Working Paper 2/2012

Second-best climate policy

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The CREE Centre acknowledges financial support from The Research Council
of Norway, University of Oslo and user partners.
ISBN: 978-82-7988-111-7
ISSN: 1892-9680
http://cree.uio.no

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January 26, 2012

Abstract

Countries with an active climate policy often use several other policy instruments in addition to a price on carbon emissions, such as subsidies to renewable energy. An obvious reason for subsidizing alternatives to carbon energy is that the price of carbon emissions is "too low". The paper derives implications for a second-best climate policy if for some reason the price of carbon emissions is lower than the Pigovian level, and also discusses reasons policy makers might have for setting the tax rate at an inefficiently low level. Even if the current tax rate is optimally set, governments cannot commit to future tax rates. In some cases this inabilty to commit may justify subsidies to investments in renewable energy.

Keywords: carbon tax, subsidies, commitment

JEL classification: Q42, Q48, Q54, Q58

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[*] While carrying out this research I have been associated with CREE -Oslo Centre for Research on Environmentally friendly Energy. CREE is supported by the Research Council of Norway.

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1 Introduction

It is widely recognized among economists that a price on carbon emissions, trough a carbon tax or a price on tradeable emission permits, is the most important policy instrument to reduce such emissions. Standard economic reasoning also implies that in the absence of other market failures, an appropriately set carbon price is the *only* instrument needed to achieve an efficient climate policy. In practice, however, most countries having an active climate policy use several other policy instruments in addition to a price on carbon emissions. One of the most frequently used instruments is explicit or implicit (e.g. through portfolio standards) subsidies to alternatives to carbon energy. This may be justified by economic theory if there are market failures associated with the supply of such alternatives. For instance, it is often assumed that there are various market imperfections associated with the development of new technology. However, governments often subsidize alternatives to carbon energy even when this has no obvious effect on the development of new technology and there are no other obvious market failure associated with the supply of the alternative. The focus of this article is on such subsidies, which should not exist in a first-best social optimum.

One obvious reason for subsidizing alternatives to carbon energy is that price of carbon emissions (henceforth called a carbon tax) is "too low", i.e. lower than the Pigovian rate (equal to the marginal environmental cost of carbon emissions). Sections 2-7 discuss the implications for a second-best climate policy if for some reason the price of carbon emissions is lower than the Pigovian level. The simplest case of one type of carbon energy and one alternative is considered in sections 2-4. In this section the optimal subsidy is derived. The results of these section are in section 5 generalized to the case of many types and uses of carbon energy, and many alternatives. The implications of some or all carbon emission being regulated trough a cap and trade system are discussed in section 6. Finally, in section 7 it is shown that if the tax on some type of carbon emissions for some reason is below the

Pigovian level, it is optimal for taxes on other carbon emissions also to be below the Pigovian level, even if there is no constraint on these taxes. It may in some cases even be second-best optimal to set the tax on one type of carbon emissions below the constrained tax rate of another type of carbon emissions, even though the latter is "too low".

An obvious question is why the carbon tax is set below the Pigovian level. This is discussed in section 8, together with an analysis of the welfare costs of setting the carbon tax lower than the Pigovian level.

To achieve a first-best optimum, it is not sufficient to set the current carbon tax equal to the Pigovian level. Since many current decisions related to abatement and investment in renewable energy have long-term consequences, a first-best optimum requires that also future carbon tax rates are set equal to the future Pigovian levels. This would require that governments were able to commit to a future tax path. In reality, this is not possible. Without commitment, market agents who make investment decisions must base their decisions on what they expect about future climate policies. In policy debates, it is often argued that lack of commitment may lead to inefficiently low emission reducing investments, and that emissions therefore will be higher than they would be with commitment. For instance, Stern (2007, p. 399) argues that "lack of certainty over the future pricing of the carbon externality will reduce the incentive to innovate".

With this motivation, the simple model of section 2 is in section 9 extended to a two-period model. It is shown that the inability of governments to commit to future tax rates in some cases *may* justify a subsidy of renewable energy.

Section 10 offers some concluding remarks.

2 A simple model for analyzing climate policy

The simple model presented in this section has the following features:

- The model ignores all issues related to the international dimension of the climate issue.
- There is one final good and two endogenous inputs in the production function for this good. These inputs can be interpreted as fossil fuels and renewable energy.
- Carbon in the atmosphere is not explicitly treated as a stock pollutant.

Aggregate consumption is given by

$$c = F(x, y) - px - b(y) \tag{1}$$

where $F(x, y)$ is gross output. Of this output, px is needed to produce (or import) fossil fuels, called carbon henceforth, in the amount x. Moreover, $b(y)$ of the gross output is needed to is needed to produce renewable energy in the amount y.[1]

Social welfare is given by

$$W = c - vx \tag{2}$$

where vx represents environmental damage from using carbon energy, and v is exogenous and positive. Henceforth, v will be called the valuation of emissions (or emission reductions). A more general damage function would have

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[1] The reason for letting the cost of y be given by $b(y)$ instead of a linear function qy as was assumed for x is that the analysis is simplified if corner solutions can be ruled out. If both x and y have linear cost functions corner solutions may occur, and will typically occur if x and y are perfect substitutes.

damages as a non-linear function of x. Even with a more general function, environmental damages would only depend on the flow of carbon emissions. It is more realistic to let damages depend on the stock of carbon in the atmosphere, and not on the flow. Most analyses of climate policy assume that damages depend on the stock of carbon. The assumption (2) used in the present analysis must hence be interpreted as a simplification. Note, however, that if environmental damages are linear in the stock of carbon, it can be shown that this is equivalent to having a damage function as in (2), i.e. with damages proportional to the flow of carbon emissions, see e.g. Hoel (2011) for details.

The following assumptions are made:

1. F is strictly increasing and strictly concave in its arguments, and $F_{xy} < 0$.
2. p and v are exogenous and positive, and $p+v$ is assumed to be so small that a first-best optimum is characterized by $x > 0$.
3. $b(0) = b'(0) = 0$; $b'(y)$ and $b''(y)$ are positive for $y > 0$.

3 The first-best optimum

Using (1) and (2), social welfare is given by

$$W = F(x, y) - (p + v)x - b(y) \tag{3}$$

Maximizing W with respect to x and y gives (since the assumptions above imply an interior solution)

$$\begin{aligned} F_x(x, y) &= p + v && (4) \\ F_y(x, y) &= b'(y) && (5) \end{aligned}$$

The interpretation of these two equations is straightforward: The marginal productivity of using carbon should be equal to its total marginal cost, which consists of production (or import) costs plus climate costs. The marginal productivity of using renewable energy should be equal to the marginal cost of producing this energy.

The optimal (x, y) depends on the climate cost parameter v, and it is straightforward to see that x is lower the higher is v. Moreover, the assumption $F_{xy} < 0$ implies that y is higher the higher is v.

The market outcome follows from profit maximization of producers. Aggregate profits are

$$\pi = F(x, y) - (p + t)x - (b(y) - sy)$$

where t is a carbon tax and s is a subsidy on renewable energy. Profit maximization gives

$$\begin{aligned} F_x(x, y) &= p + t \qquad (6) \\ F_y(x, y) &= b'(y) - s \qquad (7) \end{aligned}$$

Comparing with (4)-(5) it is immediately clear that the market outcome will coincide with the social optimum if $t = v$ and $s = 0$. In other words, the social optimum is achieved be setting a carbon tax at the Pigovian rate, and not subsidizing renewable energy.

4 A second-best subsidy

Assume now that for whatever reason, the tax rate t is set lower than v. The next section will give some discussion of why policy makers might find it difficult or impossible to set $t = v$, and therefore instead set t to some level below v (perhaps zero).

It follows from (6)-(7) that both x and y will be determined once t and s

are chosen. The second-best optimal $s-$ for an exogenous values of $t-$ is the value of s that maximizes (3). To solve this maximization problem, it is useful to consider y as a policy variable. The interpretation is the following: Once t is given, it follows from (6) that the government can obtain whatever x it wants by choosing a suitable y. Once x and y are both given, the appropriate subsidy follows from (7).

From the discussion above, it is clear that the second-best value of y must maximize

$$W = F(x(y,t),y) - (p+v)x(y,t) - b(y) \tag{8}$$

where $x(y,t)$ is implicitly defined by (6). From (6) it follows that

$$\begin{aligned} x_y(y,t) &= \frac{F_{xy}}{-F_{xx}} < 0 && (9) \\ x_t(y,t) &= \frac{-1}{-F_{xx}} < 0 && (10) \end{aligned}$$

It follows from (3) that maximizing W with respect to y gives

$$(F_y - b') + (F_x - p - v)\, x_y(y,t) = 0$$

or, using (6) and (7)

$$s = (v-t)(-x_y(y,t))$$

Since $-x_y > 0$, it follows that the second-best optimal subsidy is positive if $t < v$, and is larger the larger is $v - t$.

5 Heterogeneous carbon taxes and many types of renewable energy

An obvious extension of the model is to assume that fossil fuels are used for different purposes in the economy, so that there is an input vector $\mathbf{x} = (x_1, ...x_m)$ instead of only one input x. Similarly, there may be many types and uses of renewable energy and other inputs affecting the use of carbon, so that there is an input vector $\mathbf{y}$ =$(y_1, ...y_n)$ instead of only one input y. With this modification, (1) is replaced by

$$c = F(\mathbf{x}, \mathbf{y}) - p\Sigma_i x_i - \Sigma_i b_i(y_i)$$

Social welfare is a before given by (2), but with x replaced by $\Sigma_i x_i$.

It is straightforward to see that the first-best optimum is now given by

$$\begin{aligned} F_{xi}(\mathbf{x}, \mathbf{y}) &= p + v && (11) \\ F_{yj}(\mathbf{x}, \mathbf{y}) &= b'_j(y_j) && (12) \end{aligned}$$

and that the market outcomes given by

$$\begin{aligned} F_{xi}(\mathbf{x}, \mathbf{y}) &= p + t_i && (13) \\ F_{yj}(\mathbf{x}, \mathbf{y}) &= b'_j(y_j) - s_j && (14) \end{aligned}$$

As before, the first-best optimum can be achieved by $t_i = v$ for all i, and $s_j = 0$ for all j.

Consider now the second-best problem for which carbon tax rates are exogenous and at least some of them are below v. As before, the vector $\mathbf{y}$ may be considered as a policy variable, with the corresponding subsidies following form (14) once $(\mathbf{x}, \mathbf{y})$ is determined. The second-best value of $\mathbf{y}$ must maximize (where $\mathbf{t} = (t_1, ...t_m)$)

$$W = F(\mathbf{x}(\mathbf{y}, \mathbf{t}), \mathbf{y}) - (p + v)\Sigma_i x_i(\mathbf{y}, t_i) - \Sigma_j b_j(y_j) \qquad (15)$$

where $\mathbf{x}(\mathbf{y}, \mathbf{t})$ is implicitly defined by (13). It is straightforward to verify that maximizing W with respect to $\mathbf{y}$ gives

$$\left(F_{yj} - b'_j\right) + \sum_i \left(F_{xi} - p - v\right) \frac{\partial x_i(\mathbf{y}, \mathbf{t})}{\partial y_j} = 0$$

or, using (13) and (14)

$$s_j = \sum_i (v - t_i) \left[-\frac{\partial x_i(\mathbf{y}, \mathbf{t})}{\partial y_j} \right] \tag{16}$$

The optimal subsidy of each type of renewable energy thus depends on how it affects all different types of carbon use. However, it is *not* the sum of reduced carbon that matters for how large the subsidy should be. The reduction of each type of carbon use following an increase in the use of a renewable energy (or any other input) must be multiplied by the difference between the valuation of emissions and the tax rate for this type of carbon use.

Since it is only the sum of emissions that matters for the climate, it might seem strange that it is not simply the sum of emissions that enters the expression for the optimal subsidy (16), but a weighted sum. To understand this it is useful to rewrite (16) as

$$s_j = v \left[-\frac{\partial \left[\Sigma_i x_i(\mathbf{y}, \mathbf{t}) \right]}{\partial y_j} \right] - \sum_i t_i \left[-\frac{\partial x_i(\mathbf{y}, \mathbf{t})}{\partial y_j} \right] \tag{17}$$

The first term in this expression measures the environmental benefit of increasing the use of y_j, and depends on the sum of reduced carbon emissions the increase in y_j leads to. However, there is also a non-environmental indirect *cost* of increasing the use of y_j. By increasing the use of y_j the use of goods causing carbon emissions declines. For each such good, this reduction gives a social cost, since the marginal benefit of using the good exceeds the marginal cost of supplying the good. This difference is due to the carbon tax on the good, since the carbon tax is identical to the difference between the user and producer price of the good when the good is measured in terms of carbon emissions it causes. If different goods (or different uses of the same good) have different carbon taxes, these cost terms (second term in (17)) differ between goods and thus carbon emission sources. Hence, the optimal subsidy of each type of renewable energy depends not only on how total emissions are affected, but generally also on how each emission source is affected.

So far, nothing has been said about the geographical location of carbon emissions affected by renewable energy. The most obvious interpretation is that the analysis is for a single country, and that all x_i and y_j refer to carbon emissions and renewable energy in this country. However, the use of renewable energy in a specific country may also affect foreign carbon emissions in other countries. If e.g. Norway increases its production of wind power, this may due to electricity trade across borders reduce coal power production, and hence carbon emissions, in e.g. Germany. The effect on foreign emissions should be included in the formula (16) or (17) in a similar way as domestic emissions. However, there are two important differences between foreign and domestic emissions.

First, if the country under consideration is concerned about its own welfare and not that of other countries, the last term in (17) should not be included for foreign emission. For domestic emissions this term represents lost welfare due to taxation. There are similar welfare costs for the foreign countries if they have carbon taxes. However, these costs are irrelevant for a country that is only concerned with its own welfare.

Second, it is not obvious that a country values domestic and foreign emissions equally. Since it is only total global emissions that matter for the climate, one could certainly argue that all carbon emissions ought to be valued equally. However, there may be political reasons why a county is more concerned about its own domestic emissions than foreign emissions. An example of this is Norway: In 2008 the Norwegian Parliament set a policy goal for Norway to contribute to a reduction of 24 million tonnes CO_2 equivalents from 1990 to 2020. In *addition* to this goal of global emission reductions, the Parliament set a goal of domestic emission reduction for the same period equal to 15 million tonnes CO_2 equivalents. This additional domestic goal implies a much higher cost to Norway of achieving the global goal than if the global goal could be achieved in a cost-effective manner through international quota trade (see e.g. Fæhn, 2010). It is difficult to interpret this concern for domestic emissions in any other way than that $v_D > v_F$, where the subscripts D and F stand for domestic and foreign.

With the two points above in mind, the formulas (16) and (17) should be generalized to

$$\begin{aligned} s_j &= \sum_i (v_D - t_i) \left[-\frac{\partial x_i(\mathbf{y}, \mathbf{t})}{\partial y_j} \right] + v_F \left[-\frac{\partial x_F(\mathbf{y}, \mathbf{t})}{\partial y_j} \right] && (18) \\ s_j &= v_D \left[-\frac{\partial \left[\Sigma_i x_i(\mathbf{y}, \mathbf{t}) \right]}{\partial y_j} \right] + v_F \left[-\frac{\partial x_F(\mathbf{y}, \mathbf{t})}{\partial y_j} \right] - \sum_i t_i \left[-\frac{\partial x_i(\mathbf{y}, \mathbf{t})}{\partial y_j} \right] && (19) \end{aligned}$$

where x_F stands for foreign emissions and x_i as before stands for domestic emissions of type i.

The analysis above suggest that all inputs that influence carbon emissions should in principle be subsidized (if s_j from (16) is positive) or taxed (if s_j from (16) is negative). In practise, subsidies will be restricted to only a few inputs that affect the use of carbon. There are (at least) two reasons for this. First, there are probably quite large administrative costs associated with each subsidy, and these costs are likely to be independent of how large the subsidy is. This implies that one should restrict subsidies to a limited number of inputs. A second reason for limiting the use of subsidies is that the terms in square brackets in (16) for many inputs are extremely difficult – probably impossible – to calculate with any reasonable degree of confidence. This suggests limiting subsidies only to the inputs for which one feels reasonably confident about the size of the terms in square brackets.

6 Emission regulated by tradable quotas

So far, it has been assumed that emissions are regulated by a carbon tax. An alternative type of regulation is tradable emission quotas. Consider first the simple model of section 2 with only one type of carbon emissions and one type of renewable energy. The market outcome is as before given by (6)-(7). However, (6) must now be interpreted as the demand for emissions, and thus for quotas, with t being the quota price. The supply of quotas, and thus emissions, is a policy variable. For any given quota supply $\bar{x}$ and subsidy s of renewable energy, (6)-(7) and the quota market equilibrium $x = \bar{x}$ will determine y as well as the endogenous quota price t. Hence, $t = t(\bar{x}, s)$, and by differentiating (6)-(7) it is straightforward to verify that[2]

$$\begin{aligned} t_{\bar{x}}(\bar{x}, s) &= -\frac{F_{xx}F_{yy} - F_{xy}^2}{b'' - F_{yy}} < 0 \\ t_s(\bar{x}, s) &= -\frac{-F_{xy}}{b'' - F_{yy}} < 0 \end{aligned}$$

This first-best optimum is achieved by setting $s = 0$ and $\bar{x}$ equal to the value of x following from (4)-(5). This gives a quota price equal to v, making the market outcome (6)-(7) identical to the social optimum (4)-(5).

If the government for some reason sets the quota supply $\bar{x}$ higher than the socially optimal x-value but keeps $s = 0$, the equilibrium quota price will be lower than than v, since $t_{\bar{x}} < 0$. If the government also introduces a subsidy on renewable energy this will imply an additional reduction in the quota price, since $t_s < 0$. The interpretation is straightforward: Increased supply of quotas reduces the equilibrium quota price, as does reduced demand for quotas due to more renewable energy as a consequence of the subsidy.

In section 2 it was demonstrated that it would be optimal to have a positive subsidy on renewable energy if $t < v$. If the supply of quotas is set so high that the equilibrium quota price is below v, one might expect that

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[2] Strict concavity of F implies $F_{xx} < 0$, $F_{yy} < 0$ and $F_{xx}F_{yy} - F_{xy}^2 > 0$.

it also in this case is optimal to subsidize renewable energy. However, this is not the case. Social welfare is as before given by (3), but with x exogenously given. Maximization of W with respect to y therefore gives $F_y(x, y) = b'(y)$, which is achieved by $s = 0$, no matter what value x has.

Moving to the more general model of section 3, with several sources of carbon emissions, a relevant case is for some carbon sources to be regulated by quotas, while other sources are regulated by a carbon tax or not at all. An example of this is the EU quota system, where emissions from the power sector and from manufacturing industry are regulated through a common EU quota system. Most other sources of carbon emissions are regulated at the national level, in many cases through carbon taxes. The sum of emissions regulated by quotas is politically determined through the supply of quotas, and is therefore not affected by changes in the use of renewable energy. In the expression (16) for optimal subsidies it is therefore only those sources of carbon emissions that are not regulated by the quota system that should be included.[3]

If the quota system is a common system covering several countries, such as the EU quota system, an increase in domestic renewable energy may move domestic emissions covered by the quota system to abroad. If domestic emission reductions are valued higher than foreign reductions (see the discussion leading to (18)), the expressions for the optimal subsidy may include terms for emissions covered by the quota system. Let x_Q be domestic emissions covered by the quota system. If an increase in y_j reduces x_Q and thus increases foreign emissions by the same amount[4], the expression (18) must be

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[3] If some of the carbon sources in the quota sytem also are taxed (such as CO_2 from the Norwegian petroleum sector), and an increase in y_j reduces these emissions and increases untaxed emissions elsewhere in the quota system, the reduction in taxed emissions represents a social cost for the reasons given in section 5. These social costs should be included in the expression for the optimal s_j, tending to make s_j lower than it would have been if all emissions in the quota system were untaxed.

[4] General equilibrium effects from increased foreign carbon emissions covered by the quota system to other foreign carbon emissions are for simplicity ignored.

modified to

$$s_j = \sum_i (v_D - t_i) \left[-\frac{\partial x_i(\mathbf{y},\mathbf{t})}{\partial y_j} \right] + v_F \left[-\frac{\partial x_F(\mathbf{y},\mathbf{t})}{\partial y_j} \right] + (v_D - v_F) \left[-\frac{\partial x_Q(\mathbf{y},\mathbf{t})}{\partial y_j} \right]$$

The last term in this expression represents the value of moving carbon emissions from domestic sources to abroad. While it may seem strange that a government should place a value on such a move of emissions, it is a direct consequence of the government being more concerned about its domestic emissions than emissions in other countries.

7 Second-best carbon taxes

So far, it has been assumed that all carbon taxes are exogenous, and perhaps lower than their optional levels. Section 5 considered the case of many sources of carbon emissions, each with an exogenous tax rate. But even if there is an exogenous maximal tax rate for each emission source, and these tax rates are all below v, it is not obvious that it is optimal to set each tax rate at its maximal level. To analyze this, the model of section 2 is modified by assuming that y instead of renewable energy is a second source of carbon emission (in addition to x). Social welfare is in the present case hence given by

$$W = F(x,y) - px - b(y) - v(x+y) \tag{20}$$

and the social optimum is characterized by

$$F_x(x,y) = p + v \tag{21}$$

$$F_y(x,y) = b'(y) + v \tag{22}$$

The market outcome follows from profit maximization of producers. Aggregate profits are

$$\pi = F(x,y) - (p + t_x)x - (b(y) + t_y y)$$

where t_x and t_y are the carbon tax rates for x and y. Profit maximization gives

$$\begin{aligned} F_x(x,y) &= p + t_x && (23) \\ F_y(x,y) &= b'(y) + t_y && (24) \end{aligned}$$

Comparing with (21)-(22) it is immediately clear that the market outcome will coincide with the social optimum if both carbon tax rates are equal to v. In other words, the social optimum is achieved be setting a uniform carbon tax at the Pigovian rate.

Assume now that the tax on x for some reason is constrained not to exceed some maximal level, but that there is no constraint on the tax on y. It is easily verified that it in this case is optimal to set t_x equal to its maximal level, henceforth t_x is therefore regarded as given (equal to its maximal level). Since t_y is assumed to be unconstrained, is not obvious what its optimal value is. In the first-best optimum t_y is equal to both t_x and v, which in this case are identical. When $t_x < v$ it is not obvious what t_y should be. The optimal value of t_y may be derived using a similar analysis as in section 2.

As before $x = x(y, t_x)$, and maximizing

$$W = F(x(y,t_x), y) - px(y,t_x) - b(y) - v(x(y,t_x) + y)$$

gives

$$(F_y - b' - v) + (F_x - p - v)\, x_y(y, t_x) = 0$$

or, using (23) and (24)

$$t_y = v - (v - t_x)(-x_y(y,t)) \qquad (25)$$

As before, it is assumed that $F_{xy} < 0$, implying $x_y < 0$. An obvious example of this is if the two carbon sources are coal and gas, both used for power generation. More of one of these fuels reduces the demand for the other. A second example is taxation of fuels for two competing means of transportation. A lower price of fuel for one type of transportation reduces the demand for the other form of transportation, and hence also the other use of fuel.

It is clear from (25) that if $t_x < v$, the optimal value of t_y is also below v. The intuition is the same as the intuition for a subsidy to a non-carbon alternative: In both cases (a non-carbon and a carbon alternative) one should set the price of the alternative (i.e. of y) below the social cost of the alternative, in order to encourage production and thereby reduce the use of x.

It is not obvious whether t_y should be set above or below t_x, since (25) implies

$$t_y - t_x = (v - t_x)(1 + x_y(y, t))$$

Hence, $t_y > t_x$ if $-x_y < 1$ and $t_y < t_x$ if $-x_y > 1$, where x_y is given by (9). To understand this result, consider an initial situation with $t_y = t_x < v$. In this situation emissions are too high, but since $t_y = t_x$ the allocation of emissions between the two sources is cost-effective. A small change in t_y at this starting point only has a zero order effect on cost-effectiveness. Therefore, one should change t_y in the direction making total emissions decline (since total emissions initially are too high). Total emissions decline if t_y is reduced if $-x_y > 1$, while t_y must be increased to reduce total emission if $-x_y < 1$.

Clearly, if $-F_{xy}$ is sufficiently small, $-x_y < 1$ and $t_y > t_x$. To see that $t_y < t_x$ is possible, it is useful to consider the case of perfect substitutes in more detail.

Let $\tilde{x}$ and $\tilde{y}$ be two carbon sources measured in energy units, and let gross output only depend on the sum of energy, i.e. $\tilde{F}(\tilde{x} + \tilde{y})$. Carbon emissions are $x = a_x \tilde{x}$ and $y = a_y \tilde{y}$, so

$$\tilde{F}(\tilde{x} + \tilde{y}) = \tilde{F}(\frac{x}{a_x} + \frac{y}{a_y}) = F(x, y)$$

Using (9) it follows that

$$-x_y = \frac{F_{xy}}{F_{xx}} = \frac{\tilde{F}'' \frac{1}{a_x} \frac{y}{a_y}}{\tilde{F}'' \left(\frac{1}{a_x}\right)^2} = \frac{a_x}{a_y}$$

Carbon emissions per unit of energy are higher for coal than for natural gas. So if x is coal and y is natural gas, the expression above gives $-x_y > 1$, implying that the optimal tax per unit of carbon from natural gas should be lower than the exogenous tax per unit of carbon for coal.

8 Implications for social welfare and government revenue

Returning to the simple model of section 2, the first-best optimum is achieved by setting $t = v$. Obviously, placing a restriction of t being exogenous and lower than v gives a welfare loss. How large this welfare loss is will depend on the functional forms of F and b and on the size of p, t and v.

A slightly simpler comparison between first-best and second-best is the case in which there instead of an environmental damage function vx is a given emission target $\bar{x}$ that is lower than the "Business as Usual" (BaU) emission levels, i.e. emissions following from (6)-(7) with $t = s = 0$. The first-best way to achieve this goal is to use a carbon tax of an appropriate size and no subsidy. Although this is rather obvious, it may be useful to derive this formally. Given $\bar{x}$, the first-best maximal social welfare is given by

$$W^F = \max_y \{F(\bar{x}, y) - p\bar{x} - b(y)\} \tag{26}$$

implying $F_y = b'(y)$. From (7) this implies $s = 0$, so t follows from (6)-(7) with $x = \bar{x}$ and $s = 0$.

Assume instead the carbon tax is zero. Then y must be set so $x(y, 0) = \bar{x}$ giving a specific value $y(\bar{x})$. Second-best social welfare is therefore

$$W^S = F(\bar{x}, y(\bar{x})) - p\bar{x} - b(y(\bar{x})) \tag{27}$$

The difference between W^F and W^S may be high. An example of a numerical comparison is Fischer and Newell (2008). They use a simple numerical model for the US electricity sector, and consider the costs of achieving a target $\bar{x}$ that is 5% below below BaU emission levels. They calculate the costs of achieving the target, defined as the reduction in W compared with BaU- case (ignoring climate costs). In particular, they compare the case of a carbon tax as the only instrument with the case of a renewable subsidy as the only instrument. They find that the cost in the subsidy case is two and a half times higher than the cost in the tax case.

Whether a tax or a subsidy is used as the policy instrument also has implications for the government's budget balance. With a carbon tax, the government gets a revenue increase equal to

$$t\bar{x} = F_x(\bar{x}, y^F)\bar{x}$$

where y^F is the solution to (26).

With a subsidy, the government gets an expenditure equal to

$$sy(\bar{x}) = [b'(y(\bar{x}) - F_y(\bar{x}, y(\bar{x}))]\, y(\bar{x})$$

It is not obvious which is the larger of these two. It clearly may be the case that $t\bar{x} < sy(\bar{x})$. This must hold if $\bar{x} = 0$, giving zero tax revenue and probably very large subsidy expenditures. However, it is also possible that $t\bar{x} > sy(\bar{x})$. In the study of Fischer and Newell (2008), this is the case. In this analysis $t\bar{x}$ is about three and a half times as large as $sy(\bar{x})$.

Since a carbon tax adds revenue to the governments budget balance, while subsidies add expenditures, one might believe that governments would prefer a carbon tax to a subsidy on renewable energy. However, this need not be the case. Even if the government intends to fully recycle the revenues from a carbon tax, each voter may focus on the visible tax increase and not trust that the revenue from the carbon tax will be recycled in a way compensating him or her. Moreover, some persons will be hurt more by the carbon tax than others; this will typically be those who consume more than the average share of fossil fuels due to their current preferences or earlier investments (e.g. a large house with a long commuting distance). On the production side, some industries will bear a disproportionately high share of the total costs from the carbon tax. Consumers with a high use of fossil fuels as well as workers and owners in such high emission sectors will often be successful in lobbying against a carbon tax.

In contrast, the costs of subsidizing renewable energy are likely to be less visible to the typical voter and also be more evenly shared by everyone in the economy. Sectors in the economy producing renewable energy or inputs to this production will gain from a subsidy to renewable energy, and might thus engage in lobbying for the use of such subsidies instead of a carbon tax. These arguments suggest that it might be easier to obtain political support for a subsidy to renewable energy than for a carbon tax.

9 Uncertain future carbon tax

So far, the time aspect for the production of renewable energy has been ignored. However, many types of renewable energy require large upfront investments, and once the capital is installed operating costs are so low that the production will be at full capacity. Examples of this are hydro, wind and solar energy. Clearly, for such renewable energy the future price of this energy is important for the current investment decision. Since the future price of renewable energy will depend on the future carbon tax, this tax rate is important for investment decisions. This would not be of any importance if the government could commit to a carbon tax path far into the future. However, such commitment is in practice impossible, implying that decisions on how much to invest in renewable energy must be based on market agents' expectations about future carbon taxes. It is sometimes argued that this impossibility of governments to commit to future carbon tax rates leads to uncertainty, and that this uncertainty in turn leads to less investments in renewable energy than what is socially efficient. If this argument is correct, it would be an argument for subsidizing investments in renewable energy (but simultaneously having a carbon tax).

The argument above is discussed in detail in the present section within the framework of a simple two-period model. Some of the points made here were previously made by Ulph and Ulph (2011), but in the context of a different model.

It is useful first to present the model when there is no uncertainty. Formally, the model is almost identical to the model presented in section 2. In period 1 (the present), the carbon tax is assumed to be set optimally, and investments in renewable energy take place at a cost $b(y)$. In the second period (the future) the renewable energy is used. Social welfare is thus

$$\tilde{W} = \left\{ \max_z \left[f(z) - (p+v)z \right] - \tilde{b}(y) \right\} + \frac{1}{1+r} \left\{ F(x,y) - (p+v)x \right\}$$

where r is the discount rate. Since the optimal amount of carbon in the present (z) is chosen optimally and does not depend on y, this part of the social welfare function can be ignored. Maximizing $\tilde{W}$ is therefore equivalent to maximizing

$$W = \{F(x,y) - (p+v)x\} - b(y)$$

where $b(y) = (1+r)\tilde{b}(y)$. This expression for social welfare is identical to

(3). Hence, the social optimum is as before given by (4)-(5), and the market outcome is as before given by (6)-(7). Setting $t = v$ and $s = 0$ thus gives the social optimum in the present case.

The problem in the current case is that the government does not have the ability to set t already at the time when investments in renewable energy take place. Investment decisions must therefore be made in the presence of an uncertain future t. Assuming first risk neutral producers, expected profits are

$$E\pi = E\left\{\max_{x}\left[F(x,y) - (p+t)x\right]\right\} - (b(y) - sy) \tag{28}$$

Notice that future use of carbon need not be decided before t is known. However, the choice of y must be made in the presence of t being uncertain.

Maximizing $E\pi$ gives (using the envelope theorem)

$$EF_y(x(y,t),y) = b'(y) - s \tag{29}$$

For any probability distribution of the stochastic variable t, this gives a particular value of y.

To compare the market outcome to the socially efficient outcome two types of uncertainty are considered:

- "Scientific uncertainty", implying that v is uncertain for the present and future government
- "Political uncertainty", implying that v of the future government is uncertain

The first type of uncertainty is the uncertainty of how harmful a given amount of carbon emissions is for the economy. The magnitude of this harm is currently not known, but will perhaps be better understood in the future. In the present model it is assumed that the true damages, and thus the v to be used in the future, will be known with certainty in the future. Moreover, market agents are in this case assumed to correctly anticipate that whatever v turns out to be, the future carbon tax rate will be set equal to this value.

The second type of uncertainty is different. It is assumed that the current government knows that its valuation of future carbon emissions is v. However, the current government also knows that there might be a different government in the future, and that this government may have a different valuation of emissions than itself, say $\tilde{v}$. Seen upon from the present there is a probability distribution over $\tilde{v}$. As above, market agents are assumed to correctly anticipate that whatever $\tilde{v}$ turns out to be, the future carbon tax rate will be set equal to this value.

9.1 Scientific uncertainty

In this case the government maximizes

$$EW = E\left\{\max_{x}\left[F(x,y) - (p+v)x\right]\right\} - (b(y)$$

where v is a stochastic variable. This gives

$$EF_y(x(y,v),y) = b'(y) \tag{30}$$

For any probability distribution of the stochastic variable v, this gives a particular value of y. Since it is assumed that $t = v$ whatever v turns out to be, this outcome is identical to the market outcome provided $s = 0$. In this case uncertainty about the future carbon tax is in itself thus not an argument for subsidizing investments in renewable energy.

The result above required that producers were risk neutral. It is widely believed that various imperfections of risk and capital markets imply that producers behave as if they have risk aversion. Risk aversion implies more weight on "bad" outcomes. Bad outcomes for those investing in renewable energy are outcomes with low prices on renewable energy, i.e. outcomes with a low carbon tax. Giving these outcomes more weight in the optimization gives an equilibrium condition

$$\theta E F_y(x(y,t),y) = b'(y) - s \tag{31}$$

where $\theta \in (0,1)$ is a term representing risk aversion. For (31) to coincide with the social optimum given by (30) it is clear that

$$s = (1-\theta)b'(y)$$

which is positive if there is risk aversion. With risk aversion there is thus a case to be made for subsidizing investments in renewable energy. Notice however that the same argument can be made for any long-run investment with an uncertain future price, provided producers are risk averse.

9.2 Political uncertainty

The current government in this case knows that its own valuation of future emissions is v. However, the future government's valuation $\tilde{v}$, and hence the future carbon tax $t = \tilde{v}$, is uncertain. Given this uncertainty, the current government maximizes a similar expression as (8), except that t now is uncertain:

$$EW = E\left\{F(x(y,t),y) - (p+v)x(y,t)\right\} - b(y)$$

Notice that v is known with certainty, while t is uncertain with some probability distribution. Maximization of EW with respect to y gives

$$E\left\{\left(F_x - p - v\right)x_y(y,t) + F_y\right\} = b'(y)$$

Whatever value t turns out to have, users of carbon will choose x so $F_x = p+t$. Inserting this into the expression above gives

$$E\left\{\left(t - v\right)x_y(y,t) + F_y\right\} = b'(y)$$

or

$$E(t-v) \cdot E(x_y) + covar(t, x_y) + EF_y = b'(y) \tag{32}$$

Comparing with (29) it is clear that the market outcome coincides with the social optimum if

$$s = E(t-v) \cdot E(x_y) + covar(t, x_y) \tag{33}$$

Since $x_y < 0$ (from (9)), the first term is positive if and only if $Et < v$. In other words, if the current government expects the future government to have a lower valuation of emission reductions than it itself has, this tends to make the optimal subsidy positive. However, if there is uncertainty about t but nevertheless $Et = v$, this term is zero.

The sign of the second term is not obvious. The function $x(y, t)$ was defined by (6). This function is illustrated in Figure 1 for two values of t. The curve is downward sloping and lower the higher is t. However, it is not obvious what the curvature is. For the special case of perfect substitutes, i.e. $F(x, y) = \tilde{F}(x + y)$, it follows from (9) that $x_y = -1$, since $F_{xx} = F_{xy}$ in this case. Provided $x(y, t^{\max}) > 0$, where $t^{\max}$ is the highest t in the probability distribution, the second term in (33) is therefore zero.

In Figure 1 the curve for $x(y, t)$ becomes gradually flatter as y increases. Intuitively, this seems reasonable: The smaller x is, the more difficult it is to obtain a further reduction in x by increasing y. For the same reason, it seems reasonable that the higher t is, and thus the lower x is, the the more difficult it is to obtain a further reduction in x by increasing y. In Figure 1 this is represented by the curves for $x(y, t)$ becoming flatter as one moves vertically downwards. If the steepness measured positively, i.e. $-x_y$, is smaller the higher is t, x_y is higher the higher is t, i.e. $covar(t, x_y) > 0$. If this is the case it is thus optimal to have a positive subsidy even if $Et = v$.

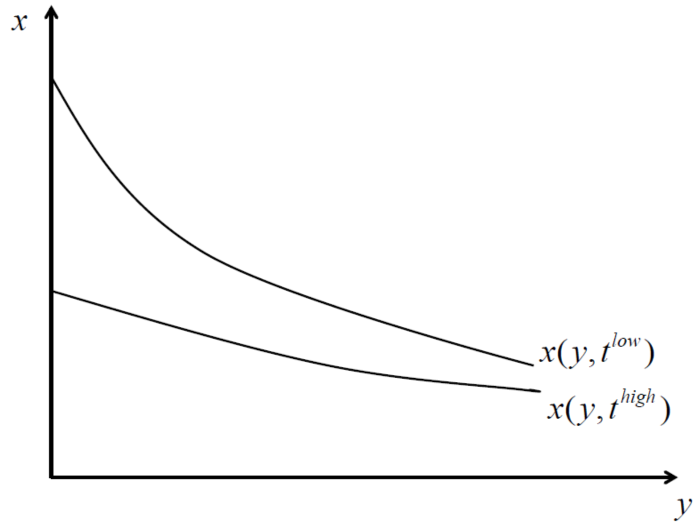

Figure 1

10 Concluding remarks

The paper has focused on renewable energy as an alternative to carbon energy. However, the analysis and results are valid for a much broader class of activities that reduce carbon emissions. A main result is that such activities should be encouraged (by subsidies or by other means) provided they reduce carbon emissions that are taxed at a lower rate than the valuation of the emission reductions. How much such activities should be encouraged does not only depend on the total emission reductions they lead to, but generally also on how the emissions that are affected are taxed and whether the emissions are domestic or foreign. If emission reductions are at different points of time this should also be taken into consideration, as the present value of the valuation v typically will be declining over time.[5]

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[5] It seems reasonable to expect v to be increasing over time, but the present value of v to be declining over time. For a further discussion see e.g. Ulph and Ulph (1994) and

It may provide useful to conclude by illustrating these general points by an example. Consider a benefit-cost analysis of a public infrastructure investment such as a high-speed railway. Assume first that a standard benefit-cost analysis has been done, but has ignored all effects on carbon emissions. If the analysis has been done correctly, it has already included costs and benefits of changing the use of taxed goods, as represented by the third term in (19). Examples of such costs could be reduced road and air transportation, to the extent that these goods are taxed.[6] To extend this benefit-cost analysis to include climate considerations, one must calculate all emission changes. Typically, there will be some emission reductions (less road and air transportation, less emissions from lower overall consumption due to the tax increase to finance the public investment), and some emission increases (from the investment in the infrastructure in addition to its operation). All of these emission changes should be multiplied by their valuations, which may depend on when the emission changes occur and may differ between domestic and foreign emissions. Finally, all the terms should be discounted and added to the non-climate present value of the benefit-cost analysis.

Hoel et al. (2009).

[6] For a tax on such a good to be relevant in the cost-benefit analysis, the tax must exceed the social cost of non-climate externalities, such as for instance local air pollution, noice, and congestion costs.

References

Fischer, C. and Newell, R.G. (2008), “Environmental and technology policies for climate mitigation”, *Journal of Environmental Economics and Management* 55, 142-162.

Fæhn, T. (2010), ”Tenke globalt; handle lokalt eller handle kvoter? En makroøkonomisk analyse av å gjennomføre Klimaforlikets mål for 2020.” Økonomiske analyser 3/2010.

Hoel, M. (2011), "The Supply Side of CO2 with Country Heterogeneity", *Scandinavian Journal of Economics* 113,pp 846–865.

Hoel, M., Greaker, M., Grorud, C. and Rasmussen, I. "Climate policy - costs and design" TemaNord issue 550, Nordic Council of Ministers.

Stern, N. (2007), *The Economics of Climate Change. The Stern Review.* Cambridge University Press.

Ulph, A. and Ulph, D. (1994), "The optimal time path of a carbon tax", *Oxford Economic Papers* 46, pp 857-868.

Ulph, A. and Ulph, D. (2011), "Optimal Climate Change Policies When Governments Cannot Commit", Department of Economics, University of St. Andrews, Working Paper 1104.