Working Paper 2/2011

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The CREE Centre acknowledges financial support from The Research Council of Norway, University of Oslo and user partners.
ISBN: 978-82-7988-103-2
ISSN: 1892-9680
http://cree.uio.no

VERSION OF 1 SEPTEMBER 2011

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ABSTRACT

We study the timing and interdependence between optimal innovation policy and environmental policy in a model with emission reduction (abatement) activity and cost-reducing R&D through increasing variety numbers. We find that optimal emission prices and R&D subsidies are substitutes over time. R&D subsidies for clean innovation should start high and fall over time, while optimal emission prices start low and go up. If the lifetime of patents is infinite, we replicate earlier results that suggest R&D policy to be independent of the stage of the environmental problem.

JEL codes: H21, O30, Q42
Keywords: Environmental policy, research and development, pollution taxes, innovation subsidies, patents.

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1. INTRODUCTION

Worldwide emissions of greenhouse gases are growing, and it is recognized that technology improvements are an important element for achieving the deep emission cuts that are suggested in the climate negotiations (see, e.g., surveys in Carraro et al. (2003) and Jaffe et al. (2005)). For instance, they are essential for the success of the 'climate and energy package' of the European Union that entered law in June 2009.[2] The package aims at reducing greenhouse gas emissions by 20% in 2020, compared to 1990, setting carbon prices in energy and energy intensive industries through the EU Emission Trading System and, in addition to that, setting binding targets for renewable energy sources and Carbon Capture and Storage (CCS). The question we address in this paper is whether, in general, setting the environmental prices right is sufficient to trigger the required technological developments, or whether there is need for extra policies directed specifically at the enhancement of abatement technologies. Furthermore, if the answer to the latter question is affirmative, what characterizes the profile of such policies?

Our first main result is to establish that, assuming complete and competitive markets for innovations, optimal environmental policy only needs to set the price of the pollutant at the Pigouvian level. That is, the marginal costs to the emitter should equal the present value of the future stream of marginal damages associated with the emissions.[3] Technology response to environmental policy does not change this fact, as long as the markets for innovations function perfectly, e.g., through patents with infinite lifetime. In other words, environmental policy can be set independently of innovation policy.

Various studies on environmental R&D implicitly assume such perfect markets for innovation (cf Goulder and Mathai, 2000). It is believed, though, that the market for innovations is imperfect, and it is important to extend the analysis of economic policy to imperfect economies (Stern, 2010). Nordhaus (2002), Popp (2004, 2006), and Gerlagh and Lise (2005), for example, in their numerical analyses of R&D and climate policy, assume that the social value of innovations exceeds the private value of innovations by a constant factor 4. Under these circumstances, the apparent

[2] See http://ec.europa.eu/environment/climat/climate_action.htm (accessed 18 March 2011) for details.

[3] We limit the interpretation of the Pigouvian tax to include only environmental damages. This is a choice for convenience, common in environmental economics. In this paper we specify a cumulative absorption capacity for the atmosphere and define the Pigouvian tax as the marginal social costs of meeting the target.

question becomes whether environmental policy needs to complement the Pigouvian tax with innovation policy directed at environmental technology.

The case for a dedicated environmental technology policy is not obvious, and is not immediately implied by an imperfect market for innovations. If the gap between social and private returns on innovation is identical over different economic sectors, then a generic innovation policy can correct the innovation market failure for all sectors jointly. There are, however, reasons why environmental R&D is different (Popp and Newell, 2009; Acemoglu et al., 2010). As is explicitly focused on in our paper, patents typically expire after a certain period and this creates a temporal structure that links the life-cycle of an environmental problem to the attractiveness of environmental R&D for private entrepreneurs. Private and social returns on environmental R&D may follow different cycles. The gap between social and private returns on innovation then changes over the life-cycle of an environmental problem, and environmental R&D policy may need to vary along. Hart (2008) studies how this affects the timing of CO_2 taxes, whereas Goeschl and Perino (2007) study R&D sequences when human kind is confronted with repeating cycles of environmental problems. Our paper can be considered a more detailed study of one such cycle, such as climate change.

Our second and most interesting main finding is that the optimal environmental R&D policy may have a cyclical pattern counter to the pricing policy (e.g. carbon pricing): Assuming finite and constant patent lifetime, the optimal R&D subsidy should initially be high when carbon prices are low, and then gradually decline over time while prices increase. After a certain time, near the end of the life-cycle of the environmental problem, the subsidy should increase and converge to a constant rate. The intuition is that innovations will be biased towards technologies that pay back within the patent's lifetime, so that there is insufficient support through markets to develop and improve abatement technologies when the environmental problem is emerging and (e.g. carbon) prices are still low. Thus, innovation policy should not be set independently of the stage of the environmental problem. Considering climate change and the development of alternative energy technologies, we may read our analysis as addressing the question how the analyses in Nordhaus (2002) and Popp (2004, 2006) referred to above, might change if we explicitly include the patent expiration issue in their numerical models.

The basis of our analytical framework we borrow from the early literature on endogenous growth and environmental policy. Much of the early work in this field studied balanced growth paths (cf. Bovenberg and Smulders, 1995), or transition dynamics where the environment moves from a dirty to a clean steady state (cf.

Bovenberg and Smulders, 1996). However, apart from the questions analyzed, there are two major differences in our analytical model compared to this strand of literature.

First, we do not consider a closed economy but for convenience apply a partial analysis. This choice is based on the observation that most environmental problems are associated with specific sectors. For climate change, the single most important question concerns the costs, speed, and policies required to guide the transition of the energy supply sector towards carbon neutral energy sources. Working with a closed economy model may complicate the analysis unnecessarily. On the other hand, the partial model may create a bias in results as it does not trace the effects of sector-specific policies on other sectors. Stimulating research in the abatement sector that we describe may crowd out research in other sectors outside the model, causing welfare losses not accounted for. We control for this problem by adding a crowding out parameter. A more comprehensive assessment is provided in Section 5.

Second, while most of the endogenous growth literature referred to above studies a one-directional move from a dirty to a clean state, the transition we consider is more cyclical in nature, starting from a clean state. This is based on the empirical evidence: In the context of climate change and most other environmental problems, the life-cycle of the environmental problem starts with low emission levels and a clean environment, moving to high emissions and a large pollutant stock. To prevent an ecological collapse, at some point in time, the economy must move back to a state with low emissions. Emissions thus follow a hump-shaped curve (cf. Smulders and Bretschger, 2000; Hart, 2008). At the initial stage, the Pigouvian tax will rise sharply, but after the first stage, the growth rate of the Pigouvian tax will gradually fall (Hoel and Kverndokk, 1996). The growth in abatement technologies will follow a similar pattern.

Kverndokk and Rosendahl (2007) and Gerlagh et al. (2009) find that this abatement cycle generates a high optimal subsidy rate for abatement when the abatement technology is first adopted, but the subsidy falls significantly over time as the abatement technology matures. Kverndokk and Rosendahl derive these conclusions from a numerical model with learning by doing (LbD). Gerlagh et al. find similar results in an R&D model, but their set up in some ways resembles the learning by doing set up and is somewhat different from the standard R&D model referred to above.

The main contribution of the current paper is to examine analytically the time profile of optimal environmental R&D policy within a more conventional R&D model. Only a few studies have looked into this issue before, and we are not aware of any studies using a formal, conventional R&D model, taking into account patent lifetime and also considering the long-run dynamics towards a balanced growth path, thus this is the core distinction between our R&D model and earlier R&D models in the environmental economics literature. As indicated above, our analysis seems to support the findings in Kverndokk and Rosendahl (2007) and Gerlagh et al. (2009), but only if patent lifetime is finite. This result connects two strands of literature, one relying on learning-by-research mechanisms, the other relying on learning-by-doing mechanisms for the analysis. The result we find resembles previous results from the learning-by-doing literature, while employing the mechanisms of learning-by-research.

Our formal analysis complements empirical studies on environmental R&D. Popp and Newell (2009) estimate that social returns on environmental R&D are, indeed, typically higher than social returns to other R&D investments. However, new environmental R&D may crowd out other R&D. Even if the other R&D has lower social value, any crowding out will dampen the social value of extra environmental R&D. Popp and Newell (2009) find evidence for some crowding out within the energy sector so that R&D in alternative energy technologies crowds out R&D in traditional energy technologies, but little evidence for crowding out in other (non-energy) sectors.

As we focus on the timing of environmental policies, we also connect to the literature on the timing of abatement. Various applied studies on climate change policy have concluded that there is a need for up-front investment in abatement technologies to stimulate innovation (van der Zwaan et al., 2002; Kverndokk and Rosendahl, 2007). Others have argued that this finding is an artefact of the typical models in use where innovation occurs through learning by doing mechanisms. It has been suggested that models that describe innovation through R&D would not support early abatement (Goulder and Mathai, 2000; Nordhaus, 2002). As in this strand of literature, we analyze optimal timing, but we focus on the timing of *abatement policies* rather than on the timing of *abatement levels*.

As mentioned above, central to our analysis is the fact that patents expire after a finite time, and the third strand of literature we refer to is thus the literature on optimal lifetime of patents. Patent policy has obvious welfare implications and Nordhaus (1969) is an early study on this topic. In general, an increase in the patent length is growth enhancing by raising the rate of return on R&D (Judd, 1985). On the other hand, patents create a static inefficiency as patents allow monopolistic supply by the patent holder. Longer patents thereby reduce output, and thus consumption, by increasing the portion of the monopolistic sector. Hence, patents have two opposite welfare effects. Chou and Shy (1993), in a discrete time model, contrast a one period lifetime with an infinite lifetime and find that a one period lifetime is preferred. Iwaisako and Futagami (2003) find an optimal finite patent lifetime to trade-off the two opposite effects. This is followed up in Futagami and Iwaisako (2007) where a finite patent length maximizes social welfare in a growth model that does not exhibit scale effects. These studies focus on balanced growth paths. We extend this literature by also considering optimal patent length along a transition path.

This paper is organised in the following way. In Section 2 we develop the basic model describing the evolution of knowledge through R&D, abatement output, emissions and the stock pollutant. Technological change is driven by the Romer (1987, 1990) type of endogenous growth. We analyze the social optimum, differentiating between short-run and long-run dynamics, by establishing a unique balanced growth path, and show how the optimal path of R&D would develop over time to reach this path. We are then interested in how the social optimum can be implemented in a market and describe in Section 3 the market equilibrium for abatement goods, abatement equipment and innovation. Then, in Section 4, we analyse optimal environmental and innovation policies in the first-best setting. Methodologically, the approach is similar to Hartman and Kwon (2005) and Bramoullé and Olson (2005). In Section 5 we discuss general vs. partial equilibrium effects, whereas in Section 6 we summarise results and conclude.

2. OPTIMAL ABATEMENT AND RESEARCH

We consider an economy with a stock pollutant such as greenhouse gases (GHGs). Further, we assume a given absorption capacity for cumulative emissions, following, e.g., recent climate change literature arguing that human kind needs to set a ceiling to cumulative emissions to safeguard the earth climate system.[4]

The abatement production model has a similar structure as the model in Iwaisako and Futagami (2003). It is based on Romer's endogenous growth model, with horizontal innovation of the 'love of variety' concept (Romer, 1987, 1990; Barro and Sala-i-Martin, 1995; Dixit and Stiglitz, 1977; Gancia and Zilibotti, 2005). The model explicitly describes patents as in Futagami and Iwaisako (2007), but extends their model as it has an infinite horizon with continuous time t. There is one representative abatement sector, which could either be interpreted as abatement of emissions (e.g., carbon capture and storage), or as an alternative, emission-free, resource sector (e.g., renewables). There are H_t different abatement technologies at each point of time t,

[4] The well-known objective of the United Nations Framework Convention on Climate Change (1992) is "stabilization of GHG concentrations in the atmosphere at a level that would prevent dangerous anthropogenic interference with the climate system". In recent years various authors have suggested that under limited knowledge of the climate system, this objective should be interpreted as a bound on cumulative emissions (Kharecha and Hansen 2008, Allen et al. 2009, and Zickfeld et al. 2009).

which, e.g., could be different wind mill designs (onshore/offshore), solar panels, hydro power technologies, carbon capture technologies etc. An R&D sector develops new technologies. Technological progress takes the form of expansion in the number of different abatement technologies, i.e., increased variety of abatement equipment.

The social planner aims at minimising the present value of social abatement costs, discounted at a constant rate ρ, subject to an upper bound on cumulative emissions. We can think of this upper bound as the assumed cumulative absorption capacity. Current emissions exhaust the absorption capacity, so that in economic terms, the absorption capacity acts as an exhaustible resource.

Let E_t be emissions and let S_t be the remainder of the cumulative absorption capacity. Initial absorption capacity is given by S_0, the capacity constraint by $S_t \geq 0$, and the dynamics are as follows:[5]

$$\dot{S}_t = -E_t. \tag{1}$$

This gives a cyclical pattern of the environmental problem. We start from a clean state, then emissions are positive, but they approach zero when S_t approaches zero.

The overall economy grows exogenously, and we assume that benchmark emissions Y_t increase at a fixed rate g, while emissions can be reduced by abatement effort A_t:[6]

$$E_t = Y_t - A_t \geq 0. \tag{2}$$

Production of abatement requires the input x_i of abatement equipment, where subscript $i \in [0, H_t]$ refers to variety i, and H_t is the number of equipment varieties. H_t can also be interpreted as the state of knowledge. Building on the horizontal

[5] By 2010, cumulative emissions of CO_2 have reached about 525 GtC. Annual CO_2 emissions related to fossil fuel use deforestation are currently around 8 GtC/yr. The numbers exclude other GHGs, which also provide a substantial contribution to global warming. The papers cited in the previous footnote suggest that, in order to maintain a high probability that global mean temperatures will not increase by more than 2 degrees Celsius (compared to 1900), we should keep cumulative CO_2 emissions below ca. 1000 GtC.

[6] Y can be interpreted as energy demand, which is then treated as price-inelastic throughout the analysis. The relation between emissions and benchmark emissions is specified as a linear function for convenience of notation (a common assumption, cf. e.g. Goulder and Mathai, 2000)

innovation literature (see also Goeschl and Perino, 2007, Greaker and Pade, 2009, and Gerlagh et al., 2009), abatement is produced according to:[7]

$$A_t = \int_0^{H_t} x_{t,i}^{\beta} \mathrm{d}i, \tag{3}$$

where $0<\beta<1$, i.e., each type of abatement technology has decreasing productivity when expanded. The different varieties of abatement equipment are neither direct substitutes nor direct complements to other specific equipment. That is, the marginal product of each abatement equipment is independent of the quantity of any particular other type of equipment. Examples of this may be different abatement equipments to produce alternative energy (such as wind power, hydro power and solar power). Each variety (technology) has its own ideal site specifics, but the potential of each variety is limited so that new varieties have to be developed to increase the total amount of alternative energy that can be produced at certain marginal costs. For instance, wind power may be most valuable in areas with strong wind, and offshore wind power technologies expand the potential for wind power. Further, hydro power offers potential in areas with large waterfalls, and solar power in areas with high solar radiation inflow.[8] For our analysis we assume that decreasing returns to scale for varieties are not too strong, that is, $\beta>½$. As we will see in the next section, this condition also follows by assuming that the mark up on prices under monopolistic competition, where each innovator owns his own variety, is less than 100% (which seems reasonable). Due to symmetry, we find that aggregate production becomes:

$$A_t = H_t x_t^{\beta} \tag{4}$$

Individual innovator j develops an amount $\mathrm{d}H_{t,j}$ of new varieties proportional to his individual effort $\mathrm{d}R_{t,j}$; $R_t=\int \mathrm{d}R_{t,j}\mathrm{d}j$ denotes aggregate research efforts by all innovators at time t. We assume that research crowds out the amount of new varieties

[7] We disregard any time lags between when the costs of installing abatement equipments occur, and when abatement actually takes place (e.g., installation time of new energy capacity). We also disregard time lags in the innovation process. These time lags are of course important in a short- to medium-run analysis, but of less importance in our long-term context (cf. also the horizontal innovation literature).

[8] Similar arguments can be made about carbon capture, where different technologies exist and can be used to capture CO_2 from different sources (e.g., production of coal power, gas power, steel, cement etc.). Post-combustion technologies can often be used on several sources, whereas pre-combustion technologies are more process-specific.

found by other researchers, or alternatively that research resources are scarce, so that the following production function for new knowledge applies:

$$\mathrm{d}H_{t,j} = \mathrm{d}R_{t,j}\ R_t^{\psi-1}, \tag{5}$$

where $0<\psi<1$ measures the rate of return on R&D at the aggregate level. Thus, equation (5) implies a negative externality from R_t through crowding out of current research. The externality is more severe the lower is the value of ψ.[9] On the other hand, there is a positive spillover of research unless the innovator is able to reap all future profits from production of the new variety. Thus, as we will see below, patent rules are of major importance.[10]

Aggregation of (5) gives R_t^{ψ} as the aggregate number of new innovations, or the flow of new varieties that adds to the pool of knowledge, H_t:

$$\dot{H}_t = R_t^{\psi}\ . \tag{6}$$

Comparison of (5) and (6) shows that whereas a single researcher exhibits constant returns to scale, the sector as a whole bears diminishing returns to scale. This could be motivated by congestion externalities originating from different researchers' efforts on the same product.

As we study a partial model, there is the possibility that additional research in the abatement sector goes at the expense of (i.e., crowds out) research in sectors outside the model. Let υ denote the crowding out factor. Then, the social abatement costs are the sum of the costs of abatement equipment $H_t x_t$ and the social costs of research

[9] The crowding out assumption basically means that the sector as a whole bears decreasing returns to scale within a period due to, e.g., congestion externalities originating from different researchers' efforts on the same product. This may be a reasonable assumption, as it will smooth the research path over time. Assume instead constant returns to scale, i.e., ψ=1. Then the conclusion from the optimisation problem below would be that we should delay all abatement until the pollution problem is so severe that the safe pollution threshold is reached. At this point of time, research spikes, and abatement costs and pollution levels drop close to zero.

[10] There are other imperfections of research that could have been introduced, but would not have changed the main results. For instance, this model does not specify a dynamic spillover effect based on earlier research, such as "standing on shoulders" or learning effects. This could have been introduced for instance by letting dH increase in H, see, e.g., Goulder and Mathai (2000) and Gerlagh et al. (2009). Such adjustments would probably strengthen the main results below that innovation should be stimulated strongest initially.

$(1+\upsilon)R_t$, where all unit costs are equal to one (note that all varieties are equally productive):

$$C_t = H_t x_t + (1+\upsilon)R_t, \tag{7}$$

Thus, we have crowding out effects of research both within the abatement innovation sector (ψ), and in other research sectors (υ). For notational convenience, below we will substitute κ for $1+\upsilon$, so that it measures both direct costs plus crowding out costs.

Social Optimum

The social planner minimizes the net present value of all future costs

$$V(H_0,S_0,Y_0) = \min \int_0^\infty e^{-\rho t}[H_t x_t + \kappa R_t]\mathrm{d}t, \tag{8}$$

subject to the restriction on the environmental stock $S_t \geq 0$, stock accumulation dynamics (1) and (6), and production equations (2) and (4), with x_t, and R_t as the control variables. We notice that for $H_0=S_0=0$, there exists no solution because emissions cannot be decreased to zero without a prior knowledge stock. However, as long as either knowledge is strictly positive, $H_0>0$, or the cumulative emission allowance is positive, $S_0>0$, a solution exists.

For the analysis below, it is convenient to use the intensive form of the stock variables, i.e., knowledge H_t and cumulative absorption S_t per emissions Y_t. As the equation system does not have constant returns to scale, however, the intensive form for H_t is not simply knowledge divided by emissions. We first have to assess the overall returns to scale of the abatement sector. Let us consider the change in abatement levels when both research costs and abatement expenditures increase by factor two. If research expenditures R_t increase by factor two for all time periods up to point t, knowledge H_t increases by factor 2^ψ. For abatement expenditures $H_t x_t$ to increase by factor two, the intensity of abatement equipment use x_t must increase by factor $2^{1-\psi}$. Abatement output A_t then increases by factor 2^λ with $\lambda=\psi+\beta(1-\psi)<1$ (cf. (4)). Thus, when all expenditures increase by factor 2, abatement increases by less and the abatement sector has overall decreasing returns to scale.[11] The lemma below

[11] We can also express the returns to scale the other way around, from levels of abatement to costs. Consider two paths, and in the second of these abatement is larger by factor 2. If expenditures on research and abatement equipment differ proportionally, in the second path costs will be higher by factor 2^γ with $\gamma=1/(\psi+\beta(1-\psi))>1$.

specifies the normalization that defines the analysis in intensive form. We omit time subscripts for convenience.

LEMMA 1. *There exists a function* $v(h,s)$, $v_h<0$, $v_s<0$, *such that net present value costs satisfy*

$$V(H,S;Y)=Y^{\gamma}v(\frac{H}{Y^{\gamma\psi}},\frac{S}{Y}) \tag{9}$$

with $\gamma=\frac{1}{\lambda}=\frac{1}{\psi+\beta(1-\psi)}>1$.

Proof. See the Appendix.

Lemma 1 informs us that we can conveniently analyze the dynamics using normalized variables, $h_t=H_t/Y_t^{\gamma\psi}$, $s_t=S_t/Y_t$, $\chi_t=x_t/Y_t^{\gamma(1-\psi)}$, $r_t=R_t/Y_t^{\gamma}$, $p_t=v_h(.)=Y_t^{\gamma(\psi-1)}\eta_t$ and $q_t=v_s(.)=Y_t^{1-\gamma}\theta_t$, where $\eta_t=V_H(.)$ and $\theta_t=V_S(.)$ are the shadow prices for knowledge H_t and the cumulative absorption capacity S_t, respectively. Notice that the normalization implies $h_t\chi_t^{\beta}=1$ if $E_t=0$ and $h_t\chi_t^{\beta}<1$ if $E_t>0$, and that social abatement costs become

$$H_tx_t+\kappa R_t=Y_t^{\gamma}(h_t\chi_t+\kappa r_t)\,. \tag{10}$$

On a balanced growth path, normalized variables remain constant and the social abatement costs increase at rate γg. To ensure finite net present costs, we require that the discount rate is at least equally large:

$$\rho>\gamma g. \tag{11}$$

Bellman's principle tells us that the relation (9) holds for all t, and that two optimal paths will not cross in (h_t,s_t) space. Thus, the lemma shows that the dynamics of the social optimum are fully captured through the two state variables h_t and s_t, and their dual variables p_t and q_t. Also, notice that since $\dot{S}_t\leq 0$, and $\dot{Y}_t>0$, we must have $\dot{s}_t<0$ iff $s_t>0$, and $\dot{s}_t=0$ iff $s_t=0$.

Let us consider the current value Hamiltonian, $\mathcal{H}_t$:

$$\mathcal{H}_t = H_tx_t + \kappa R_t - \theta_t\dot{S}_t - \eta_t\dot{H}_t - \varepsilon_tE_t - \lambda_tS_t, \tag{12}$$

where ε_t and λ_t are the dual variables for the non-negativity constraints for E_t and S_t, respectively. We have changed sign for θ_t and η_t such that they are positive and can be interpreted as the shadow prices for the absorption capacity and knowledge, respectively. The first-order conditions read (where we omit the time subscripts):

$$0 = \mathcal{H}_x = H - \beta(\theta - \varepsilon)Hx^{\beta-1} \tag{13}$$
$$0 = \mathcal{H}_R = \kappa - \psi\eta R^{\psi-1} \tag{14}$$
$$\dot{\theta} = \rho\theta + \mathcal{H}_S = \rho\theta - \lambda \tag{15}$$
$$\dot{\eta} = \rho\eta + \mathcal{H}_H = \rho\eta - (\beta^{-1} - 1)x \tag{16}$$
$$\lambda S = 0;\ \varepsilon E = 0 \tag{17}$$

Note that equation (16) is derived by using (1), (2), (4) and (13). We can rewrite the first-order conditions in intensive form, with $p_t = Y_t^{\gamma(\psi-1)}\eta_t$ and $q_t = Y_t^{1-\gamma}\theta_t$. For completeness, we will also write the dynamics (1) and (6) in intensive form. We note that we do not need to normalize ε and λ, as these are co-state variables for which only the complementarity with $e_t = E_t/Y_t$ and $s_t = S_t/Y_t$ matters.

$$\dot{s}_t = -gs_t - (1 - h\chi^{\beta}) \tag{18}$$
$$\dot{h}_t = r_t^{\psi} - \psi\gamma g h_t \tag{19}$$
$$\chi_t = (\beta q_t)^{1/1-\beta} - \varepsilon \tag{20}$$
$$r_t = (\psi p_t/\kappa)^{1/1-\psi} \tag{21}$$
$$\dot{q} = [\rho + (1-\gamma)g]q - \lambda \tag{22}$$
$$\dot{p} = [\rho - (1-\psi)\gamma g]p - (\beta^{-1} - 1)\chi \tag{23}$$
$$\lambda s = 0;\ \varepsilon e = 0 \tag{24}$$

Long-term dynamics

We first establish properties for the long run, when $s_t=0$, and thus also $e_t=0$. Define the time T as the earliest time at which $s_T=0$. In the long-run, we only need to analyze the dynamics for the other state variable, the knowledge stock h_t, and its co-state variable p_t. Since emissions are zero, we have $Y_t = H_t x_t^{\beta}$, which we can rewrite as $h_t\chi_t^{\beta}=1$. By substitution of (21) in (19), and of $h_t\chi_t^{\beta}=1$ in (23), we find the two-equation dynamics for the state-co-states h_t and p_t:

$$\dot{h}_t = (\psi p_t / \kappa)^{\psi/(1-\psi)} - \psi\gamma g h_t \tag{25}$$
$$\dot{p}_t = [\rho - (1-\psi)\gamma g]p_t - (\beta^{-1} - 1)h_t^{-1/\beta} \tag{26}$$

The state-co-state dynamics produce the phase diagram depicted in Figure 1. The locus for $\dot{p}_t$=0 lies in the positive quadrant and is downward sloping because $(1-\psi)\gamma g<\gamma g<\rho$ (cf. (11)).

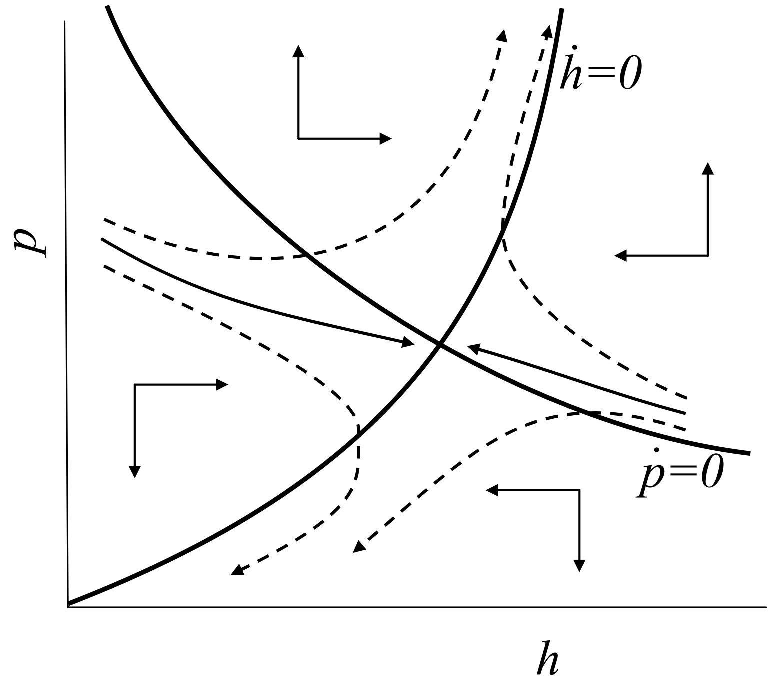

FIGURE 1. *Phase diagram for the long-run optimal path*

It is immediately clear from the phase diagram that a unique balanced growth path exists where the normalized variables h_t, χ_t, p_t and q_t are constant. Furthermore, the balanced growth path has saddle-point stability, and the unique saddle path to the balanced growth path has h increasing and p decreasing, or the other way around. Thus, when the initial knowledge stock is below the balanced growth level, $h_t<h^*$, the balanced growth path is approached from the upper-left and the price of knowledge p_t decreases. Along this path, the growth rate of h_t will decrease as it is increasing in p_t and decreasing in h_t (cf. (25)). From $h_t\chi_t^{\beta}=1$, it then follows that χ_t and x_t will have an increasing growth rate. We summarize this in the proposition below.[12]

PROPOSITION 1. *A unique balanced growth path exists with* s_t=0, h_t=h^*. *Off the balanced growth path, if* s_T=0 *and* $h_T<h^*$, *then for all* $t>T$:
$\dot{H}_t/H_t > \psi\gamma g$ *and* $\dot{H}_t/H_t$ *is decreasing (< and increasing if* $h_T>h^*$*),*
$\dot{x}_t/x_t < (1-\psi)\gamma g$ *and* $\dot{x}_t/x_t$ *is increasing (> and decreasing if* $h_T>h^*$*).*

[12] The lower and upper bounds for the growth rates of H_t and x_t follows from the definitions of h_t and χ_t.

The proposition above completes the analysis of the long-run. We now turn to the short-term dynamics in state space (h_t, s_t). The main idea of the short-term analysis is to show that when the initial knowledge stock is small, say h_0=0, then throughout time the knowledge stock will remain small (in a precise way defined below), and when the absorption capacity of the environmental stock is exhausted, s_T=0, the balanced growth path is approached from below. This property will then enable us to sufficiently characterize the short- plus long-run dynamics so as to establish all required properties regarding the private and social value of knowledge.

Short-term dynamics

To analyze the short term, we run the dynamics of (18)-(24) backwards in time. That is, we take some pair (h_T, p_T) on the stable manifold of Figure 1, and let $\lambda_T=\varepsilon_T=0$. Then we consider what happens if $\lambda_t=\varepsilon_t=0$ for all $t \leq T$. q_t increases exponentially at rate $\rho-(\gamma-1)g>0$ up to $t=T$ (cf. (22)), and so χ_t increases at rate $[\rho-(\gamma-1)g]/(1-\beta)>0$ (cf. (20)). Thus, if the path enters balanced growth at $t=T$, so that (25) and (26) are zero for $t=T$ and $h_T=h^*$, it follows that the right-hand-side of (23) is positive for $t<T$. That is, p_t increases for $t<T$. It then follows that the right-hand-side of (25) is negative, so that h_t decreases for $t<T$. The path is depicted as line *B* in Figure 2. If we include the dynamics for s_t in (18), we can construct a corresponding path $\{(s_t, h_t)\}$ that goes backwards in time from t=T to t=0. From Bellman's principle it is then obvious that any element on this path can be taken as initial condition. This path is depicted as line *B* in Figure 3. The next proposition describes the features of this line.

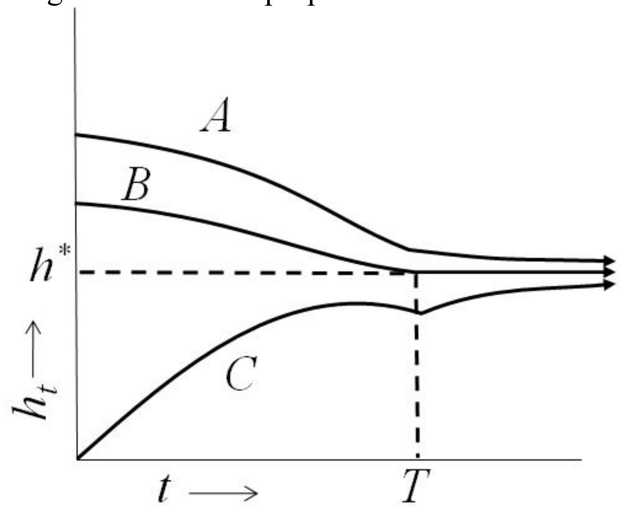

FIGURE 2. *Dynamics of knowledge*

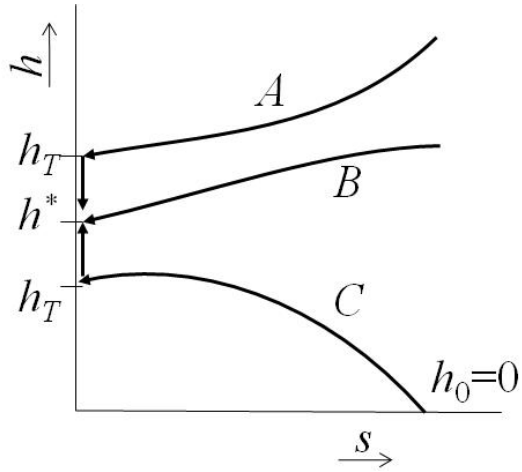

FIGURE 3. *Dynamics in state-space*

PROPOSITION 2. *For any $s_0>0$, there is a unique h_0, with $\partial h_0/\partial s_0>0$, such that the optimal paths for initial conditions (s_0,h_0) enter balanced growth in finite time.*

Bellman's principle also informs us that in state space as shown in Figure 3, optimal paths cannot cross. Therefore, as all paths move to the left (cf. (18)), any initial condition (s_0,h_0) with h_0 below (above) line *B* will reach $s_T=0$ in finite time with $h_T<h^*$ $(h_T>h^*)$, cf. line *C* (*A*) in Figure 3. This proves the next proposition.

PROPOSITION 3. *For initial conditions $s_0>0$, $h_0=0$, when the optimal path enters the long term dynamics at $t=T$, we have $s_T=0$ and $h_T<h^*$.*

We have now established that if we start without initial specific abatement knowledge, the knowledge stock will still be below the balanced growth level when we enter the long-run dynamics. The result is intuitive and it will be essential to establish how the value of knowledge develops over time.

3. MARKET EQUILIBRIUM

We now take a look at how we can implement the first best allocation through research subsidies, or changing the lengths of patents. Thus, we first explore the precise structure of innovation.

The producers of the abatement equipment own patents and, therefore, receive monopoly profits. However, they have to buy the innovations from the R&D sector, where innovators are competitive and use research effort as an input.[13] We assume that patents have a certain lifetime L_t, and that the equipment can be produced free of charge by anyone after expiration of the patent. Notice that we allow for the patent lifetime to change over time, and to be used as a policy instrument. Free entry is assumed in all markets, including the market for innovation. By assuming a fixed durable input in production which is normalized to one, producers may still make profits. As mentioned before, we assume negative externalities from aggregate current research through crowding out of research effort. Thus, in this model there are four imperfections related to innovations: Too little production of patented abatement equipment due to monopolistic competition, positive spillovers of innovation after the expiration of the patent as innovators maximise profits over the patent lifetime only, negative spillovers of total research effort on new innovations, and crowding out of innovations in other sectors. The level of innovations supported by the market may therefore exceed or fall short of the social optimal level. As innovation is taking place in private firms, the role of the government is to create incentives to achieve the social optimal levels of innovation.

We distinguish between two different types of equipment; those with patents expired ($y_{t,i}$), and those with running patents ($z_{t,i}$). The number of varieties with expired patents is denoted M_t, and the number of varieties with running patents is denoted N_t. Adding up both gives the total knowledge stock

$$H_t = N_t + M_t. \tag{27}$$

All varieties have the same unit production costs. The varieties with expired patents are produced competitively, and sold at unit price. Because of symmetry between the varieties, in equilibrium the same quantity will be employed of each equipment with expired patent, i.e., $y_{t,i}=y_t$. The varieties with running patents are produced by the patent holder, and sold at a mark up price $w_{t,i}$. Again, because of symmetry, we have $w_{t,i}=w_t$ and $z_{t,i}=z_t$ for equipment with running patents.[14] The abatement production identity then becomes:

$$A_t = M_t y_t^{\beta} + N_t z_t^{\beta}. \tag{28}$$

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[13] Alternatively we could assume that the innovators are producing the abatement equipment, so that they own the patents and receive the monopoly rent. This would not change the arguments or conclusions of the analysis.

[14] In the following we will therefore omit the subscript *i*.

The flow of new varieties R_t^{ψ} adds to the pool of patented knowledge, N_t, but after a period L_t these varieties leave the pool of patented knowledge and enter the pool of patent-free knowledge M_t:

$$\dot{M}_t = R_{t-Lt}^{\psi} \tag{29}$$

$$\dot{N}_t = R_t^{\psi} - R_{t-Lt}^{\psi} \tag{30}$$

We now describe the market equilibrium, given a set of policy instruments. In the next section we search for the first-best policy.

Abatement goods

The public agent implements an emission tax τ_t, or more generally an environmental policy that induces a cost of emission in the market. From (2) we see that this translates into a market price for abatement A_t, as E_t and A_t are perfect substitutes. Equipment with running patents is subsidized at rate ω_t to correct for market power.[15] The abatement producer maximises the value of production minus the input costs:

$$\text{Max } \tau_t A_t - M_t y_t - N_t(1-\omega_t) w_t z_t, \tag{31}$$

subject to (28) , where y_t and z_t are the control variables.

The first order conditions of this maximisation problem determine the abatement producer's demand for patent-free and patented varieties, respectively:

$$y_t = (\beta\tau_t)^{1/(1-\beta)}, \tag{32}$$

$$z_t = (\beta\tau_t/(1-\omega_t)w_t)^{1/(1-\beta)}, \tag{33}$$

The first order condition for patent-free varieties y_t in (32) is similar to the corresponding condition under the social optimum given by (13), with the exception that the social price of abatement, θ_t, is replaced by the market price of abatement, τ_t (recall that ε_t=0). In other words, the Pigouvian tax is replaced by the emission tax. For patent-holding varieties z_t, the market equilibrium (33) can be matched to the social optimum if we set a subsidy $\omega_t = 1-1/w_t$ jointly with implementing the Pigouvian tax, i.e., $\tau_t=\theta_t$.

[15] Other policy instruments such as licensing and contracts could also be used to correct for market power due to the patent system, see, e.g., Maurer and Scotchmer (2006) and Scotchmer (1991).

Monopolistic supply of abatement equipment

Acting as monopolists, the producers of patented abatement equipment maximise profits at each point in time, π_t, taking into account the falling demand curves for abatement equipment (again we omit subscript i):

$$\text{Max } \pi_t = z_t(w_t - 1), \tag{34}$$

subject to (33). We notice that 'profits' refer to the rent value of the patent and not to a surplus. Free entry ensures the zero-profit condition: net revenues from selling the equipment minus production costs equal the rent that the monopolist pays to the patent holder.

The first order condition from maximising (34) with respect to w_t determines the price of the abatement equipment:

$$w_t = w = 1/\beta. \tag{35}$$

From (33) and (35) we find the market equilibrium level of z_t:

$$z_t = (\beta^2 \tau_t/(1-\omega_t))^{1/(1-\beta)}. \tag{36}$$

Using (34) we find the rent value of a patent:

$$\pi_t = (\beta^{-1} - 1) z_t. \tag{37}$$

The value of a patent can now easily be calculated as the net present value of the future patent rents, over the patent lifetime L_t:

$$V_t = \int_0^{L_t} e^{-\rho t} \pi_{t+u} \, du = (\beta^{-1} - 1) \int_0^{L_t} e^{-\rho t} z_{t+u} \, du. \tag{38}$$

Notice that the value of a patent increases with the patent lifetime, the deployment subsidy and the emission tax, as the demand for equipment increases with both the subsidy and the tax (cf. (36)). Thus, all these policy instruments affect the incentives for research.

Markets for innovation

The innovators maximise profit with respect to research effort, where the price of the innovation equals V_t, i.e., the net present value of the patent over its lifetime. The government subsidizes research expenditures at a rate σ_t. Thus, the innovators' maximization problem is:

$$\text{Max } V_t \mathrm{d}H_{t,j} - (1-\sigma_t)\mathrm{d}R_{t,j}, \tag{39}$$

subject to (5) .

First order conditions give that the unit cost of research, which is set equal to one, is equal to the value of the patent, V_t, multiplied by the productivity of $\mathrm{d}R_{t,j}$, $R_t^{\psi-1}$. Due to the zero-profit condition, in equilibrium the value of all patents is equal to the value of all research effort:

$$V_t \, R_t^{\psi} = (1-\sigma_t)R_t. \tag{40}$$

The eight equations (28), (29), (30), (32), (36), (37), (38) and (40) define a market equilibrium through the variables A_t, M_t, N_t, y_t, z_t, π_t, V_t, R_t, for a given environmental tax policy τ_t, subsidies ω_t and σ_t, and patent lifetime L_t. It is straightforward to see that given a path for the policy instruments, the equilibrium exists and is unique; this is a prerequisite for the public agent to steer the economy towards the efficient allocation. Equations (32) and (36) determine the equipment inputs y_t and z_t, respectively. Substitution of (36) in (37) provides π_t, and subsequent substitution in (38) gives an unambiguous value for a new patent at time t, V_t, as dependent on future taxes and deployment subsidies. Subsequently, (40) determines the research effort dependent on the current research subsidy, and (29) and (30) determine the state of knowledge for all t. Finally, (28) determines the abatement level.

4. FIRST-BEST R&D POLICY

Note that innovations depend on the tax and subsidy policies for the coming L_t periods. When patent lifetime L_t goes to infinity, innovators take into account benefits over the full future horizon. On the other hand, when patent lifetime is finite, then innovators are short or medium-sighted, and thus there is a positive externality from innovations. This feature is the core distinction between our R&D model and earlier R&D models in the environmental economics literature.

We now compare the social optimal research effort (14) with the market equilibrium research effort (40). We rewrite the latter as (using (38)):

$$R_t^{1-\psi} = (1-\sigma_t)^{-1}(\beta^{-1}-1)\int_0^{L_t} e^{-\rho u}\, z_{t+u}\, du \tag{41}$$

A comparison with (14), using (16) and $x_t = z_t$, quickly reveals the optimal research subsidy level:

$$\sigma_t = 1 - (\kappa/\psi) \int_0^{L_t} e^{-\rho u} z_{t+u}\, du \; / \int_0^{\infty} e^{-\rho u} z_{t+u}\, du. \tag{42}$$

Note that the subsidy rate may be negative if negative externalities from abatement research, i.e., crowding out of other abatement research ($\psi<1$) and research in other sectors ($\kappa>1$), dominate the positive externalities that appear after the patent has expired.

Comparing the social optimum in equation (13) with the market equilibrium in (32) and (33), and using the market price defined by (35), we find the optimal policy instruments to be $\tau_t=\theta_t$ and $\omega_t = 1-\beta$ when emissions are positive. When emissions are zero, the tax is set exactly such that abatement equals benchmark emissions, while the optimal subsidy remains the same.

We are now able to define the first best policy to obtain the social optimum. Through a tax on emissions equal to the Pigouvian tax, $\tau_t=\theta_t$, a subsidy on patented abatement equipment equal to $\omega_t = 1-\beta$, and a patent lifetime L_t combined with an R&D subsidy/tax σ_t that satisfies (42), the first-best outcome can be implemented. The reasoning is clear. There are three groups of imperfections in the model; i) emissions, ii) imperfect competition in the market for patented abatement equipment, and iii) positive and negative externalities of research effort. Remember that the last group of imperfections comprises three externalities, one positive and two negative (crowding out effects). Therefore, we would need three (combinations of) policy instruments to implement the social optimum: a tax on emissions, a subsidy to production of patented abatement equipment, and a combination of research subsidy/tax and patent lifetime. Policy makers can choose to either fix the patent lifetime and adjust the research subsidy, or to fix the research subsidy and adjust the patent lifetime.

In order to shed light on the optimal combination of patent lifetime and research subsidy given by (42), we will consider three specific cases. As noted in the introduction, we are particularly interested in the dynamics of the instruments. First, the following proposition considers the implications of having patents that remain valid infinitely.

PROPOSITION 4. *For patents with infinite lifetime, $L_t \to \infty$, the efficient R&D subsidy/tax that implements the first-best outcome is constant for all t*: $\sigma_t = 1-\kappa/\psi$.

The proof follows straightforwardly from (42) and looks simple, but its meaning is more subtle. If innovation markets are complete, i.e., infinite lifetime of patents, innovation policy can be separated from environmental policy. That is, the stage of the environmental problem has no effect on the R&D subsidy. As mentioned in the introduction, this result resembles the typical assumption in integrated assessment models with R&D (Nordhaus 2002, Popp 2004, 2006, Gerlagh and Lise 2005). The level of the subsidy now depends on the crowding out effects in the abatement sector (ψ), and the costs or benefits of pulling research effort from other sectors (κ). With infinite patents, the private sector captures the entire social value of knowledge. However, as innovators increasingly develop the same knowledge as other innovators when their expenditures increase, research has a negative externality and a tax is appropriate. On the other hand, if other sectors have similar research externality characteristics, we should expect that $\kappa<1$, reducing the optimal tax level. The proposition suggests that, in the case of infinite patents, abatement research should face the same tax or subsidy as other research activities, given that the different research activities have similar characteristics. Indeed, this also seems intuitive when abatement is not a different type of activity when compared to other sectors.

As noticed in the introduction, the abatement sector differs from other sectors through its cyclical behaviour as studied through the short-term analysis of the previous sections. In the case of finite patents, that is, when innovation markets are incomplete, the cyclical behaviour is cause for a non-constant subsidy level. This case is highly relevant, as real-world patent lifetime is not infinite.[16] The following proposition states that if patents have *constant* but finite lifetime, we must dynamically adjust the research subsidy to implement the first best.

PROPOSITION 5. *Consider the case that patents have constant finite lifetime, $L_t=L<\infty$, and the initial knowledge stock is zero, $h_0=0$. Then there is a t^* with $T–L<t^*<T$ such that the research subsidy that implements the first-best decreases monotonically for $0\leq t\leq t^*$, and increases afterwards (for $t\geq t^*$).*

The full proof is provided in the appendix, but the conceptual mechanisms are readily understood, using the figures below.

[16] For instance, patent lengths in the US and the EU are 20 years.

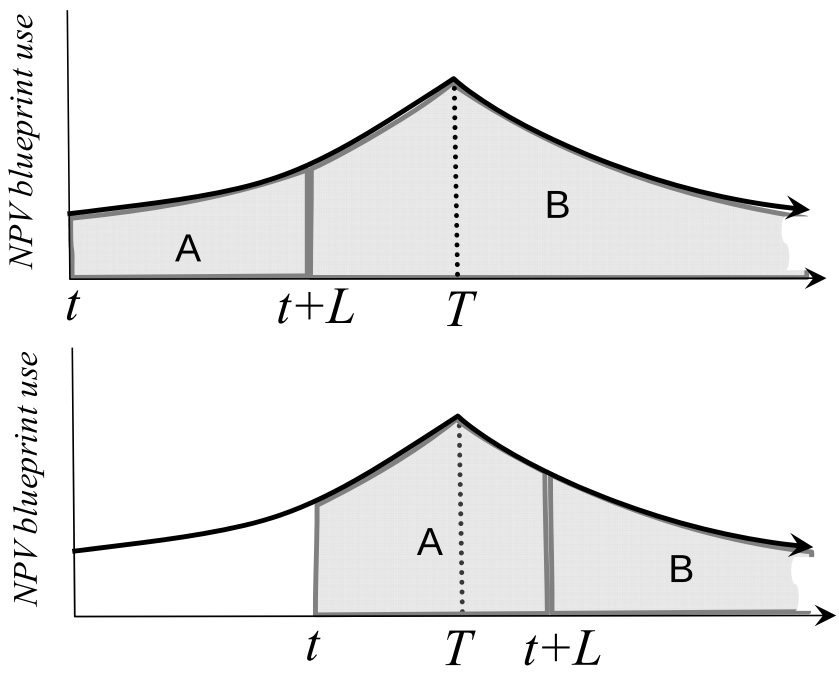

FIGURE 4. *Dynamics of Private versus Social value of blueprints*

The figure shows schematically the rent value of a blueprint for abatement technology, i.e., π_t in (34), evaluated at time zero (discounted). In the early stages, the price of emissions and the use of blueprints are low, so that the rent value is low. As the emission price grows rapidly, faster than the interest rate, the net present value rent goes up from t=0 to t=T. After the first phase of rapid growth, from time T onwards, the growth of abatement drops to the growth of benchmark emissions Y_t. The intensity in the use of knowledge grows slower and the net present value decreases. In the figure, at time t, the private value of a new patent is equal to the aggregate rent value over the next L periods, that is, area A. The social value is equal to the private value plus the rent value after expiration, A+B, where B extends to the right to infinity.

It is immediately clear from the top diagram that in the early phase, the private value A is small compared to the social value A+B. With finite patent lifetime, the private benefits of innovation will typically be low compared to the social benefits. Consequently, the optimal subsidy should be relatively high.

As time passes, and we move from the top to the bottom diagram, the share of private value A in total social value A+B increases. That is, the main benefits of the technology come at later stages, when the price of emissions has risen. Innovations developed during this stage yield a high rent value to the innovators, during the lifetime of the patent, and thus the need for research subsidies diminishes. A straightforward interpretation of our results is that initially environmental policy should focus on knowledge development, while employment of abatement technology becomes relatively more important at a later stage of the policy cycle.

To understand why optimal research subsidies go up again after t^*, we need a more subtle argument. Innovations rapidly increased the knowledge stock during the first phase, but at time T, the level of knowledge has still not reached the balanced growth level. This means that the growth rate of knowledge is still high and decreasing, and consistently the intensity of knowledge use, which is the rent value of blueprints, is rapidly decreasing. But if the rent value is rapidly decreasing, that means that the current rents, which make up the private value, are high compared to future rents, which make up the social value. Over time, as the knowledge stock reaches its balanced growth path, the private versus social value of knowledge goes down and converges to a constant ratio.[17] The subsidy, inversely, goes up and converges.

From this last argument, it also becomes clear that the last part of the proposition is reversed if the initial knowledge stock h_0 is sufficiently large so that knowledge at $t=T$ exceeds the balanced growth level, $h_T>h^*$, In that special case, the research subsidy that implements the first-best decreases monotonically for all time t.

Rather than varying the research subsidy over time, we could instead adjust the patent lifetime. Though there may be various practical problems to dynamically adjust the patent lifetime, the question of the optimal patent lifetime is considered a relevant question in the literature (cf Futagami and Iwaisako, 2007). For completeness we thus translate the above result to the dynamic lifetime context:

PROPOSITION 6. *Consider the case with constant research subsidies, $\sigma_t=\sigma$, and a varying patent lifetime L_t. Then there is a t^* with $T-L_t<t^*<T$ such that the first-best patent lifetime decreases monotonically for $0\leq t\leq t^*$, and increases afterwards (for $t\geq t^*$).*

Proof: Similar as for Proposition 5, see the appendix, but with L_t instead of ratio V_t/η_t. ■

We notice that granting longer patent lifetime is not without social costs. As they grant longer monopoly power, they distort future production, or alternatively, require future public funds to correct for market power in the market for abatement equipments. On the other hand, the need for public funding of R&D is reduced accordingly.

[17] The appendix shows (Figure 5) and discusses the profile of the growth rate.

Together, Propositions 4-6 show that policy makers have some flexibility in their choice of research policy. They can either choose an infinite patent lifetime combined with a fixed subsidy/tax on research (Proposition 4), or, if they want to avoid infinite patent lifetime, they can pick a constant research subsidy *or* patent lifetime, and adjust the other instrument in line with the stage of the environmental problem (Proposition 5 and Proposition 6). That is, with incomplete innovation markets, there is a clear link between the first-best innovation policy and the stage of an environmental problem.

5. GENERAL VS PARTIAL EQUILIBRIUM

A more precise interpretation takes into consideration the partial equilibrium context of our analysis. In general equilibrium, there is competition for inputs to research and development, which means that an increase in abatement-related research may crowd out other research and dampen overall growth. Also, we do not explicitly model the distortionary effects of taxes needed to pay for research subsidies. Such questions are in the domain of general equilibrium models (Bovenberg and Smulders 1995), and our analysis cannot be read as a suggestion that clean R&D should receive special treatment compared to R&D in other sectors, per se. Specifically, we do not suggest that subsidies for clean innovations always provide a double dividend. The new insight from our analysis is complementary to the insights from general equilibrium models in the sense that we explicitly consider the dynamic and cyclical nature of many environmental problems, and the associated cyclical nature of optimal policies. Proposition 4 informs us that, if patents have infinite lifetime in all sectors, then the cyclical nature of environmental problems has no traction on optimal policy. Proposition 5 informs us that, if patents have constant finite lifetime, then clean innovation policy should be dynamically adjusted.

The general equilibrium analysis may tell us that, in balanced growth, clean innovations should or should not be treated differently from other innovations. The analysis of this paper informs us that, though this insight may hold true in the long run, optimal policy may need to deviate in the short run. When a cluster of clean technologies need a quick start to address an emerging environmental problem, the private value of patents is relatively low compared to the social value of the increase in knowledge, and more public support for innovation is warranted. At a later stage, when the clean technology has matured, it needs less support. For dirty technologies, an inverse pattern may hold. When the use of a particular cluster of dirty technologies will drop as part of a policy to address an environmental problem, then the private value of patents might still be high due to the expected use of the technology in the next couple of decades, but the social value of knowledge is relatively small, because of the expected reduction in the use of the technology in the longer term.

In the context of climate change policy, the analysis suggests a more favourable fiscal treatment of research for renewable energy and a less favourable fiscal treatment for research in fossil fuel exploration, but only for the coming decades. The effect on economic growth of directed innovation policy is probably limited. The major effect of climate policy on growth probably does not pass through the relocation of researchers, but through higher energy prices.

Another caveat of the analysis is its limited attention to practical constraints on innovation policies. It may prove difficult to implement the first-best policy, especially in the early stage of development when the optimal subsidy rate (patent lifetime) may be very high (long). For instance, in reality the public agent cannot provide near 100 per cent subsidy to research firms, without strict control of the research effort carried out.[18] Infinite patents may also be difficult to implement in practice, or the research firms may fear that their patent rights are weakened before the sales of their equipment kicks off. Note, however, that an initially high subsidy rate or very long patent lifetime will only affect the very first technologies that are developed, and thus have a limited bearing on the majority of equipments developed and used for abatement in the later stages when substantial abatement levels are required (as opposed to the case in Proposition 4 where infinite patent lifetime applies to technologies developed at any time). An alternative policy at the early stages of development may be to undertake public R&D, through dedicated research on abatement technologies, and then let private R&D firms take over when the environmental problem develops. However, regulating public research to achieve a first-best research effort may also prove difficult, e.g., due to lack of monetary incentives. To sum up, stimulating research effort in an optimal way may be particularly difficult initially, long before most of the abatement takes place.

6. CONCLUSION

In this paper we have studied the links between (the timing of) innovation policies and abatement policies under different assumptions of innovation policies such as the possibilities to use patent life time as a policy instrument. The latter follows from a core distinction between our model and earlier R&D models in the environmental economics literature, namely that the lifetime of patents is finite. Our analysis is based on an R&D model supplemented with emission-abatement-pollution dynamics, and four imperfections related to innovations; too little production of patented abatement equipment due to monopolistic competition, positive spillovers of innovations due to finite patent lifetime, negative spillovers through crowding out effects within the

[18] In the EU there is an upper limit to the legitimate rate of R&D subsidy.

abatement technology sector, and crowding out effects in other sectors. Innovation policy instruments may include deployment subsidies to patented equipment, research subsidies (or taxes), and the lifetime of patents. Our main result demonstrates that the positive spillovers of innovations due to finite patent lifetime are particularly strong at the early phase of an environmental problem, and to account for this, optimal innovation policy needs to adjust dynamically.

Our results share the tone of earlier papers on the timing of abatement *efforts* in the sense that we find a focus on technological development in the early phase of an environmental cycle, while the *use* of abatement technologies mainly occurs at later stages when the technology is more mature. But in terms of *policies*, our findings sketch a different picture, in a subtle way. The efficient environmental tax should equal the Pigouvian tax, so that, in this sense, environmental *policy* is independent of innovation dynamics. Innovation policy, however, changes with the environmental cycle. If the patent lifetime is finite, the optimal subsidy starts at a high level, giving an incentive to accelerate R&D investments, and then falls over time as the environmental problem becomes more mature. In a similar way, if the research subsidy is constant, the optimal lifetime of a patent should be very high initially and then fall. This result on innovation policy signals an important difference with previous energy-emissions-environment models with innovation, where implicitly infinite patents are assumed.

The intuition behind our results is that, at the early stages when an environmental problem emerges, the private incentives for innovation are modest only, while the social benefits are large. Over time, private incentives increase more relative to the social value. The reason for the modest private incentives is that, typically, the price of emissions and the value of total abatement activity is low, initially. As a result, the private value of owning technological knowledge is modest, and the incentive to innovate is low. Yet, the social value of knowledge also includes gains further in time, beyond the patent expiration date. When time passes and the future benefits of knowledge enter the patent's lifetime horizon, they increase the private incentive and the need for fiscal compensation decreases.

Thus, an emerging environmental problem calls for public intervention, not only through emission or resource use taxes, but also through subsidies or other measures that stimulate clean innovation. Possible measures include public R&D, targeted subsidies on private clean R&D, or longer lifetime patents.

ACKNOWLEDGMENTS

This is a totally revised version of Memorandum 10/2008, Department of Economics, University of Oslo, and FEEM working paper 5308/2008 called "Linking environmental and innovation policy". Comments from Scott Barrett, Rolf Golombek, Michael Hoel, Olli Tahvonen and many other participants in the project "Environmental economics: policy instruments, technology development, and international cooperation" at the Centre for Advanced Study (CAS), are gratefully acknowledged. We also thank Rob Hart, Sjak Smulders, Asbjørn Aaheim, Simone Valente, Lars Olson, participants from several conferences and seminars for their comments and two anonymous referees. The research for this paper started at the Centre for Advanced Study (CAS) at the Norwegian Academy of Science and Letters in Oslo in 2005/2006. The financial, administrative and professional support of the Centre to this project is much appreciated. Gerlagh also thanks the Dutch NWO Vernieuwingsimpuls program for support, and Kverndokk and Rosendahl thank the programme RENERGI at the Norwegian Research Council for financial support. While doing the revisions of this paper we have been associated with CREE - Oslo Center for Research on Environmentally friendly Energy. CREE is supported by the Research Council of Norway.

APPENDIX: PROOFS

Proof of LEMMA 1. *There exists a function* $v(h,s)$, $v_h<0$, $v_s<0$, *such that net present value costs satisfy*

$$V(H,S;Y)=Y^{\gamma}v\left(\frac{H}{Y^{\gamma\psi}},\frac{S}{Y}\right) \tag{9}$$

with $\gamma=\dfrac{1}{\lambda}=\dfrac{1}{\psi+\beta(1-\psi)}>1$.

Proof. We make two notes about notation. First, in the lemma we omit the t=0 subscript, as the equation holds for any value of H, S and Y. For the proof though, it is convenient to consider the values as initial conditions, so that we will use the t=0 subscript. Second, whereas the intensive form for the absorption capacity is unambiguously defined by S/Y, for the knowledge intensive form we could equally use $H^{1/\gamma\psi}/Y$, or $H/Y^{\gamma\psi}$. It turns out more convenient to use the latter.

Consider initial conditions (H_0,S_0,Y_0) and (H_0^*,S_0^*,Y_0^*) with $S_0^*/S_0=Y_0^*/Y_0=\lambda$, and $H_0^*/H_0=\lambda^{\psi\gamma}$, where λ is the scale ratio between the two initial conditions. We must now show that costs satisfy $V^*/V=\lambda^{\gamma}$. To do this, we show that for any feasible path (x_t,R_t) for the initial conditions (H_0,S_0,Y_0) with associated net present value costs V, we can construct a solution (x_t^*,R_t^*) for (H_0^*,S_0^*,Y_0^*) with net present value costs $V^*/V=\lambda^{\gamma}$. The construction guarantees that $V^*/V\leq\lambda^{\gamma}$. Using the inverse construction provides the weak inequality the other way around.

We construct the path (x_t^*, R_t^*) that maintains the ratios $H_t^*/H_t=\lambda^{\psi\gamma}$ and $S_t^*/S_t=\lambda$ for all t. For this purpose, take $R_t^*=\lambda^{\gamma}R_t$, which ensures that $H_t^*/H_t=\lambda^{\psi\gamma}$ throughout. Furthermore, we take $x_t^*=\lambda^{(1-\psi)\gamma}x_t$, so that the abatement ratio is proportional to the gross emissions ratio, $A_t^*=H_t^*(x_t^*)^{\beta}=\lambda^{\psi\gamma+\beta(1-\psi)\gamma}H_t\,(x_t)^{\beta}=\lambda A_t$, while costs increase by factor λ^{γ}: $H_t^*x_t^*=\lambda^{\psi\gamma+(1-\psi)\gamma}H_t\,x_t\;=\lambda^{\gamma}H_t\,x_t$. Q.E.D.

Proof of PROPOSITION 5. *Consider the case that patents have constant finite lifetime, $L_t=L<\infty$, and the initial knowledge stock is zero, $h_0=0$. Then there is a t^* with $T-L<t^*<T$ such that the research subsidy that implements the first-best decreases monotonically for $0\leq t\leq t^*$, and increases afterwards (for $t\geq t^*$).*

Proof: From (42), we can see that $\dot{\sigma}_t>0$ iff V_t/η_t decreases. To study optimal research subsidies, we thus must understand the dynamics of the value of innovations, both the social perspective (η_t) as from the private perspective (V_t). The innovation value depends on the development of the use of equipment, x_t, over time. From (13) and (15) we have that in the short-run ($\varepsilon_t=\lambda_t=0$), i.e., for $t<T$,

$$\dot{x}_t/x_t=\rho/(1-\beta). \tag{43}$$

Combining Proposition 1 and Proposition 3, we know that if $h_0=0$, then at $t=T$, $h_T<h^*$ and the growth rate for x_t will sharply drop and then slowly increase towards a level below the initial growth rate (since $\rho/(1-\beta)>(1-\psi)\gamma g$). The growth in the use of abatement equipment path looks as in Figure 5, where the dotted line denotes the balanced growth level $(1-\psi)\gamma g$.

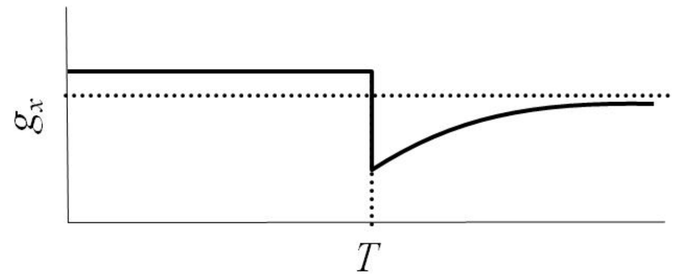

FIGURE 5. *Dynamics of abatement intensity growth, $\dot{x}_t/x_t$*

Given the growth profile for abatement intensity x_t as depicted in Figure 5, we can derive the ratio between future intensity x_{t+L} and x_t as in Figure 6.

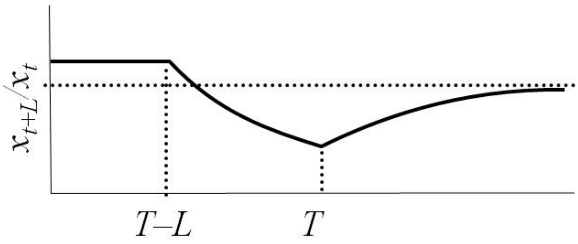

FIGURE 6. *Dynamics of* x_{t+L}/x_t

From (16) and (38) we have that

$$\frac{\dot{\eta}_t}{\eta_t} = \rho - (\beta^{-1} - 1)\frac{x_t}{\eta_t} \text{ and } \frac{\dot{V}_t}{V_t} = \rho - \frac{x_t - e^{-\rho L} x_{t+L}}{V_t} \tag{44}$$

and from here we get the dynamic development of V_t/η_t

$$\frac{\partial}{\partial t}\frac{V_t}{\eta_t} > 0 \Leftrightarrow \frac{x_t - e^{-\rho L} x_{t+L}}{V_t} < \frac{(\beta^{-1} - 1)x_t}{\eta_t} \Leftrightarrow \frac{x_{t+L}}{x_t} > e^{\rho L}\left(1 - \frac{(\beta^{-1} - 1)V_t}{\eta_t}\right). \tag{45}$$

Thus, we can draw the dynamics of V_t/η_t in a sort of phase diagram where we consider the dynamics of V_t/η_t versus x_{t+L}/x_t.

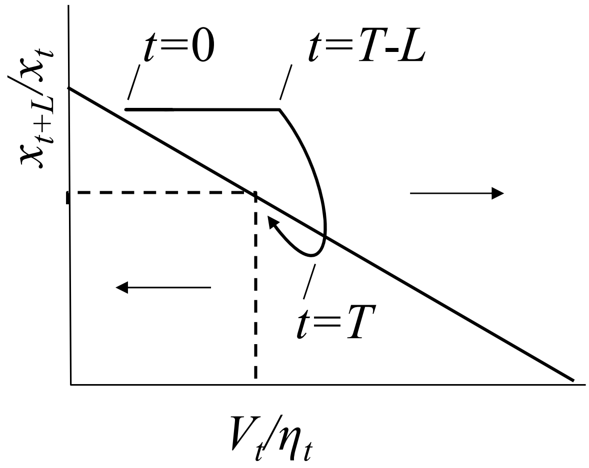

FIGURE 7. *Dynamics of* V_t/η_t

There is a downward sloping locus such that V_t/η_t is constant. Above the locus, the ratio of the private value versus the social value of innovations is increasing, below the locus, the ratio is decreasing. If the equilibrium path crosses the locus, it will cross it vertically turning clockwise. The long-term steady state will be on the locus. From the dynamics of x_{t+L}/x_t in Figure 6, we know that the steady state will be approached from below. Combining this insight with the phase dynamics, the path must approach the steady state from south-east. The path cannot cross the locus for $t>T$. At $t=T$, x_{t+L}/x_t has a minimum, and thus, going back in time, the path moves north-east. Going further back in time, from $t=T$ to $t=T–L$, the ratio x_{t+L}/x_t increases up to a level above the steady state, and thus, the path must cross the locus and move north-west. Finally, from $t=T–L$ to $t=0$, the ratio x_{t+L}/x_t is constant and the path must move horizontally. The path as drawn in Figure 7 follows. We see that the V_t/η_t increases from $t=0$ onwards until the path crosses the locus for some t^* with $T–L<t^*<T$, after which the ratio V_t/η_t decreases. The research subsidy follows an inverse pattern. Q.E.D.

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