution of investments differs, however, between the two cases: In the deterministic case, investment in coal power is 304 GW, which is 55 GW higher than under uncertainty. Under certainty investors know for sure the profitability of new coal power plants, and undertake all projects with non-negative profitability. Under uncertainty, investors know cost of investment, but cost of operating a new coal power plant is uncertain because the future price of coal is uncertain. Thus, (part of the) new coal power capacity will not be used if the input price turns out to be too high. This explains why investment in coal power is lower under uncertainty than in the deterministic case.

As seen from Table 2, both in the deterministic case and under uncertainty there is no investment in oil power. In the deterministic case, investment in oil power capacity is simply not profitable. Under uncertainty, there is a chance that the oil price will turn out to be so low that investment in oil power will be profitable. However, at the point in time where investment has to take place (2000) this probability is too low to ensure a positive expected profit. Hence, also under uncertainty there will be no investment in oil power.

Because investment in coal power is higher in the deterministic case than under uncertainty (see discussion above), and demand for electricity in the deterministic case does not differ much from expected demand under uncertainty, there is room for additional investment in electricity production capacity under uncertainty relative to the deterministic case. Table 2 shows that under uncertainty, investment in hydro power is only marginally higher than in the deterministic case, investment in gas power is around 50 percent higher than in the deterministic case, and investment in wind power is roughly 200 percent higher than in the deterministic case. In general, investment in different electricity technologies depends on their long-run marginal cost of production. In LIBEMOD these are increasing in the equilibrium quantities; either because it is assumed that the best locations are taken first (reservoir hydro and wind), or because there are increasing costs of providing more of the input (natural gas and bio mass). Our results reflect that reservoir hydro has a much stepper long-run marginal cost curve than wind power.

In the deterministic case, the entire new electricity production capacity will for sure be used in at least one time period. In contrast, under uncertainty part of the new production capacity may not be used at all; if, for example, the coal price turns out to be very high, it may not be profitable to run any of the new coal power plants.

Because fuel prices differ between scenarios, also production of electricity will differ between scenarios; total production of electricity varies under uncertainty between 3721 TWh and 4809 TWh, and has an expected value of 4571 TWh. Note that in the deterministic case, total electricity production is 4851 TWh, that is, slightly above the highest level of production under uncertainty. From the discussion above we know that total investment in electricity production capacity is almost identical in the two cases. Hence, the difference in total production reflects higher rates of capacity utilization in the deterministic case.

Whereas the reported results in Table 2 for scenario aggregation reflect three sources of uncertainty, there are definitely more possible sources of uncertainty, for example, the weather. To test the importance of weather uncertainty, we introduced uncertain precipitation and wind (number of hours it blows in a season) in Scandinavia. Using data for a period of 21 years to generate 21 scenarios, we found that the equilibrium under this type of uncertainty was only marginally different from the deterministic outcome. Note that under the reasonable assumption that weather uncertainty is stochastically independent of uncertainty in oil prices, coal prices and GDP growth rates, we would have obtained the same result if weather uncertainty had been “added” to our three sources of economic uncertainty.

Monte Carlo

We now turn to the Monte Carlo simulations. Table 3 shows investment in electricity technologies by scenario. As explained above, in each scenario there is no uncertainty: Agents know for sure in 2000 (period 1) which scenario that will materialize in 2030 (period 2). Hence, if agents know that there will be high growth rates, they will tend to investment more; if they know the price of coal will be low, they will tend to invest in coal power; and if they know the price of oil will be low, they will tend to invest in oil power – all solutions are tailor-made.

Investment in coal power under Monte Carlo varies between 54 GW and 367 GW. The weighted Monte Carlo average is 250 GW (see Table 2),[6] which is almost identical to the investment level under uncertainty (249 GW). The weighted average of the Monte Carlo simulations does not, however, provide a good estimate of investment in other electricity technologies under uncertainty. For gas power, the difference is around 30 percent, whereas for wind power the difference is roughly 10 percent.

In the Monte Carlo simulations, there is investment in oil power in scenario 5 only; this scenario is characterized by a very low oil price. Here the level of investment is as high as 211 GW. The weighted Monte Carlo average of oil power investment (13 GW) therefore exceeds the optimal level under uncertainty (zero). However, under Monte Carlo the overshooting of oil power investment is of the same magnitude as the undershooting of gas power and wind power investments. Therefore, the Monte Carlo average of total investment in electricity production capacity differs by only one percent from the optimal solution under uncertainty. The difference in total production of electricity is somewhat higher (3 percent), which reflects that under Monte Carlo investors know which scenario that will materialize. In contrast, under uncertainty the rate of capacity utilization depends on which scenario that will materialize, see the discussion above.

Finally, the high level of oil power investment in scenario 5 takes place in a few countries only. In these countries it is profitable to invest in international electricity transmission capacity in order to export part of the domestic electricity production. This is the main reason why average Monte Carlo investment in electricity transmission lines is about 20 percent higher than the solution under uncertainty, see Table 2.

[6] We use the scenario probabilities as weights.

Table 3 Monte Carlo. Uncertain oil price, coal price and GDPs in 2030. Investments (GW) in 2010.

Scenario	Total power capacity	Hydro power capacity	Gas power capacity	Coal power capacity	Oil power capacity	Bio power capacity	Wind power capacity
1	432	13	45	262	0	23	89
2	405	10	27	330	0	16	22
3	418	10	15	367	0	14	12
4	323	13	71	138	0	22	79
5	371	8	5	137	211	9	1
6	377	8	10	348	0	10	1
7	292	10	55	189	0	16	22
8	398	7	3	379	0	9	0
9	304	10	51	216	0	14	13
10	238	12	90	54	0	21	61

4.3 Political uncertainty

In this case there is one source of uncertainty, namely whether the future carbon tax will be zero or 90 USD/tCO2. In general, a carbon tax enhances the competitive position of non-fossil fuel electricity technologies, and weakens the competitive position of fossil fuel electricity technologies. Because emissions of CO2 per unit of energy (measured in toe) is larger for coal than for oil, and larger for oil and than for natural gas, coal will typically be the big loser if a carbon tax is imposed, whereas natural gas comes in an intermediate position: it improves its position relative to other fossil fuels, but weakens its position relative to non-fossil fuels. The net effect is therefore ambiguous, and may depend on a number of factors like the market share of natural gas in end-user demand and in electricity generation, and also on the carbon tax rate itself; if the tax is increased marginally from a low level, say zero, or marginally from a high level, say 90 USD/tCO2, the effect on natural gas may be very different.

Uncertainty vs. no uncertainty

In the present case of an uncertain carbon tax, future fossil fuel prices are unknown in period 1 (2000). Although the materialization of the uncertainty will not differ across countries – all countries will face (almost) the same producer prices of fossil fuels – the impact of high (or low) producer prices of fossil fuels will differ somewhat across counties. First, fossil fuel end-user prices will differ across countries because of country differences in costs of transport/distribution and taxes, and the impact of end-user prices differs across countries because of differences in end-user demand (the CES utility functions). In addition, the market share of fossil fuels, as well as the composition of fossil fuels, differ across countries both in end-user demand and in electricity generation. These effects resemble the impact of uncertain oil and coal prices under economic uncertainty, see above. However, under economic uncertainty both the future level of demand, as well as differences in demand across countries, were important factors to understand why investments differ between the case of uncertainty and the deterministic outcome. We therefore expect the difference in investment to be much lower in the case of political uncertainty than under economic uncertainty.

Table 4 Uncertain CO2 tax rate in 2030. Investments in gas transmission capacity (mtoe), electricity transmission capacity (GW) and electricity production capacity (GW) in 2000.

	International gas transmission capacity	International electricity transmission capacity	Total power capacity	Hydro power capacity	Gas power capacity	Coal power capacity	Bio power capacity	Wind power capacity
Deterministic	146	19	336	14	66	120	25	111
Stochastic	136	20	360	14	72	144	25	105
MC 1	157	5	365	9	30	304	13	9
MC 2	174	55	365	17	105	0	33	210
MC average	165	30	365	13	67	152	23	110

Table 4 shows investment in electricity technologies under the four different cases; Deterministic (a carbon tax of 45 USD/tCO2 will for sure be imposed), Stochastic (uncertain carbon tax), Monte Carlo 1 (a carbon tax will for sure not be imposed) and Monte Carlo 2 (a carbon tax of 90 USD/tCO2 will for sure be imposed). As seen from Table 4, under uncertainty, investment in coal power is lower than in the deterministic outcome (144 GW vs. 120 GW). For other technologies, the difference is smaller: 105 GW vs. 111GW for wind power and 72 GW vs. 66 GW for gas power. Hence, investment under uncertainty differs somewhat from investment in the deterministic outcome. For total investment in power capacity, the difference is around 10 percent (360 GW under uncertainty vs. 336 GW under no uncertainty).

Table 4 also shows that under uncertainty, investment in international electricity transport capacity is only marginally higher than in the deterministic outcome. The difference in investment in international gas transmission capacity is somewhat larger; 136 mtoe under uncertainty vs. 146 mtoe under no uncertainty.

Table 5 shows production of electricity (in 2030) by technology in the deterministic case, if there is uncertainty about the carbon tax and it turns out that no carbon tax is imposed (Stochastic 1), and if there is uncertainty about the carbon tax and it turns out that a 90 USD/tCO2 tax is imposed ("Stochastic 2"). The deterministic outcome is typically between Stochastic 1 and Stochastic 2; the only exception is for new renewable, reflecting that renewable production capacity is higher in the deterministic outcome than under uncertainty. Note, however, that for production in old fossil fuel plants, the deterministic outcome is slightly below the high value under Stochastic 1, whereas for production in new fossil fuel plants, the deterministic outcome is slightly above the low value under Stochastic 2. Hence, the deterministic outcome is not just a constant average of Stochastic 1 and 2; the average varies by fuel and country.

Table 6 provides information on use of natural gas and steam coal (in 2030) by sectors (households, industry and power generation) in the deterministic case, the two outcomes under uncertainty and the two Monte Carlo outcomes. Use of natural gas does not differ much between Stochastic 1 and 2, and the average of Stochastic 1 and 2 is close to natural gas use under certainty. For use of coal, the average of Stochastic 1 and 2 is somewhat (13 mtoe) lower than in the deterministic outcome, but now there is a significant difference between Stochastic 1 and 2. Primarily, this difference reflects that with a high carbon tax (Stochastic 2) most of the old coal power plants are not profitable to operate, whereas if there is no carbon tax (Stochastic 1) then the entire old capacity of coal power plants is profitable to run. In addition, production in new coal power plants (as well as investment in new coal power plants) is higher in Stochastic 1 than in Stochastic 2.

Table 5 Uncertain CO2 tax rate. Production of electricity (TWh) in 2030.

	Total	Old hydro	Old fossil fuel	Old renewable	Old nuclear	New hydro	New fossil fuel	New renewable
Deterministic	4126	432	717	91	836	43	1472	535
Stochastic 1	4346	432	727	87	836	42	1703	518
Stochastic 2	3644	432	264	91	836	42	1461	518
MC 1	4851	432	697	92	836	29	2632	134
MC 2	3467	432	363	79	836	52	825	880

Table 6 Uncertain CO2 tax rate. Use of energy (Mtoe) in 2030.

	Households	Industry	Power generation	Total
Natural gas				
Deterministic	173	108	120	401
Stochastic 1	178	110	121	409
Stochastic 2	158	100	131	389
MC 1	192	131	68	391
MC 2	150	90	166	406
Steam coal				
Deterministic	3	11	252	267
Stochastic 1	4	19	294	318
Stochastic 2	3	9	179	190
MC 1	4	18	541	562
MC 2	3	9	26	37

Monte Carlo

Surprisingly, total investment in electricity production capacity does not differ between Monte Carlo 1 and 2. This reflects two counteracting factors: On the one hand, under Monte Carlo 2 production of electricity (in 2030) in fossil fuel plants that existed in 2000 ("old" plants) is roughly half of the production under Monte Carlo 1 – this simply mirrors the difference in the carbon tax between Monte Carlo 1 and 2 (no carbon tax vs. 90 USD/tCO2). A small production of electricity in old fossil fuel plants in Monte Carlo 2 tends to increase the price of electricity, thereby providing an incentive to expand the electricity production capacity. On the other hand, the high carbon tax in Monte Carlo 2 will lower demand, thereby making it less profitable to invest. By chance, the net effect of these two factors is zero when Monte Carlo 2 is compared to Monte Carlo 1. In both cases investment amounts to 365 GW, but the composition is different: Under Monte Carlo 1 new renewable electricity capacity amounts to 31 GW (260 GW under Monte Carlo 2), whereas new fossil fuel electricity capacity amounts to 334 GW (105 GW under Monte Carlo 2).

Under uncertainty, investment in coal power, gas power and wind power are close to the Monte Carlo averages, and total investment under uncertainty is only two percent lower than the Monte Carlo average. Hence, the averages of the Monte Carlo simulations provide good estimates of investment under uncertainty.

Table 4 shows that investment in international electricity transmission capacity is lower under uncertainty than the average Monte Carlo investment. The high Monte Carlo average reflects substantial investment in Monte Carlo 2, where numerous wind power stations are set up, particularly in Scandinavia, accompanied by investment in international electricity transmission.

As stated above, new electricity production capacity does not differ much between the Monte Carlo level (365 GW) and the outcome under uncertainty (360 GW). Still, the difference in production of electricity is significant. If there is uncertainty about the carbon tax and it turns out (in 2030) that no tax is imposed ("Stochastic 1" in Table 5), then total production of electricity is roughly 10 percent lower in Stochastic 1 than in Monte Carlo 1. The reason is that the composition of electricity capacity is significantly different in the two cases: In Monte Carlo 1, most of the new capacity is fossil fuel based, which has a maximum rate of utilization of 90 percent. In Stochastic 1, around one third of the new capacity is found in wind power, which has a maximum rate of utilization below 40 percent.

Similarly, if there is uncertainty about the carbon tax and it turns out that a 90 USD/tCO2 tax is imposed ("Stochastic 2" in Table 5), then total production of electricity is roughly 5 percent higher in Stochastic 2 than in Monte Carlo 2. Again, the difference reflects the composition of new electricity production capacity: in Monte Carlo 2, there is no investment in coal power, but substantial investment in wind power (which has a low rate of utilization).

Average electricity production under uncertainty is almost 5 percent below the Monte Carlo average, whereas it is almost 10 percent above the production level in the deterministic outcome. Hence, the Monte Carlo average provides a better estimate of the outcome under uncertainty than the deterministic outcome. This reflects the benefit of using the scenarios in stead of applying the expected value when one falsely assume there is no uncertainty.

We now turn to Table 6, which provides information on use of natural gas and steam coal (in 2030) by sectors (households, industry and power generation) in the five cases. If it turns out that no tax is imposed, then use of natural gas in power generation is almost twice as high under uncertainty (Stochastic 1) than under Monte Carlo 1. If the tax turns out to be 90 USD/tCO2, use of natural gas in power generation is roughly 25 percent lower under uncertainty (Stochastic 2) than under Monte Carlo 2. However, on average total use of natural gas under uncertainty and in the Monte Carlo simulations are almost equal.

For steam coal, total use is almost twice as high in Stochastic 1 than in Stochastic 2, whereas total use of coal is almost 15 times higher in Monte Carlo 1 (no carbon tax) than in Monte Carlo 2 (90 USD/tCO2); there is no investment in coal power in the latter case, see discussion above. Thus, the outcome in Monte Carlo *i*, *i*=1,2, are poor estimates for the outcome in Stochastic *i*, and the average values under uncertainty and Monte Carlo differ by as much as 20 percent.

5 Conclusions

The stochastic scenario method can be used to solve numerical equilibrium models with several agents simultaneously maximizing their payoff under uncertainty. The method has been implemented in a stochastic version of LIBEMOD, a numerical multi-market model of the European energy market. We find that replacing the uncertainty with the expected value, which in the paper was referred to as the deterministic outcome, leads to large deviations from the optimal solution under uncertainty. Monte Carlo simulations that apply the same scenarios as the ones used when solving for the optimal solution under uncertainty, approximate the optimal solution much better, at least for aggregate numbers.

The current version of LIBEMOD is not dynamic, but by imposing investments to be the same in all scenarios while energy use is scenario specific, we impose a structure where investments are decided before the uncertainty is revealed while use is decided after the uncertainty is revealed. Our approach can be extended to dynamic multi-period models with learning. The information available in different periods would then be represented by partitions of the set of scenarios; the decision makers in a period do not yet know the exact scenario that will materialize in the future, only which subset the true scenario will belong to. Typically, decisions made in the first period will have to be the same in all scenarios, while decision in a later period will be the same within a subset of scenarios, but different across subsets. In the last period the exact scenario will be known. Learning is represented by the gradually finer partition of the set of scenarios.

We can also account for risk aversion, either by assuming that investment is decided by the firms' owners who are diversified in the financial market, or that investment is decided by risk-averse managers. In the first case, probabilities are replaced by weights derived from the prices of Arrow securities, that is, contingent claims that pay 1 $ if a particular scenario materializes. After replacing probabilities with normalized prices of contingent claims, all agents will behave as if they were risk neutral. Risk aversion will be reflected in the prices on contingent claims. This approach is similar to the use of equivalent martingale measures in finance (Harrison and Kreps, 1979, Duffie, 1996). With risk averse manager a similar approach can be used, but in this case the scenario-weights will be firm specific, and thus some modest changes in the first-order conditions would be required.[7]

[7] More details on how to account for risk aversion and to extend the method to cover dynamic models can be obtained from the authors upon request.

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