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A Framework for Studying the Environmental Impact of Biofuel Policies

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Abstract

In this thesis I present a new framework for investigating the environmental impact of and optimal policies for biofuel production. The model captures the interactions between fossil fuel and biofuel in addition to the dynamic nature of the carbon cycle. Compared with the social planner optimum, two external effects that the market fails to integrate are exposed; the negative environmental impact of emission from fuel consumption, and the benefits of carbon capture through growth in the crops used for biofuel production. It is then shown that these deficiencies can be corrected through a common carbon price, where emission is taxed and carbon capture is subsidized. The socially optimal solution can also be reproduced using a tax/subsidy scheme on fossil fuel and biofuel production. The last part of the thesis investigates the effects of the most commonly used policy instrument for biofuels; a blending mandate. It is shown that a blending mandate will, as desired, tend to increase the use of biofuel and lower fossil fuel production. The optimal tax on fossil fuel, in the presence of a blending mandate, is then derived. If the total benefits of biofuel are high enough to compensate for the environmental damage of the direct emissions, the tax on fossil fuel should be lower than the first-best tax.

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I appreciate every thing you have taught me through our lively debates.
I treasure our moments of disagreement, which have made wiser.
I adore your ability to always make me laugh, even in bad days.
I love that you are always there, regardless, on my side.
I dedicate this thesis to you.

Thea M. Sletten, Oslo, May 2012

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Chapter 1

Introduction

According to the International Panel of Climate Change(IPCC), 95% of the energy used by world transport is petroleum based, and transport was responsible for approximately 23% of all greenhouse gas (GHG) emissions in 2004 [1]. The fuel consumption and thus emissions are expected to further increase during the next several decades. Vehicles powered by hydrogen or electricity are still a long way from being adequate substitutes for fossil fuel combustion engines vehicles. This leaves biofuel as the only alternative for curtailing emissions from the transport sector, where the technology is available today. Biofuel is thus an important weapon in the battle to prevent the catastrophic consequences of a larger rise in the average global temperature. Fiscal policy measures will be essential to create incentives for producing and using biofuel, as long as the real cost of producing biofuel is higher than the cost of producing fossil fuel.

First generation biofuels have been accused of increasing the food prices and have also been criticized for having high lifecycle emissions. More and more focus has thus been given to the second generation of biofuels, made from cellulosic biomass, where boreal forest has been a prime candidate for production. Unfortunately there is another issue arising when using boreal forests in biofuel production, as described by Bjart Holtsmark in the recent article "Use of wood fuels from boreal forests will create a biofuel carbon debt with a long payback time" [2]. As the title suggests, using slow growing crops in the production of biofuels will not be carbon neutral in the near future. Holtsmark's conservative estimates suggests that using boreal forest in biofuel production will create a carbon debt with a payback time of 150-200 years. This is equivalent to saying that for 150-200 years the level of atmospheric carbon will be higher than status quo, which imply that the climate effect will be negative in this period. These objections do not imply that biofuel should be disregarded as a tool for preventing climate changes, as there still exists production possibilities which ensures a reduction in GHG emissions, also in the near future. Instead it underlines the importance of taking into account dynamic effects as well as the large differences between types of biofuels, when modeling life cycle emissions. In particular it is crucial to consider these effects when adopting fiscal policy measures aimed at the fuel industry.

When considering dynamic environmental effects from fuel use, biofuel and fossil fuel are usually modeled separately, and the independent results are compared. However, in real life the the two fuel types will coexist in the same market, and their demand will thus be interlinked. Any changes in the cost, prices, taxes or subsidies of either fuel will affect the demand for both types of fuel. This will again have consequences for the environment, through changes in emission. Since fossil fuel is not a renewable resource, the optimal taxation should reflect that fact in addition to the environmental damages it causes. Biofuel is not necessarily carbon neutral in the short run, so this should be taken into consideration when subsidies or blending mandates are enforced. And lastly, when biofuel is introduced into the fossil fuel market, interaction effects may arise and should be investigated to ensure that the implemented policies will yield the desired effects.

The initial goal with this thesis was to study the dynamic environmental effects when introducing biofuels into the fossil fuel market. From this I wanted to explore how fiscal policy should be used to create incentives for the market agents to internalize the environmental costs of fuel consumption. When I started investigating what framework to use for this purpose, I found no published models which integrated all these effects. I have therefore taken upon me the task of creating a new framework for modeling environmental effects from fuel consumption, with emphasis on biofuels. The goal is to be able to capture both the dynamic nature of the carbon cycle, as well as the interaction effects arising in the fuel market. The tradeoff when creating more realistic, and thus more complex models, is that you can no longer solve them analytically. However, it is still possible to study some of the general properties of the system based on the first order conditions and steady state solutions, even though the complete dynamic solution must be obtained numerically. In the appendix I have outlined a general strategy for obtaining a numerical solution to this and similar types of models.

Chapter 2

Biofuels, for Better or for Worse?

2.1 Rationale for Biofuels

The interest for biofuels dates back to long before climate changes was put on the political agenda. If fact, Rudolf Diesel demonstrated his Diesel engine in the 1900 'Exposition Universelle' using peanut oil. Biofuel was considered a viable, competitive fuel until the oil prices started to fall in the 1940s. The production was relaunched in the US and Brazil in the 70s, when they started producing ethanol from corn and sugarcanes. However, the interest for biofuels did not spread to other countries until the rising climate issues added a new reason for investing in this industry. Today the debate about biofuel is mostly clustered around climate concerns, but there are also other compelling reasons for exploring biofuel as an alternative fuel.[3][4]

2.1.1 The end is near

The fact that oil is a non-renewable resource (at least in all relevant timeframes) has been known for a very long time. The production of oil is predicted to have already reached its peak in 2005 [5]. It has also been forecasted that the extraction of oil on average will decline by 3% per year until the stock is depleted [5]. One could argue that the current and previous generations have acted myopic in their exhaustion of this resource, as the demand for oil is still rising. This will inevitably lead to a point where production can no longer meet demand. The problem is also amplified by highly volatile petroleum prices, making energy security a concern for many countries. In particular, when the remaining oil reserves are going to be controlled by a few countries, mainly in the middle east, there will be increasing needs to secure energy sources that do not rely on the current political climate.

Today, petroleum accounts for 95% of all fuel used for transportation [1], adding up to two-thirds of the world's oil consumption. Energy consumption in the transport sector is expected to grow by 2% per year in the coming decades [1], doubling the energy use in only 35 years. In contrast to energy consumption in other parts of the economy, the increasing energy demand in the transport sector cannot be met by water, wind, solar or other acknowledged, green energy sources. While both hydrogen and electricity are promising technologies, biofuel is the only substitute for oil that is suitable for the transport sector, where the technology is available today. Without the possibility of transporting goods and people in a cost efficient manner, all countries or even cities must become more or less autarkic. When basic needs, like food and energy can not be provided at affordable prices, we are looking at a world wide crisis so comprehensive it would be hard to imagine the full extent of it.

2.1.2 Let a thousand flowers bloom

Biofuels production will lead to increasing interest and need for advancement in the agriculture sector. This may enable rural areas to develop a new livelihood and progress economically. Due to low commodity prices, farmers around the world have had problems making ends meet. Biofuels production is a new opportunity for many countries to maintain and revitalize their primary sector. New crops, as well as the ability to use residuals from food and forestry crops, will give rise to a more diversified income stream for farmers. With additional private and government investments it is also possible to evolve towards more sophisticated farming techniques in developing areas, increasing the efficiency, sustainability and profitability of farming. This might also prohibit land abandonment and some of the excessive migration to the larger cities. The most successful biofuels industry sector is found in Brazil, where the director of the Department of Energy in Brazil, Antonio J. F. Simões, claims that the Biofuels industry was responsible for 1 million direct and 6 millions indirect jobs in Brazil in 2007. This initiation of rural development have already lead hundreds of thousands out of poverty, and will hopefully continue to fuel Brazil's economy.[6][7][8][9]

2.1.3 The grass is greener

Approximately 23% of the world's GHG emissions comes from transporting people and goods, and in USA and Canada transportation is responsible for as much as 33% of the countries' emissions [10]. In addition, we have the increasing demand from up-and-coming economies. For example, China is predicted to quadruple their fuel consumption in 2025, as compared to 2002 [10]. According to the International Energy Agency, biofuels could provide 27% of the world's fuel consumption in 2050 [3]. If the new EU directives for sustainable biofuels are followed, all biofuels will have a minimum of 35% reduction of GHG emissions compared to fossil fuel, rising to a 60% reduction by 2018.[11]. Combining these numbers we see that biofuels can reduce emissions from the transport sector with

9% -16% in the relatively near future.[1]

Biofuel is by no means a one and only solution to the climate crisis, but rather a valuable tool when it comes to reducing emission from transportation in the near future. It is important to notice the emphasis on the near future, as hydrogen or other technologies will hopefully develop further down the line. However, to stay within the infamous two degree limit we need to reach the global emission peak between 2015 and 2021, and within 2050 we must have reduced emissions compared to the 2000 level by at least 48% [12]. Thus we need to invoke every conceivable GHG reducing instrument today, to be able to avoid the most calamitous consequences of the commencing climate crisis.

2.2 Technology

It is common to divide biofuel technology into two subgroups based on the maturity of the technology involved; First Generation (Coventional) biofuels, and Second Generation (Advanced) biofuels. I will not go into details about all the available production techniques, but rather give a short presentation of the most common ones. The current standing in development and commercialization of proven technologies is displayed in the figure below.

[1]These are highly uncertain numbers, and are only presented as an example and not as facts.

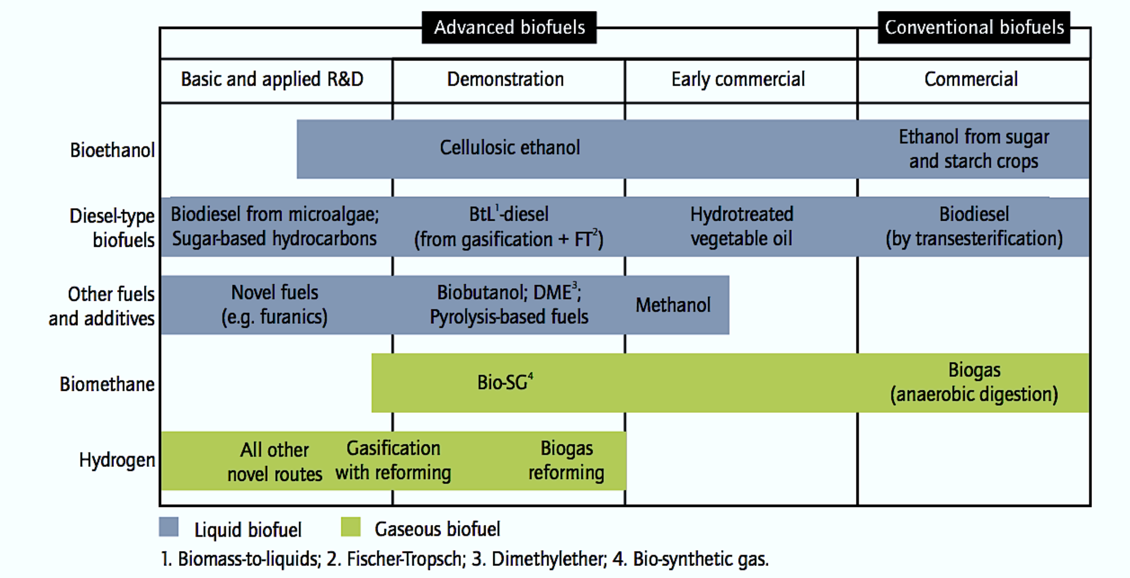


Figure 2.1: Current status of biofuels technology[3]

2.2.1 Here today, gone tomorrow

First generation bioethanol are mainly produced using food crops like corn, wheat, sugarcanes, palm oil or rape. This technology is considered mature, and is in commercial use today. Brazil and USA produce 73% of all biofuels [13], with sugarcanes and corn as their primary sources of biomass, and ethanol fermentation as the most commonly used production method. This is a process in which sugars, such as glucose, fructose or sucrose, are converted into energy, producing ethanol as one of the bi-products [14]. If starch is utilized instead of sugar, an additional step of converting the starch into glucose must be included, which makes this process more energy-intensive. After the fermentation the ethanol is recovered, and then concentrated by a variety of processes. The production costs depend on the feedstock prices, which makes the total costs and profitability highly volatile.[3]

Biodiesel is diesel fuel based on vegetable or animal fat. It is produced from raw vegetable oils, as well as animal fats and used cooking oil. The oils are reacted with an alcohol, like methanol or ethanol, and produce esters of fatty acid and glycerol [15]. The finished biodiesel consists of long-chained alkyl-esters, and the conversion rate from oil to biodiesel can be as high as 98% [16]. The profitability of conventional biodiesel depends a lot on co-products from the production, e.g. protein meal and glycerin, and it is also sensitive to feedstock prices [3].

A limiting factor for all 1st generation biofuels is the land intensive production, combined with the use of food crops. This combination means that scaling up the production of 1st generation biofuels is bound by the need for arable land for food production. Since many of the same crops are used for food, the energy crops will affect the food prices both directly through crop sales and indirectly through competition for land. In addition, the GHG reduction is fairly small for many of the food crops, in particular if land changes are taken into account.

2.2.2 Knock on wood

Second generation biofuels are made from lignocellulosic biomass[2] or woody crops, which is harder to convert to fuel than conventional food-crops. The cellulosic feedstock must first go through a biochemical conversion, making cellulose and hemicellulose into fermentable sugars. The fermentation of the sugars into bioethanol is the same process as for 1st generation biofuels.

Cellulose is the world's most widely available biological material, present in such low-value materials as wood chips and wood waste, fast-growing grasses, crop residues like corn stover, and the organic fraction of municipal solid wastes [18]. In places where waste and residual materials are easily available, hardly any additional land is required to produce biofuels. However, additional supplements of fast growing forest crops or grasses will necessarily need land. If marginally arable or degraded land is used to grow the energy crops, the resulting impact on food production may still be negligible. Using degraded land can also yield

[2]Lignocellulosic biomass is biomass composed of cellulose, hemicellulose and lignin. [17]

additional benefits through making the land more resistant to soil erosion [19]. Either way, the most important benefit is that the potential production volume as well as the GHG emission reductions are predicted to be much higher with 2nd generation biofuels [19].

There are several other 2nd generation technologies that are currently under development, but few that are equally promising in the near future. If we expand our horizon a bit further into the future, one of the more interesting projects is using algae as the source of biomass. Algae are extremely fast growing organisms, which do not require any arable land (as they live in water), and can produce as much as 100 times more fuel per area as conventional vegetable oil crops. In a report from the International Energy Agency they state that *"...“bio-jet” fuel could be produced on a landmass equivalent to the size of the US state of Maryland and be sufficient to supply the present world's aircraft fleet with 100 percent of its fuel needs [19].* These projections undeniably sound great, but the technology is still a long way from commercialization due to the extremely high production costs. However, this might change over the coming decade, and if it does, it can substantially increase the total possible production volume of biofuels.

2.3 Critique

2.3.1 Let them eat cake

The biofuel industry has been criticized for diverting agricultural production away from food crops, and thus contributing to higher food prices world wide [13]. It is usually argued that increased use of food crops in biofuel production will increase the demand and thus the price of the crops in question. This will then lead to second round effects, as more land are used to produce for example corn, leaving less viable land for producing wheat, soybeans or other food crops. The price of these crops will then start to rise, and we get a self-enforcing process of price increases. The conclusion drawn by the critiques is that increased production of biofuels leads to all-over higher food prices. Higher food prices will always hurt poor countries the most, and hunger crises may arise when people who was barely able to pay for their food no longer are able to.

There has been done a lot of research on the effects from biofuels production on food prices, and in particular on the recent spike in commodity prices (before the financial crisis). However, there seem to be little consensus about the size of these effects. The results vary much due to large variations in methods of calculation and choices of assumptions, but mostly because different combinations of commodities and crops were considered. It is difficult to draw any conclusions based on the current standing, but I can mention that biofuels have been claimed to be responsible for everything from 3% to 70% of the increase in commodity prices the last 6-8 years [20].

A last (and less politically correct) remark is that the US has a long history of subsidizing farming, initially unrelated to biofuels. The first bill which ensured permanent price support to American farmers was the Grain Futures Act, which was passed in 1922 [21]. Mainly due to the strong agriculture lobby, the subsidies have been sustained and will probably remain in place, regardless of whether the crops are used for food or fuel. This is enabling American farmers to supply their crops (in particular corn) at a substantially lower price than for example Mexico or Canada. Unable to compete with government financed corn production, farmers in other countries lost their livelihood. The increase in the commodity prices may reinstate the profitability of outside US farming, leading to less hunger and not more. UK National Farmers Union's biofuels advisor, Jonathan Scurlock, voiced this opinion in a BBC article in 2007:

> *For years, Mexican dependency on cheap American corn had ruined the Mexican maize business and millions of farmers had left the land. Now Mexicans are starting to grow maize again. It is a slow process, but it will start to reduce their dependency on the north.*[6]

Even though recent studies suggest that a combination of high oil prices, poor harvests and financial investments in the commodity market probably had a considerably higher impact on food prices than biofuel production, food security remains a critical topic in the biofuels debate [3]. It is also worth mentioning that future biofuel production is predicted to mainly be based on cellulosic materials, and depend less on food crops [10], which might resolve some of the mentioned issues.

2.3.2 The lay of the land

During the last decade land use has become one of the most disputed subjects in the biofuel debate. The first concern was the use of arable land to produce energy crops rather than food. As the world population grows there will be more need for arable land for food production, and since biofuel production competes for the same land, this might lead to hunger due to lack of food production. However, this is not a large problem today as crops grown for biomass take up less than 2% of the world's arable land [19]. The farming industry also has a lot of potential for productivity increase, in particular in developing countries, so land competition is not likely to become an issue in the near future.

There are other more pertinent issues with regards to biofuel production, concerning the environmental impact from land use changes. If virgin land is converted to crop fields, a lot of the carbon originally stored in the soil will be released into the atmosphere. If the land also had other forms of vegetation, like a forest or even worse, a rain forest, the amount of carbon emitted from converting such land would be substantial. The emissions caused by land use changes can be converted to a carbon debt, measured in years it takes to revert back to status quo. Converting peatland in Indonesian or Malaysian rain forests to produce palm oil, yields a payback time of 423 years. American ethanol production from corn range from 48 years, when the original area was abandoned farmland with forest, to 93 years, when the original area was a central grassland. The best case scenarios are seen when marginally arable land or abandoned farmland without forest is used to produce biomass. The payback times for the two latter scenarios are less than 1 year, which means that we would get an almost immediate climate gain.[13][22]

The land use changes above are often denoted direct land use changes, as the areas in question are converted directly into biofuels feedstock production. Biofuels production might also lead to so-called indirect land use changes. One example of this would be an increase in the US biofuels production from corn, leading to less corn exported as food. As discussed in the previous section, this may induce an increase in the corn price and cause second round price effects on other food crops. Because margins now are higher, it might be profitable to convert land with high carbon density into farmland for food production. So even though the land used for energy crops was existing farmland, indirect effects might cause additional emissions significantly increasing the payback time.[23][13]

I do not dispute the fact that biofuels production may increase emissions due to land use changes, but I do question the assumption that virgin land would stay untouched if biofuels was not a part of the picture. There will always be some competition over land use, whether it is food production, cattle farming, infrastructure, settlement or biofuels production which is the intended use of the land. I definitely agree that we should take land use changes into consideration, but to get the correct picture we need to consider the same effects for all other projects requiring land. For example, if a farmer chooses to use more land to produce more food, instead of the more expensive alternative of increasing the efficiency on the current land, he should have to consider the effects of land use changes. Or if we were to build out a suburban area to lower the population density within a city, that project should be held up to the same standard as the biofuel industry. My only objection to including emissions from land use changes in the life cycle assessment of biofuels is that it will give a skewed picture of reality, unless it is included in all other land intensive projects as well.

When it comes to the indirect effects one should be slightly careful about inferring causality on the basis of correlation, as the driving forces behind land use changes are very complex. As Kline and Dale states it in Science Magazine 2008:

> *... field research, including a meta-analysis of 152 case studies, consistently finds that land-use change and associated carbon emissions are driven by interactions among cultural, technological, biophysical, political, economic, and demographic forces within a spatial and temporal context rather than by a single crop market.*[24]

Even though the size of the land use effects are still debated, it seems to be broad agreement about the importance of taking land use changes into account when assessing the environmental effect from increased biofuel production.

2.4 Concluding Remarks

2.4.1 Separate the wheat from the chaff

It is not clear what the effects from increasing the production and use of biofuel will be. Some of the factors mentioned above indicates that it might actually harm the environment to expand the biofuel industry. The scattered results and opinions are mainly due to the fact that biofuel is not a homogeneous product group. From a consumer's perspective it might seem homogeneous, but from an environmental standpoint there are clear, distinctive differences. For example, biofuel made from corn and biofuel made from sugar canes are both first generation biofuels and often reviewed collectively. However, the possible emission reduction (land use changes excluded) from corn is 14% compared to fossil fuels, while sugar canes can reduce emissions by 86% [13]. Extreme differences can also be seen when comparing carbon debt and payback times for different biofuels types, which ranges from no payback time to more than 400 years. I believe that by not clearly differentiating between the types of biofuels, one can never come to an agreement about best practices. If we want to construct policies which enables sustainable development we need to start separating the wheat from the chaff.

Chapter 3

The Next Top Model

In this model only three different fuel options are available; fossil fuel and two types of biofuels. The three fuel types are perfect substitutes, but differ in production costs and environmental impact. Fossil fuel, R_t, is extracted from a non-renewable stock, $S_t \geq 0$, such that the change in the resource stock is given by the gross extraction,

$$\dot{S}_t = -R_t. \tag{3.1}$$

As the oil gets depleted it is necessary to extract from less accessible areas like deep water, or use unconventional techniques as extraction from oil sand. The technology constraint is captured in the stock dependent unit cost of extraction, increasing rapidly as the stock gets depleted: $c = c(S_t)$: , $c'(S_t) < 0$, $c''(S_t) > 0$ and $c(S_t) \to \infty$ as $S_t \to 0$. It is not feasible to achieve complete depletion of all the remaining reservoirs, and in this model it will be the increasing costs, and not the physical depletion, which will be the binding constraint.

Biofuel production is modeled as harvest of available vegetation. I will assume that only two types of crops, or input factors, are suitable for biofuels production. V_t^s, the "slow crop", have a low effective growth rate like wood from boreal forest or palm oil. The "fast crop", V_t^f, represents biofuels produced in a way which allows for more immediate environmental gain, like sugar canes or waste materials from wood or food crops. The natural growth of each crop can thus be interpreted as the average depletion rate of atmospheric carbon through growth in the crops. In real life there will be a continuum of crops with different payback times, but the main characteristics can be studied using this simplified model. Alternative land use will not be included in the model, but I assume that we are looking at suitable areas for biofuels production, such that the climate effects from land conversions are negligible compared to the direct emissions from producing and using biofuels.

In contrast to traditional forest models where the crops are only harvested after a completed rotation period, the harvesting in this model is continuous. This can be viewed as an aggregation of many areas with "rotation period" harvesting, being at different stages in the rotation period. When the number of areas gets large, the total harvest tends to a continuous function in time.

The natural growth functions will also need to be consistent with aggregation of crop areas. Picture a large population of trees of different ages, divided into smaller areas, where each area consists of trees with the same age. Assuming growth only depends on volume, each of the trees within one area has the same growth rate. If you let the areas develop without interruption, the total volume stored in vegetation will tend to a steady state, as the crop areas reach their carrying capacity. It does not matter for the growth function if the crops in the different areas have different ages or not, as we are only looking at the average growth rate across all the areas. The main reason for the aggregation argument is to allow for continuous harvest, which is easier to integrate into models with continuous time than the "rotation period" approach.

The hypothesis is that it is possible to use a population growth model, like the logistic growth model [25], to describe the crop growth. I will not constrain the model to a specific function, but assume the following more general properties: For $i = \{s, f\}$, $f_i(V_t^i)$: $f_i'(V_t^i) > 0$, whenever $V^i < V_{MSY}^i$, where

V^i_{MSY} is the maximum sustainable yield for crop i, i.e. $f_i'(V^i_{MSY}) = 0$ and $f_i(V^i_{MSY}) = max(f^i(V^i))$. Otherwise $f_i'(V^i_t) \leq 0$. In both cases $f_i''(V^i_t) < 0$ for all V^i_t. In addition $f_i(0) = f_i(\bar{V^i_t}) = 0$, which means that without harvest the two crops will stabilize at levels $\bar{V}^s$ and $\bar{V}^f$ respectively, corresponding to the maximum volumes of the two crops. The growth of the slow crops are assumed to be substantially slower than the fast crops, $f_s(V_t) << f_f(V_t)$, whenever $f'(V^f_t) > 0$. A simple sketch of two possible growth functions are given below.

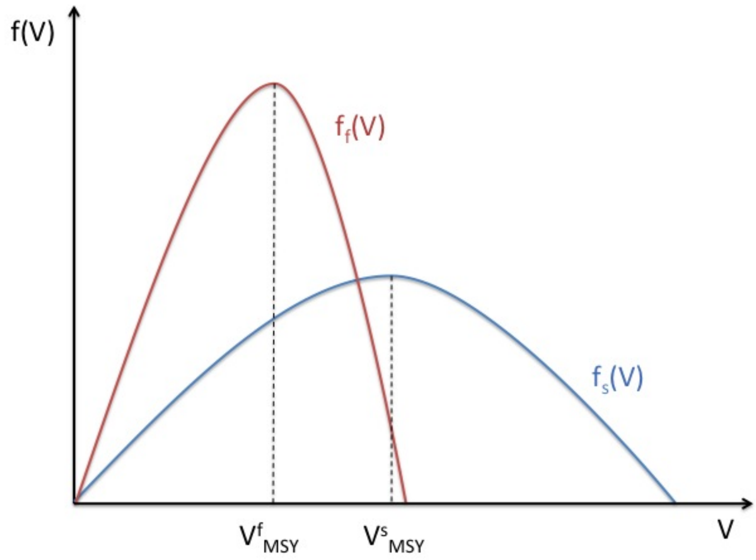


Figure 3.1: Growth of the two crops as a function of volume.

The change in the volume of the two crops will then be given by:

$$\dot{V}^s_t = f_s(V^s_t) - H^s_t \tag{3.2}$$
$$\dot{V}^f_t = f_f(V^f_t) - H^f_t, \tag{3.3}$$

where H^s_t and H^f_t represent the harvest of the respective crops at time t. The upper limit for sustainable production of biofuels might be lower than its potential value, due to immature technology or expensive production methods. This is captured through increasing marginal costs of harvest: $b_i = b_i(H_t^i) : b'(H_t^i) > 0,\ b''(H_t^i) \geq 0$.

I will follow the recommendations by David Archer when modeling atmospheric carbon and its decay. In his article "Fate of fossil fuel CO2 in geologic time", he states that "A better approximation of the lifetime of fossil fuel CO2 for public discussion might be "300 years, plus 25% that lasts forever.""[26]. I will capture this by dividing the atmospheric carbon into two repositories, A_1 and A_2, as done by Farzin and Tahvonen [27]. 75% of the emissions will go into A_1, which has a corresponding depletion rate α. The other 25% will end up in reservoir A_2, which has no intrinsic depletion rate. However, the growth of the crops will capture and store carbon from both reservoir. With total emissions given by E_t, the atmospheric carbon changes according to the equations below.

$$\dot{A}_t = \dot{A}_t^1 + \dot{A}_t^2 \quad \text{where} \tag{3.4}$$

$$\dot{A}_t^1 = \frac{3}{4}(E_t - f_s(V_t^s) - f_f(V_t^f)) - \alpha A_t^1 \tag{3.5}$$

$$\dot{A}_t^2 = \frac{1}{4}(E_t - f_s(V_t^s) - f_f(V_t^f)). \tag{3.6}$$

There will always be energy costs connected with any fuels production. Energy used in production would make less of the fuel available for consumption, but the demand would also be lowered due to increased costs. To simplify the model I will assume that energy costs are zero for both fuel types, as including these costs does not shed new light on the problems to be discussed. This way gross and net fuel values are equal, and R_t, H_t^s and H_t^f represent the fuel available for consumption, assuming a linear production function. The total net fuel production is denoted $F_t = R_t + H_t^s + H_t^f$, and will equal the total emission E_t when the units are adjusted. The social benefit of fuel meets the standard conditions for utility functions: $B = B(F_t) : B'(F_t) > 0$ and $B''(F_t) < 0$. The environmental damage from atmospheric carbon is assumed to be strictly increasing: $D = D(A_t) : D'(A_t) > 0$ and $D''(A_t) \geq 0$.

3.1 Man With a Plan

The net social welfare is given by the benefits from fuel consumption, subtracted production costs and the damage of atmospheric carbon,

$$U_t = B(F_t) - c(S_t)R_t - b_s(H_t^s) - b_f(H_t^f) - D(A_t).$$

The social planner is seeking to find the harvest and extraction paths which will maximize the discounted social welfare across all time periods.

$$\max_{\{R_t\},\{H_t^s\},\{H_t^f\}} \int_0^\infty U_t e^{-\rho t} dt$$
$$\text{subject to } (3.1), (3.2), (3.3), (3.5) \text{ and } (3.6).$$

Using classic optimal control theory, I construct the current value Hamiltonian and derive the corresponding first order conditions for an interior optimum:

$$\begin{aligned}\mathcal{H}_t =& B(F_t) - c(S_t)R_t - b_s(H_t^s) - b_f(H_t^f) - D(A_t) + \kappa_t[-R_t] + \\ & \eta_t^s[f_s(V_t^s) - H_t^s] + \eta_t^f[f_f(V_t^f) - H_t^f] + \\ & \upsilon_t^1[\frac{3}{4}(R_t + (H_t^s + H_t^f) - f_s(V_t^s) - f_f(V_t^f)) - \alpha A_t^1] + \\ & \upsilon_t^2[\frac{1}{4}(R_t + (H_t^s + H_t^f) - f_s(V_t^s) - f_f(V_t^f))],\end{aligned} \tag{3.7}$$

$$\begin{aligned}\frac{\partial \mathcal{H}}{\partial R_t} =& B'(F_t) - c(S_t) - \kappa_t + \upsilon_t^1\frac{3}{4} + \upsilon_t^2\frac{1}{4} = 0 \\ \frac{\partial \mathcal{H}}{\partial H_t^s} =& B'(F_t) - b_s'(H_t^s) - \eta_t^s + \upsilon_t^1\frac{3}{4} + \upsilon_t^2\frac{1}{4} = 0 \\ \frac{\partial \mathcal{H}}{\partial H_t^f} =& B'(F_t) - b_f'(H_t^f) - \eta_t^f + \upsilon_t^1\frac{3}{4} + \upsilon_t^2\frac{1}{4} = 0.\end{aligned}$$

The υ_t^j-values will always be negative as they represent the value of adding more carbon into the atmosphere. The negative sum of the two υ_t^i terms will thus represent the social cost of increasing the level of carbon in the atmosphere. I will denote this $\tau_t = -(\frac{3}{4}\upsilon_t^1 + \frac{1}{4}\upsilon_t^2)$, giving a more compact version of the first order conditions,

$$B'(F_t) = c(S_t) + \kappa_t + \tau_t \tag{3.8}$$
$$B'(F_t) = b_s'(H_t^s) + \eta_t^s + \tau_t \tag{3.9}$$
$$B'(F_t) = b_f'(H_t^f) + \eta_t^f + \tau_t. \tag{3.10}$$

The first order conditions (FOCs) have a well known economic interpretation: The marginal benefit of increasing fuel consumption must equal the marginal cost of increasing any of the fuel types. The cost of fossil fuel depends on the real unit cost, $c(S_t)$, the resource rent, κ_t, and the cost of atmospheric carbon, τ_t. The production costs of the two biofuel types depend on their respective real marginal costs, $b_i'(H_t^i)$, the shadow price of standing crops, η_t^i, in addition to the cost of carbon. Since all the fuel types are perfect substitutes, their marginal benefits are equal. To ensure efficiency, the volume consumed of each fuel must also be such that the marginal costs are equal across all three fuels, described by $c(S_t) + \lambda_t = b_i'(H^i) + \eta_t^i$.

The time development of the system is governed by the equations of motion, that is, the time development of the shadow prices. For $i = \{s, f\}$,

$$\dot{\kappa}_t - \rho\kappa_t = c'(S_t)R_t \tag{3.11}$$
$$\dot{\eta}_t^i - (\rho - f_i'(V_t^i))\eta_t^i = -\tau_t f_i'(V_t^i) \tag{3.12}$$
$$\dot{\upsilon}_t^1 - (\rho + \alpha)\upsilon_t^1 = D'(A_t) \tag{3.13}$$
$$\dot{\upsilon}_t^2 - \rho\upsilon_t^2 = D'(A_t). \tag{3.14}$$

The corresponding transversality conditions are necessary to ensure an internal solution of the system,

$$\lim_{t\to\infty} e^{-\rho t}\kappa_t = 0$$
$$\lim_{t\to\infty} e^{-\rho t}\eta_t^i = 0$$
$$\lim_{t\to\infty} e^{-\rho t}\upsilon_t^j = 0.$$

Combining the equations above yields

$$\tau_t = -\upsilon_t = \int_t^{\infty} (1 + 3e^{\alpha(t-t')})\frac{1}{4}e^{\rho(t-t')}D'(A_t')dt' \tag{3.15}$$
$$\kappa_t = -\int_t^{\infty} e^{\rho(t-t')}c'(S_t)R_t dt'. \tag{3.16}$$

The cost of carbon (3.15) is only dependent on the marginal damage of carbon, which is positive by assumption. This leads to the conclusion that all carbon emitted into the atmosphere give the same environmental costs, regardless of whether the carbon source is fossil fuel or one of the biofuels. The social cost of carbon reflects the damage today, as well as all future damages, of adding carbon into the atmosphere. If one unit is emitted at time t, the direct damage is given by $D'(A_t)$. If no more carbon is emitted in the future, the part stored in repository 2 will give a future discounted damage of $\frac{1}{4}e^{\rho(t-t')}D'(A_t)$, for all future times t'. In repository 1, there is also a depletion rate, so the future damage will be $\frac{3}{4}e^{(\rho+\alpha)(t-t')}D'(A_t)$. Summing up the combined damage over all times $\tau \geq t$ yields the expression in equation (3.15).

The resource rent κ_t is a reflection of the added cost of extracting oil, due to the scarcity of the resource. If you extract one unit of oil today, the immediate result is that the unit cost of oil will increase as the stock of oil decreases, making future extractions more costly. The total effect of a unit extraction today is more complex as it depends on the extraction in all future periods. The main effect is that the efficient marginal cost of extraction becomes higher than the real marginal cost when scarcity is taken into account.

It is not possible to obtain an analytical expression for η_t^i, as the effective discount rate $\rho - f'(V_i)$ is not constant. But by studying (3.12) closer, it is still possible to give the shadow price a meaningful interpretation. If you divide $\dot{\eta}_t^i$ into three terms, it is easier to see what effects are in play. $\rho\eta_t^i$ represents the necessary adjustment in η_t^i to keep up with peoples impatience or discounting. The second term $-f_i'(V_t^i)\eta_t^i$ takes into account the change in the growth rate when the volume changes, which will greatly influence the volume in the subsequent periods. The last term, $-\tau_t f_i'(V_t^i)$, includes the environmental impact, which arise due to changes in the efficient depletion rate.

3.1.1 Steady state solution

The steady state solution is characterized by $\dot{S}_t = \dot{V}_t^i = \dot{A}_t^j = \dot{\kappa}_t = \dot{\upsilon}_t = \dot{\eta}_t^i = 0$. This removes all time-dependence, and we get the long-run or steady state values of the variables. For simplicity I will choose $D'(A_t) = a \rightarrow -\upsilon = \tau =$ constant, that is, the damage of adding one more unit of atmospheric carbon is independent of the current level of carbon in the atmosphere. All steady state values are constant, indicated by removal of the time index. The equation set describing the steady state is given by: For $i = \{s, f\}$,

$$R = 0 \tag{3.17}$$

$$A^1 = 0 \tag{3.18}$$

$$A^2 = A \tag{3.19}$$

$$H^i = f_i(V^i) \tag{3.20}$$

$$B'(F) \leq c(S) + \kappa + \tau \tag{3.21}$$

$$B'(F) = b_i'(H^i) + \eta^i + \tau \tag{3.22}$$

$$\rho\kappa = -c'(S)R \tag{3.23}$$

$$(\rho - f_i'(V^i))\eta^i = f_i'(V^i)\tau \tag{3.24}$$

$$\tau = \frac{3}{4}\frac{a}{\rho + \alpha} + \frac{1}{4}\frac{a}{\rho}. \tag{3.25}$$

These 12 equations give the steady state solutions for the 12 endogenous variables $S,\ R,\ V^s,\ V^f,\ H^s,\ H^f,\ A^1,\ A^2,\ \kappa,\ \eta^s,\ \eta^f$, and τ.

The steady state solution for R is straightforward. Since fossil fuel is an exhaustible resource, it is not possible to have positive extractions in the long run, which means that we must have $R = 0$. From (3.23) we then see that the resource rent, κ, will also be zero in steady state.

Even though we can say with certainty that extraction will tend to zero, this is not the case for the stock of oil, S. Classical Hotelling models without (or with constant) extraction costs will always yield complete exhaustion of a scarce resource. However, when environmental damages and increasing marginal costs of extraction are included, we have two strong effects pulling towards zero extraction before the resource is depleted. We know that the extraction costs tend to infinity, as the stock gets depleted ($c(S) \to \infty$ as $S \to 0$). This means that the steady state level S must be strictly positive, even without taking the environmental damage into account. Including the environmental effects, represented by the cost of carbon, τ, will increase the amount of unutilized oil. $R = 0$ being a corner solution, this results in the wider constraint that the price should be *less* or equal to the marginal costs of fossil fuel in steady state. However, from an economical standpoint we know that it will never be optimal to extract the last unit of oil if the cost is higher than the price. This implies that it is optimal to stop extracting at the exact point where the next unit will make the cost higher than the price, meaning that in steady state, the price should equal the cost.

The two repositories of atmospheric carbon also has fairly trivial steady state solutions, by the looks of it. From (3.6) we see that any level of emissions exceeding the growth rate in the long run, would cause A^2 to grow for ever. This can clearly not be optimal as an ever growing atmospheric carbon stock would imply that the damage would tend to infinity. The fact that 25% of all emissions will always remain in the atmosphere implies that there can be no efficient emissions in the long run, that is, $E - f_s(V^s) - f_f(V^f) = 0$. It then follows that the steady state level of carbon in repository 1 needs to be zero, to ensure a convergent steady state solution for repository 2. Another implication of this is that without biofuels, and the natural growth of the crops, it would not be possible to have any sustainable fuel consumption in the long run.

The cost of atmospheric carbon depends on the social discount rate ρ, the depletion rate α, and the damage of atmospheric carbon, a. The first part $\frac{a}{\rho+\alpha}$ accounts for the damage of adding one more unit of carbon into repository 1, while $\frac{a}{\rho}$ embodies the cost of adding more carbon to repository 2. The cost of carbon in repository 2 is higher than the cost of carbon in repository 1, because the depletion rate reduces the damage over time in a similar manner as the discount rate. The weighted sum of these costs reflects the fact that when you increase emissions by one unit, 75% ends up in repository 1 and 25% ends up in repository 2, yielding a total cost of τ.

The steady state harvest of each crop will equal the net natural growth, which will typically be different for the two fuels. It is possible to have one or both of the crops at their maximal volumes, which means that $f_i(V^i) = H^i = 0, \eta^i = 0$. There are many different possibilities for the steady state solutions for η_i and V^i, depending on the underlying assumptions and the specific functions involved. This will be thoroughly discussed in section 3.3.

3.2 Bring Me the Producer

To be able to make any inferences about optimal use of fiscal policy instruments, we need to look at the market side of the model. I will assume a free trade market with perfect information and no cost of trade. There are many small, identical, profit maximizing firms, where no single firm has any market power. The price, $p_{m,t}$, is considered to be exogenous by each producer, even though it is endogenously determined in the market equilibrium. Since the fuels are perfect substitutes and the market is competitive, the three fuel types must have the same price. The real costs facing the firms will be the same as in the social planner problem, but the benefits are now represented through gross revenue. The assumptions above allow for aggregation of the identical firms into one representative firm, from now on called 'the producer', producing all three fuels. From the producer's point of view there is no difference between the fuel types, other than the production costs. The production function will in this case be trivial, as it equals the sum of the three input factors, F_t. The representative producer's profit function is then given by:

$$\Pi_t = p_{m,t}F_{m,t} - c(S_{m,t})R_{m,t} - b_s(H^s_{m,t}) - b_f(H^f_{m,t}). \tag{3.26}$$

The producer does not care about the environment, thus the growth in the carbon stock is not included in the market model. The growth functions of the crops and the stock of oil are unchanged,

$$\dot{S}_{m,t} = -R_{m,t} \tag{3.27}$$

$$\dot{V}^s_{m,t} = f_s(V^s_{m,t}) - H^s_{m,t} \tag{3.28}$$

$$\dot{V}^f_{m,t} = f_f(V^f_{m,t}) - H^f_{m,t}. \tag{3.29}$$

The representative firm is maximizing discounted profits over all times, by adjusting the input factors $R_{m,t}$, $H^s_{m,t}$, and $H^f_{m,t}$. I will assume that the market interest rate is equal to the social discount rate, ρ. This yields the producer's optimization problem:

$$\max_{\{R_{m,t}\},\{H^s_{m,t}\},\{H^f_{m,t}\}} \int_0^\infty \Pi_t e^{-\rho t} dt$$
$$\text{subject to } (3.27), (3.28) \text{ and } (3.29).$$

The current value Hamiltonian is constructed in the same way as before. It is important to keep in mind that the new shadow prices will generally be different from the shadow prices seen in the previous model. I have emphasized this by giving the market shadow prices different name than the corresponding prices in the social planner model. The current value Hamiltonian for the market problem then yields:

$$\begin{aligned}\mathcal{H}_{m,t} =& p_{m,t}F_t - c(S_{m,t})R_{m,t} - b_s(H^s_{m,t}) - b_f(H^f_{m,t}) + \\ & \lambda_t[-R_{m,t}] + \mu^s_t[f_s(V^s_{m,t}) - H^s_{m,t}] + \mu^f_t[f_f(V^f_{m,t}) - H^f_{m,t}].\end{aligned}$$

Differentiating the Hamiltonian with respect to the three input factors yields the producer's first order conditions for an interior optimum:

$$p_{m,t} = c(S_{m,t}) + \lambda_t \tag{3.30}$$
$$p_{m,t} = b'_s(H^s_{m,t}) + \mu^s_t \tag{3.31}$$
$$p_{m,t} = b'_f(H^f_{m,t}) + \mu^f_t. \tag{3.32}$$

As for the social planner solution, the FOCs show that the marginal benefit, i.e. the price, equals the marginal costs in optimum. The marginal cost of fossil fuel consists of the real production cost, $c(S_{m,t})$, plus the resource rent, λ_t. The marginal cost of the two biofuels consists of the real cost of harvest, $b'_i(H^i_{m,t})$, in addition to the shadow price of standing crops, μ^i_t, reflecting the changes in future yield when the volume, and thus the growth rate, changes. The only difference compared to the social planner FOCs is the exclusion of the carbon cost, τ_t, from all three equations.

The shadow prices develop according to: For $i = \{s, f\}$ and $j = \{1, 2\}$,

$$\dot{\lambda}_t - \rho\lambda_t = c'(S_{m,t})R_{m,t} \tag{3.33}$$

$$\dot{\mu}^i_t - (\rho - f'(V^i_{m,t}))\mu^i_t = 0, \tag{3.34}$$

with restrictions given by the transversality conditions:

$$\lim_{t\to\infty} e^{-\rho t}\lambda_t = 0$$
$$\lim_{t\to\infty} e^{-\rho t}\mu^i_t = 0.$$

Combining the equations above gives the market resource rent:

$$\lambda_t = -\int_t^{\infty} e^{\rho(t-t')}c'(S_{m,t})R_{m,t}dt'. \tag{3.35}$$

If we compare the equation for λ_t (3.35) with the corresponding social planner rent κ_t (3.16), we see that the two expressions are identical. However, this does not mean that the two shadow prices would be equal at any point in time. As long as the resource rent depends on the volume of the remaining stock and the extraction of oil, the social resource rent and market resource rent will only be identical when the two extraction paths (which uniquely determines the volume) are identical. How to achieve this will be discussed in Chapter 4, where the optimal policy is derived.

As in the social planner model, it is not possible to find an analytical expression for μ^i_t. Looking back at equation (3.12), the most noticeable difference is the absence of the term linking the volume of standing crops to the change in atmospheric carbon $(-\tau f'_i(V^i_t))$. The effect of taking the environment out of the equation (a little pun intended) is ambiguous, as it will depend on the standing volume of crops in both models.

3.2.1 Steady state market solution

We obtain the market steady state solution by setting: $\dot{S}_{m,t} = \dot{V}^i_{m,t} = \dot{\lambda}_t = \dot{\mu}^i_t = 0$ This removes all time-dependence, and the steady state values of the variables are given by the following conditions: For $i = \{s, f\}$,

$$R_m = 0 \tag{3.36}$$
$$H^i_m = f_i(V^i_m) \tag{3.37}$$
$$p_m \leq c(S) + \lambda \tag{3.38}$$
$$p_m = b'_i(H^i_m) + \mu^i \tag{3.39}$$
$$c'(S)R = -r\lambda \tag{3.40}$$
$$(r - f'(V^i_m))\mu^i = 0. \tag{3.41}$$

These 9 equations can be solved to obtain the steady state values of the 9 endogenous variables: S_m, R_m, V^s_m, V^f_m, H^s_m, H^f_m, λ, μ^s, and μ^f.

There are many similarities with the social planner solution. For instance, no fossil fuel will be used in the fuel production, $R_m = 0$, and the resource rent will be zero, $\lambda = 0$. The boundary solution results in the price being less or equal to the fossil fuel costs. By the same argument used for the social optimum steady state, we can conclude that the producer will stop extracting oil when the price equals the costs, as the next unit of extraction would result in a loss. Since the cost facing the producer is lower than the social cost of fossil fuel, we know that the remaining stock of oil will be less than the socially optimal one. The market steady state level of atmospheric carbon will be given by the integral of:

$$\dot{A}_{m,t} = \dot{A}^1_t + \dot{A}^2_t, \quad \text{where} \tag{3.42}$$
$$\dot{A}^1_t = \frac{3}{4}(E_{m,t} - f_s(V^s_{m,t}) - f_f(V^f_{m,t})) - \alpha A^1_{m,t} \tag{3.43}$$
$$\dot{A}^2_t = \frac{1}{4}(E_{m,t} - f_s(V^s_{m,t}) - f_f(V^f_{m,t})). \tag{3.44}$$

Here we have $E_{m,t} = R_{m,t} + H^s_{m,t} + H^f_{m,t}$, meaning that A_m will typically be different from A in the social planner solution. If fossil fuel bore the sole responsibility for emissions, we would know that the market level of atmospheric carbon would be higher than the socially optimal level. However, since including biofuels can lead to more or less emission, we cannot conclude that this will be the case (though it is more likely).

An important difference between the two steady state solutions are the shadow price-volume interactions for the two crops. In the social planner solution (3.24) the value of one more unit of standing crops will only equal zero if the volume is at the MSY level, i.e. $f_i'(V_m^i) = 0$. From equation (3.41) we see that $\mu_i = 0$ whenever the growth rate is different from the discount rate. The volumes, harvest paths, and the shadow prices of the two crops will be determined from (3.37), (3.39) and (3.41), yielding several possible solutions, which will be studied in the next section.

3.3 The Yellow Brick Road

In this section I will take a closer look at the optimal steady state for the volume and the shadow price of the standing crops, and possible saddle paths leading to these steady states. I will compare the results from the social planner solution and the market solution, making use of phase diagrams to display the dynamics.

To simplify the illustrations I will focus only on one of the biofuel types. Later, I will briefly discuss the implications of having two types of crops. Each phase diagram displays the steady state values of the volume and the shadow price of the standing crops, for a given case scenario. The saddle paths leading to the steady states are indicated, though only as rough "guesstimates" as the specific function forms are not specified.

To be able to discuss the dynamic properties, I have made some simplifications. When the system has reached the steady state, the fossil fuel production will be zero, so only the biofuel production will affect the price. However, when talking about the saddle path, this will generally not be the case. To be able to clearly display the interactions between the shadow price and volume of crops, I will disregard the interaction with fossil fuel in the discussions below.

3.3.1 Social planner solutions

To construct the phase diagrams I need to find the conditions ensuring that $\dot{\eta} = 0$ and $\dot{V} = 0$. The price equation can be found directly from the steady state solution (3.24), while the steady state volume is characterized by the harvest being equal to the growth rate of the crops (3.20). Combining this with the FOC

(3.22), we have both the $\dot{\eta} = 0$ and the $\dot{V} = 0$ loci,

$$\eta = \frac{\tau f'(V)}{\rho - f'(V)} \text{ for } f'(V) \neq \rho \tag{3.45}$$

$$\eta = B'(f(V)) - b'(f(V)) - \tau. \tag{3.46}$$

As seen from the equation above, η is not defined for $\rho = f'(V)$, and I will denote this limit volume V^ρ. Looking at the derivative of the $\dot{\eta} = 0$ curve we find $\frac{\partial}{\partial V}(\frac{\tau f'(V)}{\rho - f'(V)}) = (\frac{\rho\tau}{(\rho - f'(V))^2})f''(V)$. Remembering that $f''(V) < 0$, we can conclude that η is a decreasing function of volume. Between V^ρ and V_{MSY} the growth rate of the crops is increasing. Disregarding impatience this means that it will be beneficial, from an environmental standpoint as well as a maximum yield standpoint, to increase the volume. This indicates that the value of standing crops should be positive in this interval. The curve will cross $\eta = 0$ in V_{MSY}, and continue to decrease as the growth rate further declines. The value of standing crops will then be negative when the growth rate is decreasing, since harvesting above the growth rate will actually increase the growth rate in this region. For any volumes $V < V^\rho$, we have $f'(V) > 0$ and $\rho - f'(V) < 0$. From (3.45) we can then affirm that the $\dot{\eta} = 0$ curve is strictly negative in this area. If we start from $V = 0$ and increase V gradually, we see that the $\dot{\eta} = 0$ locus is declining asymptotically towards $V = V^\rho$. This means that η is discontinuous in $V = V^\rho$, as the curve tends to $+\infty$ when approaching V^ρ from above and $-\infty$ when approaching from below.

For all $V < V^\rho$ we have $\rho - f'(V) < 0$, and $f'(V)$ is decreasing towards V^ρ. Starting from $\dot{\eta} = 0$, and knowing that $\eta = 0$, we see that increasing V will lead to $\dot{\eta} = \tau f'(V) - (\rho - f'(V))\eta > 0$. For $V^\rho < V < V_{MSY}$, we observe that $\rho - f'(V)$, η, and $f'(V)$ are all positive. Here too, an increase in V, starting from $\tau f'(V) = (\rho - f'(V))\eta$, will lead to $\dot{\eta} > 0$, i.e η is increasing to the right of the curve. In the last region, $V > V_{MSY}$, we have $\eta < 0$, $\rho - f'(V) > 0$ and $f'(V) < 0$. As before, we start from $\dot{\eta} = 0$ and increase V, which leads to the same result as in the previous two cases: η is increasing to the right and decreasing to the left of the $\dot{\eta} = 0$ locus.

The biofuel cost function is by assumption monotonically increasing in harvest, that is, $b' > 0$ for all levels of H. The harvest is uniquely determined by the steady state volume, $H = f(V)$. Since $b'' \geq 0$ and $B'' \leq 0$, the slope $\frac{\partial}{\partial V}(B'(f(V)) - b'(f(V)) - \tau) = (B'' - b'')f'(V)$ will always have the opposite sign of $f'(V)$, and the minimum value of η will coincide with the maximum of $f(V)$ at V_{MSY}. For low volumes as well as high volumes the cost of producing biofuels is low, which means that the potential marginal social profit of increasing the production would be high. The only reason not to raise the production of biofuel at these points is if the value of standing crops is high. η is thus a measure of the alternative social profit of biofuel production as the well as the value of standing crops.

To find in what regions the volume grows and declines, it is good to start with the first order condition determining the harvest, $B'(H) = b'(H) + \eta + \tau$. The benefits, and thus the harvest, will decrease when any of the marginal costs, η or τ, increase. Rewriting the first order condition then gives, $H = H(\eta + \tau)$, where $H' < 0$. Using this equation in the growth equation for the volume yields $\dot{V} = f(V) - H(\eta+\tau)$. Starting from the $\dot{V} = 0$ curve and increasing η marginally will lead to a decrease in the harvest and thus an increase in the growth rate of the crops. Thus V is increasing above the $\dot{V} = 0$ locus, and decreasing below.

In the following paragraphs "*" indicates a steady state value, given by the intersection of the $\dot{V} = 0$ and $\dot{\eta} = 0$ curves. Initial values will be marked with "$_0$".

I will start by looking at the case where the social cost of biofuel is low, meaning that $B' > b' + \tau$ for all volumes V (figure 3.2). That is, both the real marginal costs b' and the environmental costs τ must be low, such that the total marginal cost is less than the price ($p = B'$). When it is cheap to produce biofuel, it will be optimal to choose a steady state volume that ensures a high production volume of biofuel. The highest possible steady state biofuel production is obtained when $V = V_{MSY}$, but due to discounting, the steady state volume will end up strictly less than V_{MSY}. If $V_0 < V^*$ the value of standing crops starts at a higher level than the marginal social profit ($\eta_0 > B'(f(V_0)) - b'(f(V_0)) - \tau_0$). It is then optimal to harvest below the growth rate, as this will lead to an increase in both the volume and the growth rate of the standing crops. Along the saddle path η is decreasing, and the volume will continue to increase until the steady state is reached. A similar argument can be used when $V_0 > V^*$, but then we have the opposite movements in the variables. The closer the minimum point of the $\dot{V} = 0$ curve is to zero, the closer the steady state volume gets to V_{MSY}. In the low cost case η^* will always be positive, as the $\dot{\eta} = 0$ locus is always above zero between V^ρ and V_{MSY}.

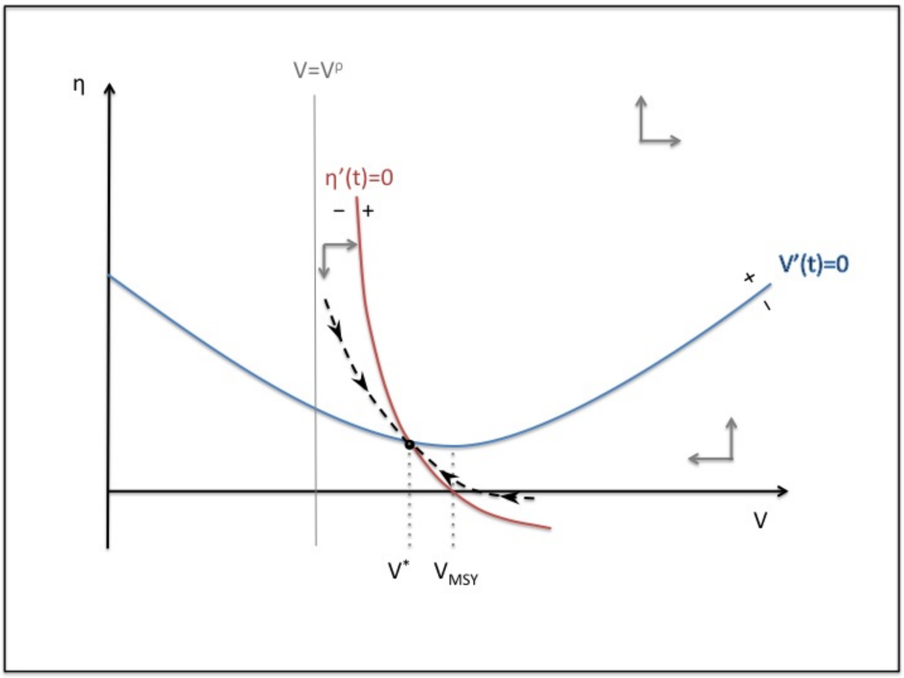


Figure 3.2: The path to steady state with low social costs of biofuel.

Moderate social cost of biofuel is characterized by the $\dot{V} = 0$ curve being both above and below zero, implying that there exists at least one point where $B' = b' + \tau$ (figure 3.3). One scenario yielding this result would be if the real costs were small, but the environmental costs (τ) were close to the price. If the initial volume was lower than the steady state volume[1], we would then get a path which

[1] I am assuming that there is only one equilibrium point in this case, even though it is possible to get multiple equilibria if the $\dot{V} = 0$ curve cross the $\dot{\eta} = 0$ curve on the left hand side of V^ρ.

gradually tends towards higher volumes, as in the low cost case. The difference here is that the higher environmental cost makes it less profitable to produce biofuel, and the system is then pushed towards a higher steady state volume than in the low cost case. The steady state volume will now be to the right of V_{MSY}, and the corresponding η will be negative.

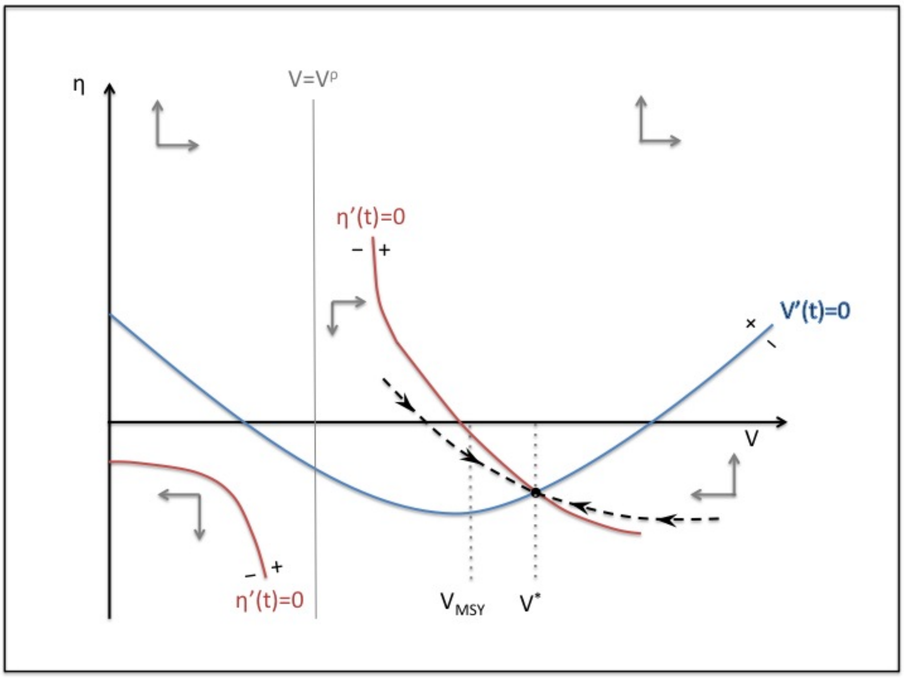


Figure 3.3: The path to steady state with moderate social costs of biofuel.

The last case deals with high social cost of biofuel, $B' < b' + \tau$, which means that the $\dot{V} = 0$ curve is below zero for all volumes V (figure 3.4). The social cost of producing biofuel is high if either the environmental costs (τ) or the real cost (b'), or both, are high. Depending on how high these costs are, there might be one or more equilibria. Figure 3.4 displays a case with three unique equilibria, and figure 3.5 shows a close-up of the saddle path leading to V^*_{low}. The common feature of all the equilibria is that the steady state harvest will be strictly less than in the two previous cases. This is not very surprising, as it is beneficial to produce biofuel when the social costs are high.

The initial volume V_0 will determine which of the steady states that will be realized. If $V_0 < V^\rho$ [2] the steady state volume will be V^*_{low}, while with $V_0 > V^\rho$ we end up in V^*_{high}. In both cases the steady state value of η is strictly negative. The last steady state is unstable, with no saddle path leading to it, so ending up in this point is deemed unfeasible.

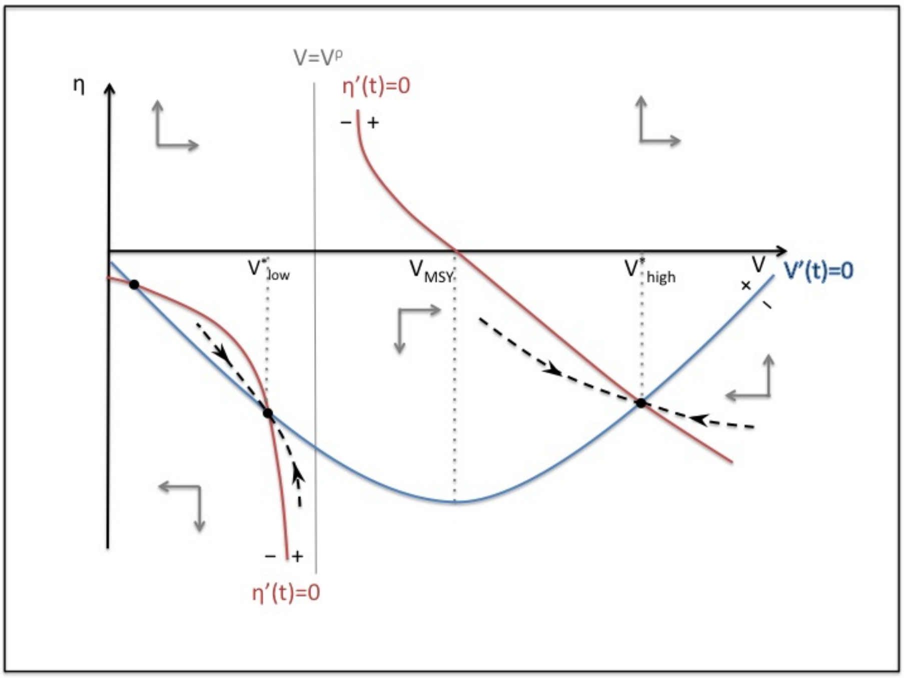


Figure 3.4: The path to steady state with high social costs of biofuel.

[2]Assuming that V_0 is larger than the unstable equilibrium volume. If V_0 was less than the unstable equilibrium volume, we would end up in a boundary solution with $H = V = 0$

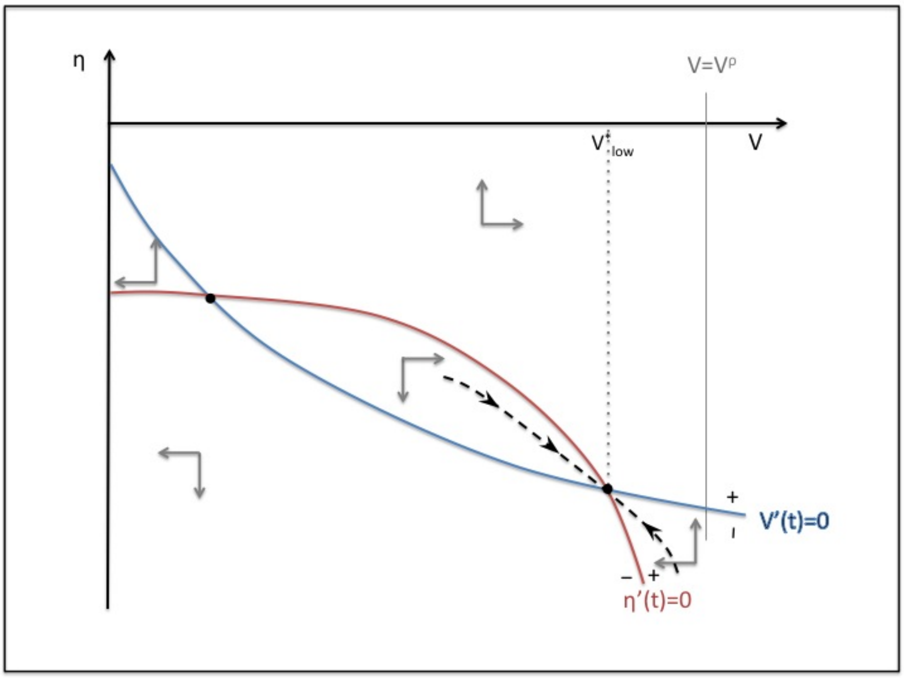


Figure 3.5: The path to steady state with high social costs of biofuel. Close-up of the low volume equilibrium.

3.3.2 Market solutions

In the same manner as for the social planner solution, I will use the steady state solutions (3.41), (3.39) and (3.37) to construct the $\dot{\mu} = 0$ and $\dot{V}_m = 0$ loci, which gives the following conditions,

$$(\rho - f'(V_m)\mu = 0 \tag{3.47}$$

$$\mu = p_m - b'(f(V_m) \quad \text{where} \quad p_m = B'(f(V_m)). \tag{3.48}$$

The $\dot{\mu} = 0$ locus is notably different from the corresponding social planner solution. There are two conditions which satisfy the constraint (3.41), resulting in two separate curves, $f'(V^\rho) = \rho$ and $\mu = 0$. If $f'(V^\rho) = \rho$ the volume is determined by the $\dot{\mu} = 0$ locus alone, but the same equation gives no information about the level of μ. The shadow price is then determined by the $\dot{V}_m = 0$ constraint. If $\mu = 0$, the $\dot{\mu} = 0$ locus gives no information about the volume, which is now governed by the $\dot{V}_m = 0$ curve. The two curves $f'(V^\rho) = \rho$ and $\mu = 0$ divide the phase diagram into four sections (most easily seen in figure 3.8). In the upper left quadrant we have $\mu > 0$ and $V < V^\rho$. Equation (3.34) then tells us that μ must be decreasing in this quadrant. The upper right quadrant is characterized by $\mu > 0$ and $V > V^\rho$, implying that μ is growing. The two bottom quadrants both have $\mu < 0$. The left quadrant have $V < V^\rho$ implying that μ is growing. Conversely, μ is declining in the right quadrant.

The producer's $\dot{V}_m = 0$ condition is very similar to the social planner's condition. The only difference is that the carbon cost (τ) is not included in the market solution. The producer only considers the price, p_m, and the real marginal cost of biofuel, b', when valuing the profit of harvesting against the value of letting the crops stand. For any given level of biofuel production, the marginal cost facing the producer will thus be lower than the social cost, due to the omission of the environmental externality.

The low cost market case, $p_m > b'$, is straightforward (figure 3.6). η will be positive, and as long as the entire $\dot{V}_m = 0$ locus is above zero, the steady state volume is completely inelastic and given by V^ρ. This means that if the government were to impose a moderate tax on biofuels, h, but keeping $p_m - b' - h > 0$, this would have no impact on the steady state volume of crops. If the government was seeking to change the steady state volume of crops, they would have to increase the biofuel tax until the cost became high enough to make the $\dot{V} = 0$ curve cross the $\mu = 0$ curve.

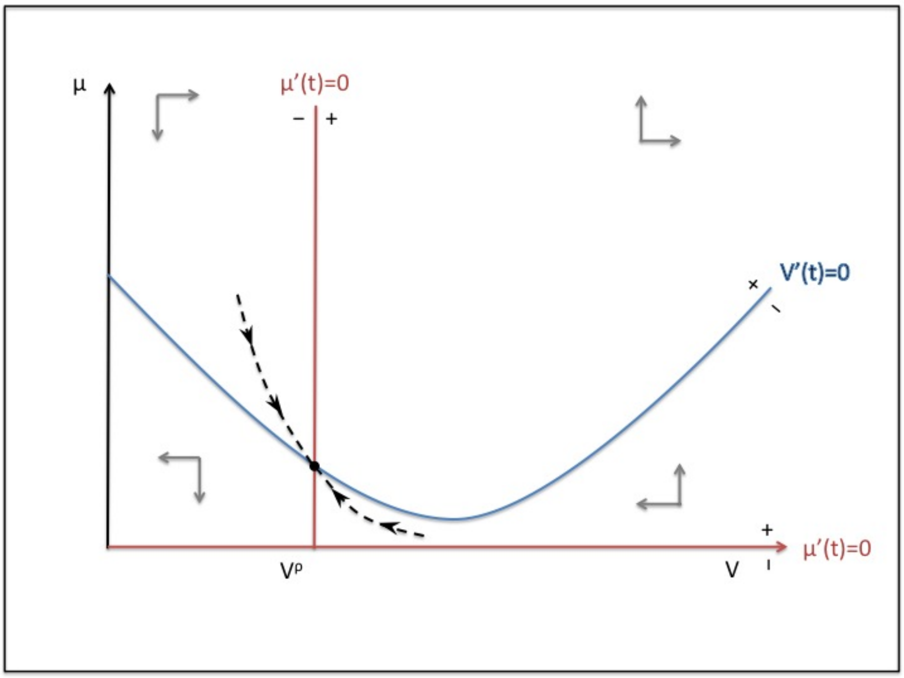


Figure 3.6: The path to steady state, with low market costs of biofuel.

The market solution has two interesting moderate cost cases, where the distinction between the two cases is given by steepness of the $\dot{V}_m = 0$ loci. A gentle slope, as displayed in figure 3.7, can have two different causes; either the marginal cost b' does not vary much when the harvest changes, or the growth rate of the crops $f(V_m)$ is unresponsive with regards to changes in the volume of standing crops. Either way it results in three equilibrium points, where two of them are stable. If the initial volume is larger than V^ρ, the only possible steady state is V^{high}, implying that we must have $\mu_0 = 0$. The saddle path moves along the x-axis (as shown in figure 3.7), indicating that $\mu_t = 0$ for all times t, including the steady state. If the initial volume is strictly less than V^ρ, we have $\mu \neq 0$, and the economy ends up in a steady state with a low value of standing crops (V^{low}). The last equilibrium point is unstable, and thus regarded unfeasible. To sum it up; the initial volume will determine the optimal μ_0, which then establishes what steady state solution the market will end up in.

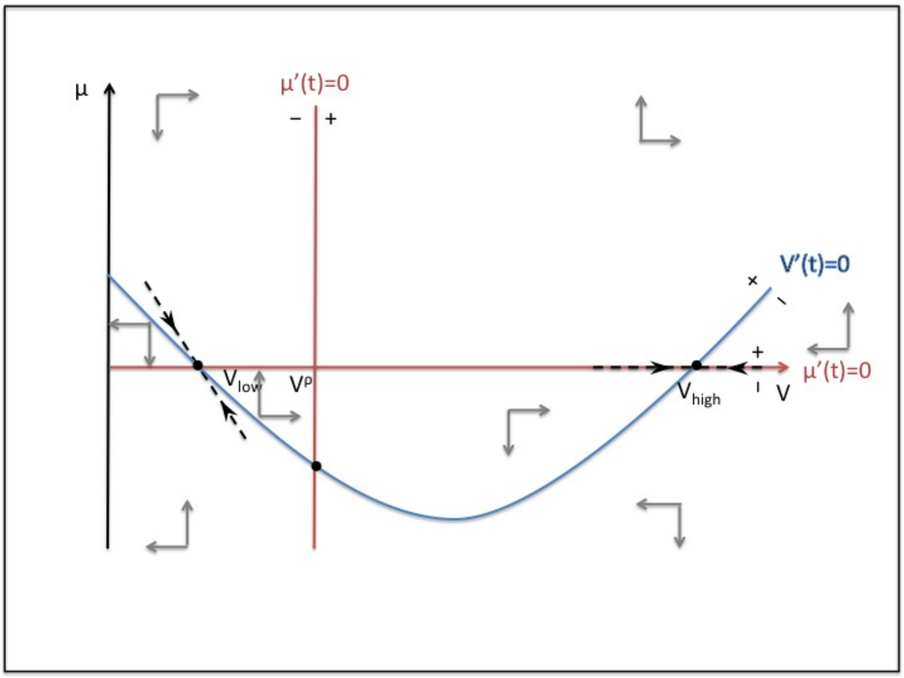


Figure 3.7: The path to steady state, with moderate market costs of biofuel.

The second moderate cost case is shown in figure 3.8, where the slope of the $\dot{V}_m = 0$ curve is much steeper (except near V_{MSY}). The cause of the steeper slope is considerable variations either in the marginal cost of biofuel (b'), or in the growth function $f(V_m)$. Again there, will be three possible equilibrium points, where two of them are stable. The difference, compared to the previous case, is that the low volume equilibrium is given by V^ρ and $\mu^* > 0$, and this is now a stable solution. If $V_0 < V^\rho$, μ_0 will be high and μ_t will be descending rapidly along the saddle path towards the steady state value. The V^{high} equilibrium will now only be realized if the initial volume is higher than V^{med} (the unstable equilibrium point), and like the previous case, $\mu_t = 0$ for all times t along this saddle path.

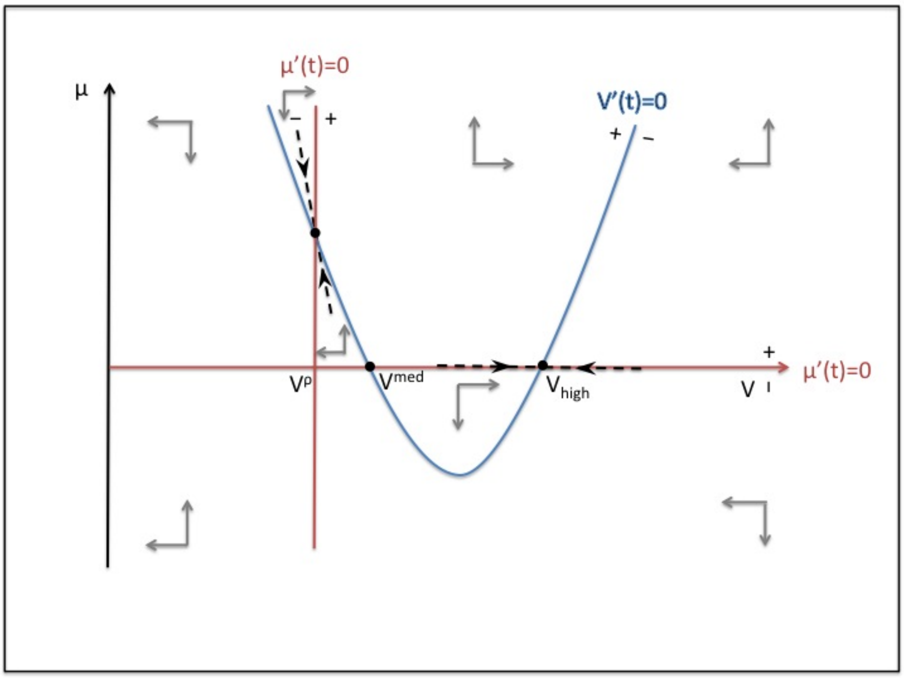


Figure 3.8: The path to steady state, with moderate market costs of biofuel.

The last market case displays an important difference between the market solution and the social planner solution: There are no stable market steady state solutions if the real cost of biofuel is high ($p_m > b'(V_m)$) (figure 3.9). The only equilibrium point is given by V^{ρ}, where the real profit of producing biofuels will be negative. Such a solution is clearly not feasible without any government interventions, so one can safely assume that the boundary solution with $H_m = f(V_{max}) = 0$ will be the market steady state solution in the high cost case.

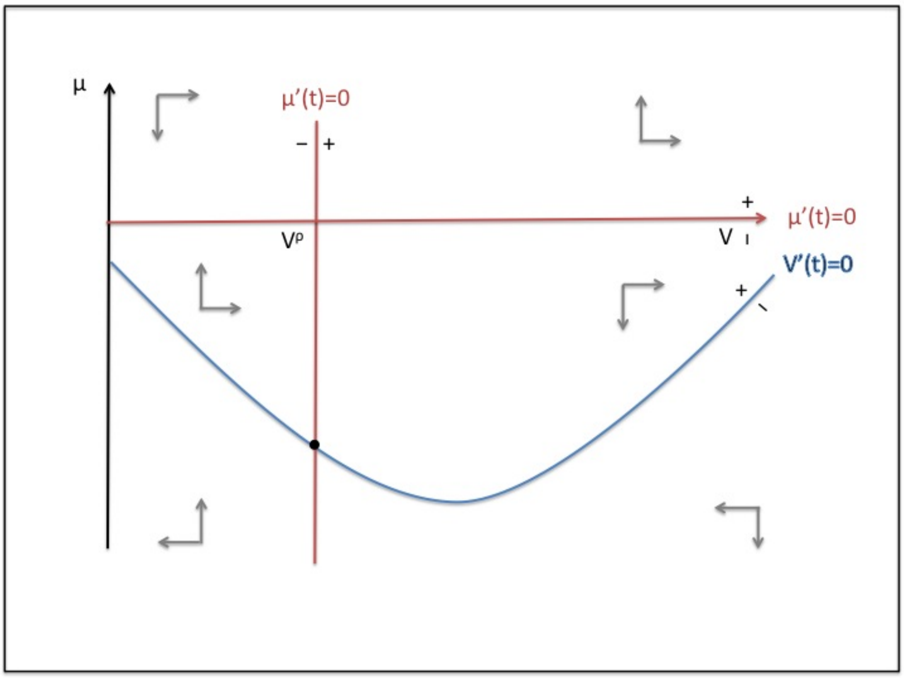


Figure 3.9: The path to steady state, with high market costs of biofuel.

3.3.3 Comparisons and observations

Returning to the case with two types of crops, we can utilize many of the same arguments as in the single crop case. Typically the real costs will be different for the two biofuel types, both due to the technological differences captured in the cost function b_i, and the growth functions f^i. The only thing that matters for the determination of the market steady state production of biofuels, is the relative real profitability between the two crops (remembering that $R = 0$ in steady state). Let us assume that the slow crop correspond to the case displayed in figure 3.7 and the fast crop correspond to figure 3.8. Also assume that the initial volume of the slow crop is $V_0^s < V^{s,\rho}$, while the fast crop starts at its maximal volume, $\bar{V}^f$. Without any government interventions, the steady state levels of the two crops will be $V^{s,low}$ and $V^{f,high}$ respectively, with $\mu_s = \mu_f = 0$. Using the same two crops in the social planner model, we will typically get variations over the case displayed in figure 3.4. The reason for this is that we are adding the environmental cost to the moderate real cost, making the total costs higher. The socially optimal steady state volumes for the two crops will in this case be given by $V^{s,*}_{low}$ and $V^{f,*}_{high}$, and both η_s and η_f will be negative.

To be able to compare the two steady state solutions it is necessary to study the underlying equations a bit more closely. Since we have now specified the initial volumes, V^s_0 and V^f_0, it is possible to solve for the steady state volumes. Setting (3.47) equal to (3.48) and using $\mu_s = \mu_f = 0$ combined with $p_m = B'(H^s + H^f)$, yields the condition $B'(f_s(V^s_m) + f_f(V^f_m)) = b'_i(f_i(V^i_m))$[3], for both crops. The corresponding social planner condition is given by $[B'(f_s(V^{s,*}) + f_f(V^{f,*})) - b'_i(f_i(V^{i,*}))](\rho - f'_i(V^{i,*})) = \rho\tau$. For the slow crop we know that $V^{s,*} < V^{\rho}$, which implies $f'_s(V^{s,*}) > \rho$. Since τ is positive and $(\rho - f'_s(V^{s,*}))$ is negative, it follows that $B'(f_s(V^{s,*}) + f_f(V^{s,*})) - b'_s(f_s(V^{s,*})) < 0$. If we then start from the market condition $B'(f_s(V^s_m) + f_f(V^f_m)) = b'_s(f_s(V^s_m))$ and increase the volume of the slow crop, we see that this increases the harvest of the slow crop, as $f' > 0$ in this area. Increasing the harvest leads to an increase in the marginal cost, b'_s, and a reduction in the marginal benefit B', which gives $B' < b'_s$. Analyzing the fast crop in the same way, knowing that the steady state volume is larger than V_{MSY} and thus $f'_f(V^{f,*}) > 0$, we get the opposite condition, $B'(f_f(V^{f,*})) > b'_f(f_f(V^{f,*}))$. Increasing the volume when $V^f > V^f_{MSY}$ leads to a reduction in the crop growth and hence a reduction in the harvest of the fast crop. Starting from the market equilibrium $B' = b'_f$, we see that increasing the volume of the fast crop will satisfy the social planner condition $B' > b'_f$.

The question is now; will both socially optimal crop volumes be larger than the corresponding market volumes, when we take into account the interaction effects? The answer is yes. If we only were to increase the volume of the slow crop (starting from the market equilibrium $B' = b'_s = b'_f$), we would get $B' < b'_f < b'_s$, which only satisfy the condition for the slow crop. If we exclusively increased the volume of the fast crop, we would get the opposite result, $B' > b'_f > b'_s$. However, by increasing both crop volumes it is possible to fulfill the conditions for both crops

[3] I am simplifying the notation by leaving out the "low" and "high" terms. Knowing that the slow crop is in the "low" equilibrium and the fast in the "high" equilibrium, this should not lead to any ambiguous results.

simultaneously, yielding $b'_s < B' < b'_f$. This leads to the conclusion that the socially optimal steady state volumes for both crops must be higher than their corresponding market steady state volumes.

The most interesting variables are not the steady state volumes, but the steady state harvest of each crop, that is, the production of the two types of biofuel. Using the results above, we see that for the slow crop $V_m^s < V^{s,*}$ entails $H_m^s < H^{s,*}$. The market is producing less than the optimal amount of the biofuel based on the slow crop. Conversely, $V_m^f < V^{f,*}$ yields $H_m^f > H^{f,*}$, in other words, the market is overproducing the biofuel based on the fast crop. This simple analysis underlines once more the importance of differentiating between biofuel types. An example would be if the government is assuming underproduction in both biofuel types, and choose to apply a subsidy to all biofuel production, this will amplify the overproduction of the fast crop. Depending on the situation, such a policy could actually make society worse off than without the subsidy.

By comparing the social planner solution and the market solution one can also draw some conclusions about the potential role of the government. A tax or subsidy on biofuel will only affect the $\dot{V}_m = 0$ curve, which limits the range of steady state volumes that can be realized using this instrument. In particular, it is not possible to steer the market towards a steady state volume between V^ρ and V_{MSY} using a constant tax on biofuel. This is because equilibrium points in this range will be unstable, so there are no saddle paths leading to these solutions, as shown in figure 3.8. However, social planner solutions within this specific range of volumes correspond to the low cost solution in figure 3.2, requiring that both the real and the environmental costs are low. We know that in real life it is generally not profitable to produce biofuels, indicating high or at least moderate production costs. From this we can infer that the low cost scenario is not realistic (at least not in the near future). We are then left with two possible scenarios with positive steady state production of biofuel, which is displayed in figure 3.3 and figure 3.4. From this we can conclude that the actual socially optimal volume will be higher than V_{MSY}. So even though it is not possible to realize all theoretical steady state values with a tax or subsidy on biofuel production, this is not likely to become an issue when we restrict the solutions to the realistic ones.

Chapter 4

The Art of Conducting

Constructing efficient fiscal policies is an art form with little appreciation in the public. Nevertheless, fiscal policy is used by governments world wide, not just to cover their budgets, but to steer market agents towards more socially beneficial behaviors. The beauty of a well constructed tax system is that it allows the government to create incentives for the market agents to behave optimally, without the micromanagement issues often associated with command and control instruments.

Many countries around the world have implemented policies to support the biofuel industry in its infant stage. Governments, including the Norwegian, have dedicated large amounts to R&D to encourage technological progress in the direction towards both cost reduction and life cycle emission reduction. However, this thesis is mainly focused on the optimal supply of biofuels, and not the technological steps needed to produce it. The discussion will thus be limited to policies directed towards production volume, and not the policies directed towards innovation and research.

4.1 Simply the best

In the previous chapters I have highlighted the differences between the socially optimal solution and the market solution. In this chapter I will investigate different policy instruments the government can use to create incentives for the market agents, with the purpose of realizing the social optimum. The goal is to get the market agents to internalize the environmental damage created by fuel consumption, as well as the benefits from carbon capture through crop growth. I will assume that the price of fuel is correct, that is, reflecting the benefits of fuel consumption ($p_t = B'(F_t)$).

In the ideal world the government can choose any type and number of policy instruments, and the most straightforward will be to tax emissions and subsidize carbon capture. Denoting the emission tax e_t, and the capture subsidy s_t, the new producer problem is to maximize

$$\begin{aligned}\Pi_t =& (p_t - e_t)F_{m,t} - c(S_{m,t})R_{m,t} - b_s(H^s_{m,t}) - \\ & b_f(H^f_{m,t}) + s_t[f_s(V^s_{m,t}) + f_f(V^f_{m,t})].\end{aligned} \tag{4.1}$$

Differentiating the new Hamiltonian using the same shadow price symbols as before, yields the new first order conditions for the market,

$$p_t = c(S_{m,t}) + \lambda_t + e_t \tag{4.2}$$

$$p_t = b'_s(H^s_{m,t}) + \mu^s_t + e_t \tag{4.3}$$

$$p_t = b'_f(H^f_{m,t}) + \mu^f_t + e_t. \tag{4.4}$$

Assuming that the transversality conditions still hold, the shadow prices now develop according to: For $i = \{s, f\}$,

$$\dot{\lambda}_t - r\lambda_t = c'(S_{m,t})R_{m,t} \tag{4.5}$$

$$\dot{\mu}_t^i - (r - f'(V_{m,t}^i))\mu_t^i = s_t f'(V_{m,t}^i). \tag{4.6}$$

I will start by looking at the fossil fuel production to investigate whether the social optimum can be reached by using an emission tax. To be able to find the optimal tax rate, we need to compare the FOCs with the social planner solution. By assumption, $p_t = B'(F_t)$, which means that in optimum we must have $c(S_{m,t}) + \lambda_t + e_t = c(S_t) + \kappa_t + \tau_t$. We want the market extraction path, and thus the remaining stock, to equal the optimal one at all times, meaning $c(S_{m,t}) = c(S_t)$. This is obtained if we set the tax $e_t = \kappa_t - \lambda_t + \tau_t$. It is important to note that the government has to be able to make a credible signal to the market about the future tax plan. If the market agents do not believe that the government will enforce its tax plan, they will deviate from the optimal path and we are back to square one. I will assume that the government is credible and able to convey its tax plan to the market. Then, because the representative firm knows that the tax will be e_t, this changes the market valuation of the remaining stock of oil. Since the market extraction path will be identical to the optimal path we get $\lambda_t = \kappa_t$. Hence the conclusion is; with a credible tax plan, the two resource rents will be identical and the optimal oil tax will equal the cost of carbon, which can be expressed as

$$\lambda_t = \kappa_t = -\int_t^{\infty} e^{\rho(t-t')} c'(S_{m,t}) R_{m,t} dt' \tag{4.7}$$

$$e_t = \tau_t = \int_t^{\infty} (1 + 3e^{\alpha(t-\tau)}) \frac{1}{4} e^{\rho(t-\tau)} D'(A_\tau).d\tau \tag{4.8}$$

The harvest paths are somewhat more complicated as they depend on both the emission tax and the carbon capture subsidy. From the FOCs we see that the optimal emission tax must be given by $e_t^i = \eta_t - \mu_t^i + \tau_t$. Using the condition $e_t = \tau_t$ implies that we must have $\eta_t^i = \mu_t^i$ at all times t. Since the producer does not take the environmental benefits of the carbon capture into account when valuing the standing crops, the subsidy needs to make up for this intrinsic difference. Studying the equations of motion for η_t^i and μ_t^i, we see that the differential equations ((3.12) and (4.6)) will have the same form if we let $s_t = \tau_t$. Using the same argument as above, and assuming a credible government tax plan, we can effectively change the producer's valuation to equal the social planner's valuation at all times.

This reproduces a well known result; the first best solution is obtainable with a common price. That is, the producer pays a price τ_t per unit emission, and he receives the same amount per unit carbon he mitigates through crop growth.

4.2 Close enough for jazz

In the real world things are never as straightforward as in the perfect model world. The first-best solution is rarely obtainable, often due to information constraints. It might be hard to measure the exact emissions, but in this model it is easily obtained from the use of the fuels. The real issue here is measuring the growth of the two crops. Since the growth depends on the standing volume of crops, which will probably change continuously, one would have to rely on information from the farmers. The farmers would have strong incentives to overstate the growth, so there is no way to obtain the first-best solution without inducing large monitoring costs, which in itself makes the solution suboptimal. Due to the information constraints it is necessary to look at second-best solutions, where the government applies policy instruments to more easily measurable variables.

4.2.1 Fuel taxes

Due to lack of information about the crop growth, the government seeks to find a way to reach its policy target. A seemingly simple way to do this would be to impose a tax or subsidy on the different fuel types. The taxes on fossil fuel and the two biofuel types are denoted o_t, h_t^s, and h_t^f.

The government's problem can be constructed as a Stackelberg game, where the government is the leader and announces the tax path which it will commit to. The producer is the follower, and will maximize profits taking the announced tax plans as given. The idea behind this game is that the government can calculate how the producer will respond to the different tax paths, and based on this, choose the tax paths yielding the highest net social benefits. The government's control variables are the tax paths, while the producer's control variables are the extraction and harvest paths as before. The new market profit function and the corresponding first order conditions are given by

$$\begin{aligned}\Pi_t =&(p_{m,t} - o_t)R_{m,t} - (p_{m,t} - h_t^s)H_{m,t}^s - (p_{m,t} - h_t^f)H_{m,t}^f - \\ &c(S_{m,t})R_{m,t} - b_s(H_{m,t}^s) - b_f(H_{m,t}^f),\end{aligned} \tag{4.9}$$

$$p_{m,t} = c(S_{m,t}) + \lambda_t + o_t \tag{4.10}$$

$$p_{m,t} = b_i'(H_{m,t}^i) + \mu_t^i + h_t^i. \tag{4.11}$$

This gives the market production of fossil fuel, $R_{m,t} = R(p_{m,t}, \lambda_t, o_t)$, and of biofuels, $H_{m,t}^i = H^i(p_{m,t}, \mu_t^i, h_t^i)$, as functions of the price, the shadow prices and the taxes. The equations of motion will be identical to the previous market model, $\dot{\lambda}_t - \rho\lambda_t = c'(S_{m,t})R_{m,t}$ and $\dot{\mu}_t^i - (\rho - f'(V_{m,t}^i))\mu_t^i = 0$. Since $p_{m,t}$ is endogenously determined by $p_{m,t} = B'(F_{m,t})$, it can be written as $p_{m,t} = p(R_{m,t}, H_{m,t}^s, H_{m,t}^f)$. Combining the market production functions and the shadow price functions yields the market response functions, only depending on the taxes and shadow prices,

$$R_{m,t} = R(\mu_t^s, h_t^s, \mu_t^f, h_t^f, \lambda_t, o_t) \tag{4.12}$$

$$H_{m,t}^i = H^i(\mu_t^s, h_t^s, \mu_t^f, h_t^f, \lambda_t, o_t). \tag{4.13}$$

Assuming for the moment that the government is informed about these response functions, they can choose the tax paths that maximize net social benefits. The new control variables for the government are $o_t, h_t^s,$ and h_t^f, and the new state variables are $S_{m,t}, V^s_{m,t}, V^f_{m,t}, A^1_t, A^2_t, \lambda_t, \mu^s_t,$ and μ^f_t [28]. This gives the new and rather messy hamiltonian for the government problem,

$$\begin{aligned}\mathcal{H}_t =& B(F_{m,t}) - c(S_{m,t})R_{m,t} - b_s(H^s_{m,t}) - b_f(H^f_{m,t}) - D(A_{m,t}) + \\ & \kappa_t[-R_{m,t}] + \eta^s_t[f_s(V^s_{m,t}) - H^s_{m,t}] + \eta^f_t[f_f(V^f_{m,t}) - H^f_{m,t}] + \\ & \upsilon^1_t[\frac{3}{4}(R_{m,t} + (H^s_{m,t} + H^f_{m,t}) - f_s(V^s_{m,t}) - f_f(V^f_{m,t})) - \alpha A^1_{m,t}] + \\ & \upsilon^2_t[\frac{1}{4}(R_{m,t} + (H^s_{m,t} + H^f_{m,t}) - f_s(V^s_{m,t}) - f_f(V^f_{m,t}))] + \\ & \theta_t[\rho\lambda_t + c'(S_{m,t})R_{m,t}] + \gamma^s_t[(\rho - f'(V^s_{m,t}))\mu^s_t] + \gamma^f_t[(\rho - f'(V^f_{m,t}))\mu^f_t].\end{aligned}$$

In the Hamiltonian, $R_{m,t}$, $H^s_{m,t}$, and $H^f_{m,t}$ are given by the market response functions (4.12) and (4.13). Differentiating with respect to the three tax paths gives the first order conditions for an internal optimum,

$$\begin{aligned}&[B'(F_{m,t}) - c(S_{m,t}) - \kappa_t - \tau_t + \theta_t c'(S_{m,t})]\frac{\partial R_{m,t}}{\partial o_t} + \\ &\sum_{i=s,f}[(B'(F_{m,t}) - b'_i(H^i_{m,t}) - \eta^i_t - \tau_t)\frac{\partial H^i_{m,t}}{\partial o_t}] = 0 \qquad (4.14)\\ &[B'(F_{m,t}) - c(S_{m,t}) - \kappa_t - \tau_t + \theta_t c'(S_{m,t})]\frac{\partial R_{m,t}}{\partial h^s_t} + \\ &\sum_{i=s,f}[(B'(F_{m,t}) - b'_i(H^i_{m,t}) - \eta^i_t - \tau_t)\frac{\partial H^i_{m,t}}{\partial h^s}] = 0 \qquad (4.15)\\ &[B'(F_{m,t}) - c(S_{m,t}) - \kappa_t - \tau_t + \theta_t c'(S_{m,t})]\frac{\partial R_{m,t}}{\partial h^f_t} + \\ &\sum_{i=s,f}[(B'(F_{m,t}) - b'_i(H^i_{m,t}) - \eta^i_t - \tau_t)\frac{\partial H^i_{m,t}}{\partial h^f}] = 0. \qquad (4.16)\end{aligned}$$

The corresponding equations of motion, determining the dynamics of the government solution, are: For $i = \{s, f\}$

$$\dot{\kappa}_t - \rho\kappa_t = c'(S_{m,t})R_{m,t} \tag{4.17}$$
$$\dot{\eta}^i_t - (\rho - f'_i(V^i_{m,t}))\eta^i_t = -\tau_t f'_i(V^i_{m,t}) + \gamma^i_t\mu^i_t f''(V^i_{m,t}) \tag{4.18}$$
$$\dot{\upsilon}^1_t - (\rho + \alpha)\upsilon^1_t = D'(A_{m,t}) \tag{4.19}$$
$$\dot{\upsilon}^2_t - \rho\upsilon^2_t = D'(A_{m,t}) \tag{4.20}$$
$$\dot{\theta}_t = 0 \tag{4.21}$$
$$\dot{\gamma}^i_t - f'_i(V^i_{m,t})\gamma^i = 0. \tag{4.22}$$

The old state equations are described by the same equations as before, and the new state equations develop according to

$$\dot{\lambda}_t - \rho\lambda_t = c'(S_{m,t})R_{m,t} \tag{4.23}$$
$$\dot{\mu}^i_t - (\rho - f'_i(V^i_{m,t}))\mu^i = 0. \tag{4.24}$$

To determine the new set of transversality conditions I will use a result derived in "Differential Games in Economics and Management Science" [28]. A definition of a controllable costate variable is needed, so I have modified "Definition 5.1" from the same book to fit this problem: *The initial value of the follower's costate variables* $\{\lambda_t, \mu^s_t, \mu^f_t\}$ *is said to be noncontrollable if* $\{\lambda_0, \mu^s_0, \mu^f_0\}$ *is independent of the leader's control path* $\{o_t, h^s_t, h^f_t\}$*. Otherwise it is said to be controllable.* This means that since $\{\lambda_0, \mu^s_0, \mu^f_0\}$ are affected by the fuel taxes, the government can "choose" these initial values by adjusting the taxes appropriately [28]. Since we know that $\{\lambda_0, \mu^s_0, \mu^f_0\}$ are controllable we can utilize the result (also modified to fit this problem): *[...] if the initial value of the state variables* $\{\lambda_t, \mu^s_t, \mu^f_t\}$ *are controllable, then we have the transversality conditions for the new shadow prices:* $\theta_0 = 0$ *and* $\gamma^i_0 = 0$.

The "old" shadow price, κ_t, η^i_t, and υ^i_t must satisfy the same transversality conditions as before,

$$\lim_{t\to\infty} e^{-\rho t}\kappa_t = 0$$
$$\lim_{t\to\infty} e^{-\rho t}\eta_t^i = 0$$
$$\lim_{t\to\infty} e^{-\rho t}\upsilon_t^j = 0.$$

The socially optimal solution was derived in chapter 3, and by definition we cannot do any better than that. The new question will be; is it possible to reproduce the social planner solution when using fuel taxes, instead of an emission tax combined with a carbon capture subsidy?

Observing the FOCs (4.14), (4.15), and (4.16), we see that there are three common factors in all three conditions: For $i = \{s, f\}$, $B'(F_{m,t}) - c(S_{m,t}) - \kappa_t - \tau_t + \theta_t c'(S_{m,t})$, and $B'(F_{m,t}) - b_i'(H_{m,t}^i) - \eta_t^i - \tau_t$. If these three factors all equal zero, the three FOCs are satisfied, and we get the conditions

$$B'(F_{m,t}) = c(S_{m,t}) + \kappa_t + \tau_t - \theta_t(c'(S_{m,t})) \tag{4.25}$$
$$B'(F_{m,t}) = b_i'(H_{m,t}^i) + \eta_t^i + \tau_t. \tag{4.26}$$

This corresponds to the social planner's FOCs, except for the extra term in the condition for fossil fuel, $-\theta_t c'(S_{m,t})$. From the equations of motion we see that $\dot{\theta} = 0$. Combining this with the transversality condition $\theta_0 = 0$, we see that $\theta_t = 0$ for all times t. An interpretation of this result is that a change in the market resource rent would not affect the social benefits, indicating that the market resource rent and the social planner's resource rent are identical. This fact was also established in the previous section about first-best policies, implying that $o_t = \tau_t$ is the optimal tax on fossil fuel, as long as we are able to realize the first-best biofuel paths as well.

The first order condition for the harvest paths, or more correctly for the biofuel tax paths, are identical to the social planner solution. However, the expressions establishing the time development of η_t^i and $V_{m,t}^i$ are different from the first best solution. The expression for η_t^i has the additional term $\gamma_t^i f''(V_{m,t}^i)\mu_t^i$, which takes into account the tax induced change in the market valuation of standing crops. In the same way as above, we use the transversality conditions of to find the time development of γ_t^i. From (4.22) we see that $\gamma_0^i = 0$ gives $\gamma_t^i = 0$ for times t. Putting this back into the transversality condition ((4.18)), the set of transversality conditions are now identical to the socially optimal set ((3.11) - (3.14)). The only remaining issue is to determine the optimal initial value of μ_t. We see that choosing taxes which impose $\mu_0^i = 0$, will reproduce the social planner solution in $t = 0$. Furthermore, this ensures that $\mu_t^i = 0$ for all times $t > 0$, which enables the realization of the socially optimal solution at all times t. We can then conclude that it is possible for the government to use fuel taxes and/or subsidies instead of the first-best solution with a common carbon price, and still realize the socially optimal solution. The only condition is that the government needs to restrict itself to biofuel taxes or subsidies which realize $\mu_0 = 0$, and then combine this with an oil tax $o_t = \tau_t$.

The motivation for using fuel taxes instead of targeting carbon emission and mitigation was information problems with regards to measuring the crop growth. Unfortunately, the government will meet information problems when using fuel taxes as well. This analysis relied on the assumption that the government was informed about the market response functions, (4.12) and (4.13). This is clearly a very strong assumption, and in real life there is no reason to believe that the government would know the exact function forms of the market response functions. This is not to say that the government is not able to find some information about these functions. Regression analysis and other types of estimation methods have been a great source of information about market reactions for many years. However, it is important to keep in mind that these will never give a 100% accurate result, so one cannot expect the implemented policies to fully replicate the social optimum.

4.2.2 Blending mandate

Biofuels are rarely used in their pure form, but are rather blended into fossil fuel in various proportions. A natural choice of policy instrument is thus a blending mandate, which dictates how much biofuels must be sold per unit fossil fuel. A blending mandate system is appealing as it requires no additional informations beyond the oil consumption, making it easier to implement. According to an article posted on biofuelsdigest.com in 2011: *Mandates currently in place provide the when and where of a 60 billion gallons (and up) biofuels market by 2022* [29]. This corresponds to 3.9Mbbl/d[1] in world wide biofuels production. As a reference, IEA reported in December 2011 that the world's total oil supply was 90Mbbl/d, while the total biofuel production was only 1.8Mbbl/d [30]. To fulfill the biofuels mandates in place, we thus need to double the production within 2022. There is no reason to believe that this strategy will yield the optimal fossil fuel and biofuel production paths. However, one could make arguments for using a blending mandate as a second best solution if the monitoring, administration or deployment costs are considerably lower than with with a tax/subsidy scheme. In the following section I will not derive the optimal blending mandate, but rather look at what effects a blending mandate will have on the market supply of biofuels and oil.

The reasoning behind a blending mandate is that the market will produce less than the optimal amount of biofuel, and that the blending mandate will force the supply up to a level closer to the socially optimal one. Since the assumption is that biofuel is undersupplied, the emissions from biofuel consumption is often exempt from emission taxes, to further encourage production. I will capture this by including a tax on oil extraction, denoted o_t, but no tax on harvest.

First I will look at the case where there is only one type of biofuel, denoted $H_{m,t}$. I will assume that the government enforces a blending mandate given by

[1]Mbbl/d is a standard unit for measuring oil production, and the abbreviation stands for *millions of oil barrels per day*. The standard oil barrel adapted by the oil industry contains approximately 159 liters of oil.

$H_{m,t} \geq qR_{m,t}$, where q is the minimum amount of biofuel which must be contained in any fuel product per unit fossil fuel. The case where the constraint is non-binding is straightforward. A non-binding blending mandate means that the market will provide more biofuels on its own than the government requires. Then the blending mandate has no impact on the market supply of biofuels, which results in the same market solution as developed in the previous chapter. Assuming that the tax on fossil fuel is unchanged, $o_t = \tau_t$, biofuel is effectively subsidized when using a blending mandate, as the cost of carbon is not included in the market cost of producing biofuel (no emission tax). This solution is bound to be suboptimal as the environmental externality will never be taken into account by the market agents.

A more interesting case is when the constraint is actually binding, which is the assumption when implementing this policy. I will assume that it is binding for all times t, which will happen if the market's preferred biofuel supply is always less than the mandated amount. Since the representative producer is bound by the enforced blending mandate, he will harvest $H_{m,t} = qR_{m,t}$ at all times. This effectively mitigates one degree of freedom for the producer, and the new profit function can be written as

$$\Pi_t = p_{m,t}(1+q)R_{m,t} - [c(S_{m,t}) + o_t]R_{m,t} - b(qR_{m,t}). \tag{4.27}$$

The producer can now only decide how much fossil fuel to produce, as the biofuel production will automatically follow. The first order condition and equations of motion are now given by

$$\begin{aligned}
&p_{m,t} = c(S_{m,t}) + \lambda_t + o_t + q[b'(qR_{m,t}) + \mu_t - p_{m,t}] \\
&\dot{\mu}_t - [\rho - f'(V_{m,t})]\mu_t = 0 \\
&\dot{\lambda}_t - \rho\lambda_t = c'(S_{m,t})R_{m,t}.
\end{aligned}$$

If $p_{m,t} \geq b'(H_{m,t}) + \mu_t$ the producer could do better by increasing the biofuel production, which is not constrained by the blending mandate. For the blending mandate to be binding we must thus have $p_{m,t} < b'(H_{m,t}) + \mu_t$, which gives

$q[b'(qR_{m,t})+\mu_t-p_{m,t}] > 0$. The first part of the first order condition is identical to the original market solution for fossil fuel, and has the same benefit-cost interpretation as before. The new cost the producer needs to consider when determining the level of oil extraction is the loss from having to increase the biofuel production when the price is below the cost. The producer sees this as being faced with a more costly oil extraction 'process' than before. This gives him incentives to produce less fossil fuel than he would without the blending mandate.

The resource rent will have the same expression as before,

$$\lambda_t = -\int_t^{\infty} e^{\rho(t-t')} c'(S_{m,t}) R_{m,t} dt', \tag{4.28}$$

and we see that the resource rent increases with increasing $R_{m,t}$ and decreasing $S_{m,t}$. This means that the second round effect will dampen the initial reduction in fossil fuel production, as the resource rent declines. The net effect will still be a reduction in fossil fuel production, but because oil is a scarce resource supplied with a resource rent, the effect will be weakened.

Going back to the case with two types of biofuels, the typical blending mandate requires that $H^s_{m,t} + H^f_{m,t} \geq qR_{m,t}$. Assuming an interior solution with positive harvest in both types of crops, will result in two first order conditions

$$p_{m,t} = c(S_{m,t}) + \lambda_t + o_t + q[b'(H^s_{m,t}) + \mu^s_t - p_{m,t}] \tag{4.29}$$

$$b'(H^s_{m,t}) + \mu^s_t = b'(H^f_{m,t}) + \mu^f_t. \tag{4.30}$$

If the constraint is binding we see that both harvest types are supplied with negative profits from the firm's perspective, i.e. $b'(H^i_{m,t}) + \mu^i_t < p_{m,t}$. The general result will be the same as with one crop; more harvest and reduced oil extraction, but now we also have an efficiency requirement determining the relative production volume of the two biofuel types.

To sum up, we see that the general effect from a binding blending mandate is that biofuels will be a larger part of the total fuel consumption. The producer will not take into consideration the difference in environmental effects between the two crop types, so production efficiency will dictate how much is produced using each crop type. We see that the harvest is effectively subsidized in both the binding and the non-binding solutions. One positive side with this type of subsidy, compared to direct subsidies, is that it is self financing. That is, the subsidy is financed through an effective increase in the oil tax and there is no need for additional government interventions.

Changing scenarios

A new question now arises; how will the optimal tax on fossil fuel change in the presence of a blending mandate? To answer this question I will construct a new Stackelberg game, where the producer's control variable is the extraction path, while the government's control is the fossil fuel tax. One could have included the blending mandate (q) as a control as well, but this is not of direct relevance for this question, so I will let the blending mandate be given from history.

From the market FOCs above ((4.29) and (4.30)), including the constraint $p_{m,t} = B'(F_{m,t}) = B'((1+q)R_{m,t})$, we get the following response functions: $R_{m,t} = R(\lambda_t, o_t, \mu_t)$. The new government hamiltonian, including λ_t and μ_t as state variables, now reads

$$\begin{aligned}\mathcal{H}_t =& B((1+q)R_{m,t}) - c(S_{m,t})R_{m,t} - b(qR_{m,t}) - D(A_t) + \\ & \kappa_t[-R_{m,t}] + \eta_t[f(V_{m,t}) - qR_{m,t}] + \\ & \upsilon_t^1[\frac{3}{4}((1+q)R_{m,t} - f(V_{m,t}) - \alpha A_t^1] + \\ & \upsilon_t^2[\frac{1}{4}((1+q)R_{m,t} - f(V_{m,t})] + \\ & \theta_t[\rho\lambda_t + c'(S_{m,t})R_{m,t}] + \gamma_t[(\rho - f'(V_{m,t}))\mu_t].\end{aligned}$$

The government has now only one control variable, o_t, as the blending mandate is given. The new first order conditions and equations of motion are then given by

$$p_{m,t} = c(S_{m,t}) + \kappa_t + \tau_t + \theta c'(S_{m,t}) + q[b'(qR_{m,t}) + \eta_t + \tau_t - p_{m,t}] \tag{4.31}$$
$$\dot{\kappa}_t - \rho\kappa_t = c'(S_{m,t})R_{m,t} \tag{4.32}$$
$$\dot{\eta}_t - (\rho - f'(V_{m,t}))\eta_t = -\tau_t f'(V_{m,t}) + \gamma_t \mu_t f''(V_{m,t}) \tag{4.33}$$
$$\dot{\upsilon}_t^1 - (\rho + \alpha)\upsilon_t^1 = D'(A_t) \tag{4.34}$$
$$\dot{\upsilon}_t^2 - \rho\upsilon_t^2 = D'(A_t) \tag{4.35}$$
$$\dot{\theta}_t = 0 \tag{4.36}$$
$$\dot{\gamma}_t - f'(V_{m,t})\gamma = 0. \tag{4.37}$$

Due to the blending mandate the total fuel consumption is given by $F_{m,t} = (1 + q)R_{m,t}$, where $R_{m,t}$ and $qR_{m,t}$ is the amount of fossil fuel and biofuels respectively. Since the harvest, extraction, crop volume and oil reserves are determined in the market, these variables will have the same values in the producer's and the government's FOCs. Combining the two sets of FOCs to solve for the optimal oil tax, we find

$$o_t = \tau_t + \theta_t c'(S_{m,t}) + q(\tau_t + \eta_t - \mu_t). \tag{4.38}$$

The first term τ_t is the carbon cost connected to the consumption of fossil fuel. The $\theta_t c'(S_{m,t})$ term is the induced change in the market resource rent, from a one-unit increase in the tax on fossil fuel. And lastly, $q(\tau_t + \eta_t - \mu_t)$ represents the effects of linking the biofuel production to the fossil fuel production through the blending mandate. In this last term, τ_t accounts for the emissions from biofuel consumption, while $\eta_t - \mu_t$ adjusts for the difference between the market and the social planner value of the standing crops.

The discounted values of the "old" shadow prices, κ_t, η_t, υ_t^1 and υ_t^2, are still required to tend to zero in the long run, while the "new" shadow prices must satisfy the transversality conditions $\theta_0 = 0$, and $\gamma_0 = 0$, as in chapter 4.2.1. Following the same arguments as used in 4.2.1, the government is free to choose the optimal value of μ_0 by adjusting the oil tax. The only way the government is able to fully control the extraction path is when the volume of standing crops is not affected by a varying market valuation, μ_t. As before, choosing $\mu_0 = 0$ will realize $\mu_t = 0$ for all t, and in this case give a fossil fuel tax rule only depending on known variables,

$$o_t = \tau_t + q(\tau_t + \eta_t). \tag{4.39}$$

τ_t is given by the same expression as before, and η_t is given by backward induction from the steady state value, as studied in section 3.3.1. We know that we are in the high-cost case, as the initial assumption was that the blending mandate was binding, and this implies that $p_{m,t} < b'(H_{m,t})$. We see that the realized steady state volume is high while η_t is strictly negative (figure 3.4). Lowering the volume of standing crops (through increasing the harvest) will in this case lead to an increased growth rate, $f(V_{m,t})$, and thus higher uptake of atmospheric carbon. If the added value from increasing the harvest is lower than the environmental damage of the direct emission from biofuel consumption, $|\eta_t| < \tau_t$, the second-best tax on fossil fuel will be higher than the first-best tax. If the environmental damage is less than $|\eta_t|$, the second-best tax on fossil fuel will actually be lower than the first-best tax. The reason for this is that a reduction in the fossil fuel tax will not only increase the fossil fuel production, but also the production of biofuel. So if the total benefit of biofuel is high enough to compensate for the direct emission, it is beneficial to encourage the use of both types of fuels by lowering the tax on fossil fuel.

I started this chapter by assuming that the government used a blending mandate in addition to a tax exemption for biofuel production. An interesting question is whether or not this policy has an effect on the market's extraction path. I assume that the government would like to use a general emission tax, e_t, in combination with the blending mandate. With no tax exemption for biofuel, the market's first order condition when maximizing profits is:

$$p_{m,t} = c(S_{m,t}) + \lambda_t + e_t + q[b'(H^s_{m,t}) + \mu^s_t + e_t - p_{m,t}]. \tag{4.40}$$

The government's first order condition and equations of motion will be unchanged, (4.31) - (4.37). The price is determined by the market equilibrium and is thus identical in both FOCs, which means that we can find the required emission tax by solving $c(S_{m,t}) + \kappa_t + \tau_t + \theta c'(S_{m,t}) + q[b'(qR_{m,t}) + \eta_t + \tau_t - p_{m,t}] = c(S_{m,t}) + \lambda_t + e_t + q[b'(H^s_{m,t}) + \mu^s_t + e_t - p_{m,t}]$. When using the optimal shadow prices as outlined above, this yields the following emission tax

$$e_t = \tau_t + \frac{q}{1+q}\eta_t. \tag{4.41}$$

Since the producer only considers fossil fuel and not biofuel as a decision variable, he is only concerned with the total fossil fuel production costs. With the tax exemption the producer will pay $o_t = \tau_t + q(\tau_t + \eta_t)$ in tax per unit fossil fuel he produces. The emission tax system yields the following tax costs per unit fossil fuel: $e_t + qe_t = \tau_t + \frac{q}{1+q}\eta_t + q(\tau_t + \frac{q}{1+q}\eta_t) = \tau_t + q(\tau_t + \eta_t)$. The tax costs facing the producer is identical in both cases, and will thus lead to the same extraction paths. The result from this analysis shows that optimizing the tax on fossil fuel will simply add the exempted carbon tax to the fossil fuel tax instead. This means that the tax exemption has no real effect as long as the blending mandate is binding, and could safely be removed.

Chapter 5

This is it

In this paper I have developed a framework for studying environmental impact of and optimal policies for biofuel production. The key elements of the model are the incorporation of fossil fuel as a non-renewable resource, varying depletion rate of atmospheric carbon and distinction between biofuel types. The main difference between the two biofuel types included in the model are the growth processes of the crops used in the production, which is the determinant factor for carbon debt accumulation. The framework uses generic functions with intuitive properties, which makes it easily adaptable to real world problems. The main strength of the model is its ability to capture the interactions between non-renewable and renewable resources, as well as the dynamic nature of the carbon cycle.

Based on information found in various reports and articles (presented in the first two chapters), I conclude that biofuel is not just a failed science project. It is possible to produce and use biofuel in a sustainable manner, but it requires extensive knowledge about all the factors contributing to the total life cycle emissions. As emphasized throughout this paper, due to the extreme variations in the environmental impact, it is crucial to distinguish between the different types of biofuels.

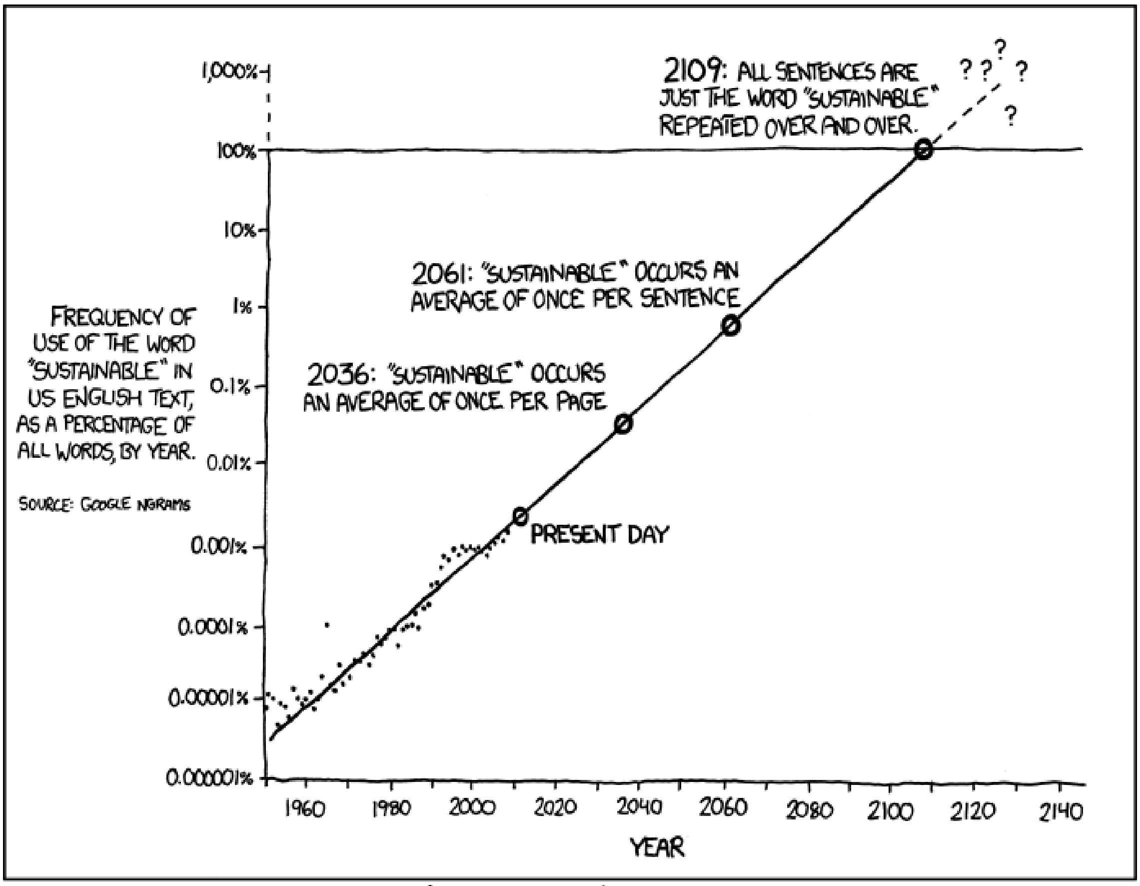
To be able to study various policy strategies, the model was solved both as a social planner problem and as a market problem. A comparison of the two sets of first order conditions was used to derive the first-best policy scheme, where emission was taxed and carbon capture was subsidized. The first-best solution reproduced a well known result; an enforced common carbon price will internalize the environmental externalities, and realize the socially optimal solution. It was also shown that the optimal solution could be reached by using a tax on fossil fuel and a tax/subsidy on both biofuels. The best approach will depend on the information at hand, that is, whether it is easier to measure emissions and carbon capture, or if it is more feasible to obtain the market response functions, which is necessary when using the latter strategy.

Studying the use of a blending mandate showed that the desired effects of reducing fossil fuel production and increasing biofuel production will be realized, as long as the blending mandate is binding. However, if the blending mandate is only targeting the total biofuel production, the composition of the biofuel types may be unfortunate, as the market does not consider the environmental differences between the crops used in production. The market only considers real costs, so the cheapest biofuel will always dominate. Translated into real life this could mean an overproduction of cheap palm oil at the expense of the more environmentally friendly, but more expensive, European rapeseed oil. Again, this underlines the importance of enforcing type specific policies on biofuel production.

Introducing a blending mandate will also affect the optimal tax on fossil fuel. If there is no carbon tax on biofuel consumption, the carbon cost should be included in the tax on fossil fuel instead. The second-best tax on fossil fuel also needs to account for the benefits of the crop growth, as the consumption of both fuel types are now directly linked. If the benefits of the carbon capture is large enough, the tax on fossil fuel should be lower than the first-best tax, while the converse is true if the carbon capture benefits are low. However, the results in chapter 4 show that when using a *binding* blending mandate, it is unnecessary to grant a tax exemption for biofuel, as this will have no real effect. If we follow this rule, the second-best tax on fossil fuel should be lower than the first-best tax.

The added value of this model is that it provides a slightly more nuanced picture of biofuels. For many years biofuels were just assumed to be carbon neutral, which is not just wrong, but also dangerous due to the induction of short term emissions. On the other hand, dismissing biofuels as harmful, either on the background of increased food prices or the introduction of a carbon debt, is also too easy. I am convinced that biofuels can bring great value, as long as we are able to acquire the knowledge necessary to create the right incentives for sustainable production.

Sustainability



From the beloved xkcd.com [31]

Appendix A

Back to the Future

As mentioned in the introduction, it is not possible to obtain a complete analytical solution of this model, as is a well known issue when dealing with more complex optimal control theory models. Like in most sciences, economists need to rely on collected data and numerical simulations to be able to draw firm conclusions based on our theories and hypotheses. In this section I will therefore present a quick step-by-step on how to go about solving the model numerically.

A.1 Numerical solution

Large scale optimal control theory problems can be quite problematic to compute numerically. The reason for this is not necessarily that the problem itself is conceptually difficult to solve. Using indirect methods is straightforward when you already have the first order conditions and equations of motion. The only thing you need to do then, is to solve the resulting boundary value problem by utilizing the initial conditions and transversality conditions. For instance, Matlab has a solid boundary value problem solver (bvp4c), which is easy and intuitive to use. However, the drawback with this solver is that Matlab is a high-level language and is not built to maximize the computation speed (except for matrix based problems). In addition, this simple brute-force strategy would require a lot of CPU time for large scale problems, regardless of the underlying code. I tested the bvp4c solver for this problem, but quickly found that it would not be possible to get even close to the accuracy needed, with my available data resources.

To be able to circumvent the CPU time problem, we can use more complicated algorithms like the Finite Element Method, pseudospectral methods, local discretization methods or others. Fortunately there are several available programs, some of them even free, which have done this for you. One promising example is the open-source tool, $\mathcal{PSOPT}$ Optimal Control Solver, written by Dr. Victor M. Becerra. The program is written in C++, which usually makes it considerably faster than higher level alternatives. It is also beneficial to use an open source tool, as this allows you to modify the code to make a better fit for the specific problem, if that should be necessary. On the other hand, the user manual is 400 pages long, so you can expect to spend some time in preparation when using this type of tool. [32]

Regardless of what tool you choose to utilize, there are some important decisions and calculations which needs to be attended to before you can start the numerical simulations.

1. First and foremost, you need to decide on a set of specific functions to be used in the model. The general functions gives a good guideline, but it will require some research to find the most reasonable and realistic function forms to describe the model.

2. When you have the complete model with all the functions specified, the next step will be to find good approximations to the parameter values. This can either be done by investigating research done on the area combined with simple computation, or more rigorously, by using real data and econometrics to find the best fitted parameter values. My background in science and mathematics tends to make me slightly biased towards the latter approach.

Besides the obvious advantage in accuracy, using real data also makes the results more easily adaptable to real world problems, which I feel makes up for the slight elongation of the calculation time.

3. Simulating an infinite time problem is naturally not an option, so it is necessary to reformulate the problem to fine time. Typically one assumes that the economy ends up in the steady state solution, and use the steady state values as boundary conditions. To convert this to a finite time problem you can decide the acceptable level of accuracy, measured in the deviation from the steady state. You can then calculate the final time that will allow you to be just within the acceptable error margin. This time will then take the role as the lower boundary for your final time, which is a good starting point for the simulations.

   Another strategy for determining the final time is to look at the problem from a more realistic perspective. If we choose a very large final time, say several thousand years, the uncertainty about the state of the world and the economy would be so high that the results would be meaningless after a certain point. Due to rapidly changing technological and thus economic outlooks, I would feel very uneasy about inferring anything about the state of the economy more than a hundred years from now. However, if a hundred years produce a large deviance from the steady state solution, it might be more beneficial to go with the first approach.

   A last solution to the infinite time problem is to let the final time be free. You can then require, as a boundary condition, that the solution must end up in the steady state. The final time will now be one of the control variables used in the optimization.

4. Almost all optimal control solvers require initial guess functions for the different variables. There is no particular procedure which will ensure that you have 'good enough' initial guesses for any given problem. However, there are a couple of steps which often produce good results.

   If the underlying model and the first order conditions are similar, though not identical, to a problem with a known solution, you can use the function form of the known solution as an initial guess. For example, the solution to the classic Hotelling model for exhaustible resources (the Hotelling rule) could serve as an initial guess function to the extraction path in this model. The underlying mechanisms are very similar, thus one could expect that the solution for this model would have elements of the Hotelling model solution.

The second approach is to start with a very simple initial guess, e.g. a linear function, and then modify the function in an iterative process of multiple reruns of the optimal control solver. To be successful with this strategy, one need to have a clear and structured plan for how the modifications should be done, so it is possible to detect patterns which results in better (or worse) results. The most reliable approach would naturally be a combination of the two, where both 'smart' guesses and multiple edits are utilized. In some solvers, like $\mathcal{PSOPT}$, there are also specific functions added to help the user with initial guess refinement, making the job a lot easier.

It can be varying preparation requirements between the different solvers, but the steps above are usually always included in some way. Before entering into the land of numerical errors, you should spend a good portion of time to understand the underlying algorithm in your tool of choice. This will enable you to fully utilize the tool at hand, and avoid subtle but possibly crucial mistakes.

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