# Productivity and Management of Renewable Resources – Why More Efficient Fishing Fleets Should Fish Less.


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## Abstract

This article analyses the effect of productivity improvements on optimal fisheries management. It is shown that when harvest costs are independent of resource stock and the stock is below its steady state level, then for any given stock it is optimal to reduce harvest levels in response to a productivity increase unless optimal harvest rate is already zero. If harvest costs are stock dependent this result is modified; for stock dependent harvest costs there exists an interval of stock sizes below the steady state where it is optimal to reduce the harvest rate for any given stock size whereas if the harvest rate is close to an economically optimal steady state it is optimal to increase the harvest rate.



Key words: Fisheries, optimal control, productivity, renewable resources.

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# 1. Introduction

Many of the world's fisheries are in dire straits because of poor management and poorly delineated property rights with a third of these fisheries labelled unsustainable by the Food and Agriculture Organization of the United Nations, FAO (2018). This poor state of affairs has been with us for some time and the costs are substantial, Clark (2005). The problem of overfishing is likely to be exacerbated by technology. It is an unfortunate fact that improved technology often implies more aggressive harvesting practices in harvesting regimes such as open access fisheries or fisheries subject to the tragedy of the commons, Whitmarsh (1990), Squires and Vestergaard (2013), Squires and Vestergaard (2018). There are several examples where improved technology has led to the collapse of a fish stock, Hannesson *et al* (2010), Gordon and Hannesson (2015). Kvamsdal *et al* (2016) point out that even in managed fisheries, "…increased technical efficiency and progress, usually lead to overcapacity in national fishing fleets. Overcapacity creates national pressure for higher quotas…." In a recent paper, Skonhoft and Quaas (2019) find that in a restricted open access or shared resource fishery if the long run steady state in a shared, but unmanaged, resource fishery is lower than the harvest rate associated with maximum sustainable yield, then improved technology may increase rents. However in larger, more accessible fisheries this condition is unlikely to hold.

The caveat in Skonhoft and Quaas (2019) not notwithstanding, it seems clear that improved technology worsens the market failure of a poorly managed fishery. This is unfortunate as improved technology has the potential to make the fishery more valuable and therefore it is even more important that fish stocks are managed as well as possible and that the role of technology is understood. Remarkably little has been written on the role of technology in an optimally managed fishery. As stated in Squires and Vestergaard (2013): "... the normative literature has yet to formally analyze the impact of changes in disembodied and embodied technical progress and technical efficiency on optimum exploitation of common renewable resources."

The analysis in the literature is mostly concerned with steady state analysis, Clark and Munro (1975), Caputo (1989), Squires and Vestergaard (2013). However steady state analysis is only locally relevant and policy advice for management of recovering stocks should not be based on steady state analysis. As we shall see, in fisheries such a strategy would be particularly ill advised as the effects of technology in the case of stock dependent costs go in completely different directions in steady state and in stock levels sufficiently below the optimal steady state. Squires and Vestergaard (2013) go some way in performing a formal analysis. Their contribution is contrasted with the results in the present paper in Section 4 below.

The results are derived in general versions of the original canonical fisheries model, Clark and Munro (1975). The modelling of improved productivity is very basic and obtained by examining how exogenous changes in a productivity parameter affect optimal policy.

Section 2 briefly reviews the canonical model of fisheries management in the case where harvest costs are independent of the size of the fish stock. In Section 3 the effect of productivity on harvest rates is derived and it is proved that if it is optimal to manage a fish stock sustainably, then for any given stock size below steady state improved technology decreases the optimal harvest rate. Section 4 examines the case where harvesting costs are dependent on the stock size. Here it is shown that even if improved technology leads to a lower steady state stock size, it also leads to decreased harvest rate if the stock is sufficiently lower than the optimal steady state. Section 5 presents concluding remarks and relates the results to current management practices.

## 2. The Canonical Schooling Fisheries Model

The following is the basic version of the fisheries model applied to a schooling fishery, Clark (1990), p 97. In a schooling fishery harvest costs are not dependent on stock size, Neher (1990), p 177. We assume that the net instantaneous benefits from harvesting is given by a continuous and strictly concave function of the harvest rate, $h$, and given by $D(h) - C(h)\alpha^{-1}$. Here $D(h)$ is a benefit function, $C(h)\alpha^{-1}$ is a cost function and $\alpha$ is a parameter with higher $\alpha$ indicating better technology. We assume that $D(0) = C(0) = 0$ and $D'(h)$ and $C'(h)$ both positive. $D''(h) - C''(h)\alpha^{-1}$ is assumed negative and $C''(h)$ is assumed positive. We also make the following important assumption:

Assumption 1.

$0 < D'(0) - C'(0)\alpha^{-1} < \infty$ .

Assumption 1 implies that we are not considering a subsistence fishery, but rather a commercial fishery or a fishery where capital and labour has alternative uses. The fish stock is assumed to be governed by

$$\dot{x} = G(x) - h, \; x(0) \text{ given} \tag{1}$$

The natural growth function $G(x)$ is taken to be strictly concave, differentiable and satisfy $G(0) = G(K) = 0$ for some $K > 0$ and positive for all $x \in (0, K)$. The specification of $G(x)$ is in line with standard biological growth functions such as the logistic, which is used in phase portraits below, but the formal results do not require a parametric growth function. We assume there exists $x \in (0, K)$ such that the derivative of the growth function equals the discount rate, $G'(x) = \rho$, which is reasonable for many commercially interesting fish species. The assumptions lead to the following optimization problem:

$$V\left(x\left(0\right),\alpha\right)=\max_{h\left(t\right)\geq 0}\int_0^{\infty}\left(D\left(h\right)-C\left(h\right)\alpha^{-1}\right)e^{-\rho t}dt$$
$$\text{subject to } \dot{x}=G\left(x\right)-h \text{ and } x\left(0\right) \text{ given} \tag{2}$$

The current value Hamiltonian for this problem is:

$$H=D\left(h\right)-C\left(h\right)\alpha^{-1}+\mu\left(G\left(x\right)-h\right) \tag{3}$$

The Hamiltonian is concave in $(h, x)$, so sufficiency theorems such as Theorem 9.11.1 in Sydsæter et al. (2005) are fulfilled. The necessary conditions become:

$$\frac{\partial H}{\partial h}=D'\left(h\right)-C'\left(h\right)\alpha^{-1}-\mu\leq 0 \quad \left(=0 \text{ if } h>0\right) \tag{4}$$

and

$$\dot{\mu}=\left(\rho-G'\left(x\right)\right)\mu \tag{5}$$

(4) follows from maximising the Hamiltonian with respect to $h$, when $H$ is a concave function of $h$ and defines $h$ as a function of $\mu$ and $\alpha$. This function is denoted: [1]

$$h=\phi\left(\mu,\alpha\right) \tag{6}$$

Note that (4) and Assumption 1 implies that $D'\left(0\right)-C'\left(0\right)\alpha^{-1}<\mu \Rightarrow \phi(\mu, \alpha)=0$.

For all $x$ where (4) holds with equality it also holds that:

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[1] It is also implicitly assumed that the value of $h$ that solves $D'\left(h\right)-C'\left(h\right)\alpha^{-1}=0$ is larger than $h_{ss}$. Without this assumption there is no need to regulate the fishery as the optimal shadow price is zero.

$$\frac{\partial h}{\partial \mu} = \phi'_{\mu} = \frac{1}{D''(h) - C''(h)\alpha^{-1}} < 0 \qquad (7)$$

$$\frac{\partial h}{\partial \alpha} = \phi'_{\alpha} = -\frac{C'(h)\alpha^{-2}}{D''(h) - C''(h)\alpha^{-1}} > 0 \text{ for } h > 0 \qquad (8)$$

$$\left(\frac{\partial \mu}{\partial \alpha}\right)_{h \text{ is constant}} = -\frac{\phi'_{\alpha}}{\phi'_{\mu}} = C'(h)\alpha^{-2} > 0 \text{ for } h > 0 \qquad (9)$$

The inequality in (8) may lead us to infer that higher productivity implies larger harvest rates. As we shall see, the inequality (9) and how it affects the shape of the isocline for $\dot{x} = 0$ is part of the reason why this is wrong.

The model assumptions ensure that there exists a steady state $(h_{ss},\ x_{ss},\ \mu_{ss}) = \lim_{t\to\infty}(h(t),\ x(t),\ \mu(t)) > 0$ which for $x(0) > 0$ will be the long run equilibrium for an optimally managed fishery, (Clark 1973, Cropper 1979). Convergence of $(h(t),\ x(t),\ \mu(t))$ to the steady state $(h_{ss},\ x_{ss},\ \mu_{ss})$ will be along a stable saddle path towards a steady state characterized by $\dot{x} = G(x) - \phi(\mu,\alpha) = 0$ and $\dot{\mu} = (\rho - G'(x))\mu = 0$. This stable path can be found in $(x,\ \mu)$ space by solving the following differential equation, Conrad and Clark (1987), Ch. 1.6.6, Judd (1998), Ch. 10.7:

$$\frac{\dot{\mu}}{\dot{x}} = \frac{\mathrm{d}\mu}{\mathrm{d}x} = \frac{(\rho - G'(x))\mu}{G(x) - \phi(\mu,\alpha)},\ \mu(x_{ss},\alpha) = \mu_{ss} \qquad (10)$$

The solution to (10) defines $\mu$ as a function of $x$ along an optimal path. We denote the stable saddle path as $\mu(x, \alpha)$. It is worthwhile to note that the stable saddle path $\mu(x, \alpha)$ is in fact the derivative of the value function, $V(x, \alpha)$. As $V(0, \alpha) = 0$ in this model, we can illustrate the value function in a phase portrait as the integral under the stable manifold. A phase portrait of the optimal solution is shown in Figure 1. (All figures are appended at the end of this manuscript.)

**Figure 1 around here.**

Inserting $\mu(x, \alpha)$ into (6) gives us optimal $h$ as a feedback control:

$$h = \phi\left(\mu\left(x,\alpha\right),\alpha\right) = \Phi\left(x,\alpha\right) \tag{11}$$

The expression in (11) is a feedback control and gives us optimal harvest rates as a function of the stock and the productivity parameter and is usually derived in a dynamic programming framework. However, by using optimal control with its explicit focus on the shadow price we gain some additional structure to the problem that enables us to construct analytical results not directly accessible with dynamic programming where interior solutions are typically assumed.[2]

[2]Alternatively we could use the optimal control conditions in (4), (5) and the differential equation for $x$ to construct a differential equation $\dot{h} = v\left(h, x\right)$, see equation (13) below. This equation would however

One such result, crucial in the analysis below, may be found in Nævdal and Skonhoft (2018) who prove that under Assumption 1 there exists a critical stock level $x_c > 0$ such that $h = \Phi\left(x,\alpha\right) = 0$ for all $x \leq x_c$. In other words; as long as the instantaneous marginal utility of harvesting is finite at $h = 0$, it is optimal for a well managed fishery to temporarily close down if fish stocks are sufficiently low.

## 3. The Effect of Increased Productivity

Here we demonstrate that for a given stock size increased productivity reduces the optimal harvest rate if $h$ is positive and the stock is below the steady state level, i.e. $\Phi(x,\ \alpha)$ is a decreasing function of $\alpha$ for all $x \in (x_c,\ x_{ss})$. This result is built on 4 propositions that are given below and proved in online appendix A. However a fairly rigorous development is given in Figure 2.

**Figure 2 around here**

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only be valid when (4) has an interior solution and is not very helpful here as much of the present analysis deals with the case where $h = 0$ is the solution to (4) along the optimal path.

**Proposition 1.**

An increase in $\alpha$ will shift the $\dot{x} = 0$-isocline in Figure 1 upwards for all $x$ except $x = 0$ and $x = K$.

**Proof:** See online appendix A.

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**Proposition 2.**

Define $Q(\alpha) = \left\{(x,\mu) \mid 0 < x < x_{ss}, \dot{x} = G(x) - \phi(\mu,\alpha) > 0, 0 < \mu < D'(0) - C'(0)\alpha^{-1}\right\}$.

Let $\alpha^{high} > \alpha^{low}$. From Proposition 1 it follows that $Q(\alpha^{high})$ is a subset of $Q(\alpha^{low})$. Then for any point $(x, \mu) \in Q(\alpha^{\mathrm{high}})$ it holds that $\left(\mathrm{d}\mu/\mathrm{d}x\right)_{\alpha^{high}} < \left(\mathrm{d}\mu/\mathrm{d}x\right)_{\alpha^{low}} < 0$.

**Proof:** See online appendix A.

■

Proposition 2 defines a set $Q(\alpha)$. This set is significant in the present context because the stable manifold, $\mu(x, \alpha)$, is contained in $Q(\alpha)$ for all $x \in (0, x_{ss})$. What this proposition does is to take an arbitrary point in $Q(\alpha^{high})$, calculate the slope of any solution that passes through that point and show that the slope is steeper when $\alpha$ is higher. Propositions 1 and 2 can be used to prove the following proposition:

**Proposition 3.**

a) $\mu(x, \alpha)$ is an increasing function of $\alpha$ for all $x \in (0, x_{ss}]$.

**Proof:** See online appendix A.

■

Roughly speaking, the proof of Proposition 3 is that since $\mu(x_{ss}, \alpha)$ is increasing in $\alpha$ from Proposition 1 and $\mu(x, \alpha^{high})$ can not intersect $\mu(x, \alpha^{low})$ because of Proposition 2, Proposition 3 follows. Given the economic interpretation of the stable manifold this also makes intuitive sense; better technology increases the marginal value of the stock given the stock size.

The critical stock level $x_c$ at and below which optimal $h$ is zero will also be affected by a productivity increase. We can prove that $x_c$ is in fact an increasing function of $\alpha$.

**Proposition 4.**

The critical $x_c$ where $h = 0$ for all $x < x_c$ is an increasing function of $\alpha$.

**Proof:** See online appendix A.

■

We can now prove the main result in this section.

**Proposition 5.**

Let $\alpha^{low} < \alpha^{high}$. Then the feedback control has the property that for all $x < x_{ss}$ it holds that $\Phi\left(x, \alpha^{low}\right) \geq \Phi\left(x, \alpha^{high}\right)$.

**Proof:** See online appendix A.

■

The result in Proposition 5 is caused by the subtle trade-offs between gains from the productivity improvement in the present and in the future. The capital theoretic explanation for the fisheries model is illustrated in Figure 3 where for simplicity it is assumed that $D(h) = ph$. The effect of a productivity improvement is deconstructed into two panels. Panel A illustrates the first order condition at two different harvest rates and the incentive to change harvest rates as a result of an increase in $\alpha$. At $h = h_{ss}$ we have that $p - C'(h)/\alpha^i = \mu(x_{ss}, \alpha^i) > 0$ for $i = high, low$. When $\alpha$ increases $h_{ss}$ remains unchanged even if the marginal cost curve shifts downwards. If $h = h_1$ and we hold $h_1$ unchanged when $\alpha$ increases then profits increase by the amount indicated by the area labelled 1. If $h = h_{ss}$ then profits increase by an amount given by the areas labelled 1 and 2. In fact for all $h < h_{ss}$ it holds that the higher $h$ is, the larger is the increase in instantaneous profits if $\alpha$ increases. This explains the incentive to reduce $h$ if $h$ is below the steady state level. By reducing $h$ one can increase $x$ faster and sooner get to levels of $x$ where profits are higher. Panel B illustrates how this works through a shift in the shadow price function $\mu(x, \alpha)$. Here the $x$-axis measures $x$ and the argument in the marginal cost curves is $h = \Phi(x, \alpha)$. Assume that productivity shifts when $x = x_1$. This shifts the marginal cost function to the right as illustrated because $x_c$ shifts from $x_c^{low}$ to $x_c^{high}$. This shift is caused by the shadow price at $x = x_1$ increasing from $\mu(x_1, \alpha^{\mathrm{low}})$ to $\mu(x_1, \alpha^{\mathrm{high}})$. The explanation is that a marginal increase in the stock will increase rents in the future as demonstrated in Panel A. Therefore the marginal value of the fish stock increases. But then $h(x_1, \mu)$ has to decrease. In fact it is only when $x$ has increased to $x = x_2$ that the shadow price has decreased to a level such that $\mu(x_2, \alpha^{high}) = \mu(x_1, \alpha^{low})$ so that harvest rates recovers to the same level as when $x = x_1$ before the productivity improvement. In Panel B one can also illustrate the value of social welfare forgone at a particular time because one does not harvest so that price equals marginal harvesting cost but manages the stock for the future. In the low productivity case the social welfare given up as an investment in future fish stocks is given by the areas marked 5 and 6 if $x = x_1$ and 6 if $x = x_2$. In the high productivity case the social welfare forgone if $x = x_1$ is given by the areas marked 5 and 6 and 5, 6, 7 and 8 if $x = x_1$.

An alternative way to understand Proposition 5 is to note that the canonical fisheries model is very similar to the well known Ramsey-Cass-Koopmans (RCK) model of economic growth. The only difference is that marginal productivity at zero use of input is assumed infinity in the RCK model, whereas the corresponding expression in the fisheries model, $G'(0)$, is assumed finite. In the analysis of the RCK model the instantaneous elasticity of substitution, also known by its reciprocal, the intertemporal elasticity of substitution is an important explanatory factor. Roughly speaking, the higher the instantaneous elasticity of substitution, the more willing is the consumer to reduce consumption in the present in order to increase consumption in the future, see Blanchard and Fischer (1989) pp 39-43 for a discussion of the instantaneous elasticity of substitution and how it affects intertemporal tradeoffs in the RCK-model. If we can show that a productivity increase has the same effect as an increase in the instantaneous elasticity of substitution we have a way of understanding and generalising the above results.

In the present model we can calculate the equivalent to the instantaneous elasticity of substitution for the integrand in (2). This expression is given by:

$$\sigma\left(h,\alpha\right) = -\frac{D'\left(h\right) - C'\left(h\right)\alpha^{-1}}{h\left(D''\left(h\right) - C''\left(h\right)\alpha^{-1}\right)} > 0 \text{ for all } h > 0 \tag{12}$$

By using (4), (5) and the differential equation for $x$ one can derive the following differential equation for $h$ as a function of $x$ which holds when (4) has an interior solution.

$$\frac{\dot{h}}{h} = \sigma\left(h,\alpha\right)\left(\rho - G'\left(x\right)\right) \tag{13}$$

From (13) it is clear that for any given $x < x_{ss}$, the higher the numerical value of $\sigma(h, \alpha)$ the faster is the rate of convergence towards the steady state value of $h_{ss}$, and therefore also towards $x_{ss}$. In light of the Propositions 3, 4 and 5 above, one would expect an increase in $\alpha$ to increase $\sigma(h, \alpha)$ so that improved technology implies that harvest is to be reduced today in order to increase the harvest in the future. Differentiating $\sigma(h, \alpha)$ with respect to $\alpha$ yields:

$$\frac{\partial\sigma\left(h,\alpha\right)}{\partial\alpha}=\frac{\overbrace{D'(h)C''(h)}^{>0}-\overbrace{C'(h)D''(h)}^{<0}}{\underbrace{h\left(C''(h)-\alpha D''(h)(h)\right)^{2}}_{>0}}>0 \text{ for } h>0 \tag{14}$$

Thus improved productivity has the same qualitative effect along the segment of the optimal path where $h > 0$ as an increase in the instantaneous elasticity of substitution has in the RCK model.

#### A More General Cost Function

The functional form used in the cost function is a special case used for convenience. In online appendix B the instantaneous elasticity of substitution is calculated for a more general cost function $c(h, \alpha)$. It is shown that if a productivity improvement lowers marginal cost for all $h > 0$, then a sufficient, but not necessary condition for $\sigma(h, \alpha)$ to be increasing in $\alpha$ is that $c'''_{hh\alpha} < 0$.

## 4. Stock dependent costs

We now look at the case where the harvest costs depend on stock size, reflecting that for many fish-species it is easier to catch a unit of fish if the stock is abundant and if the stock of the fish goes to zero, then the marginal cost of harvesting a unit of fish goes to infinity. The canonical model is now:

$$V\left(x\left(0\right),\alpha\right)=\max_{h\geq 0}\int_0^\infty\left(D\left(h\right)-C\left(h,x,\alpha\right)\right)e^{-\rho t}dt$$
$$\text{subject to } \dot{x}=G\left(x\right)-h \text{ and } x\left(0\right) \text{ given.} \tag{15}$$

It is assumed that $C_h'\left(h,x,\alpha\right)>0$, $C_x'\left(h,x,\alpha\right)<0$, $C_{h\alpha}''\left(h,x,\alpha\right)<0$ for all $h$ and $x > 0$, $C_{h\alpha}''\left(0,x,\alpha\right)=0$ and that $C_{hh}''\left(h,x,\alpha\right)>0$. This is consistent with, but not restricted to, cost functions of the form

$$C\left(h,x,\alpha\right)=k\frac{h^a}{\alpha x^b} \text{ where } a>1 \text{ and } b>0 \tag{16}$$

Thus the technology is such that an increase in α lowers the marginal cost of harvesting. It follows from $C(0, x, \alpha) = 0$ and $C_{h\alpha}''\left(h,x,\alpha\right)<0$ that an increase in α also reduces costs for all positive $h$ and $x$.

This model or models very similar to this one has been analysed in a large number of papers and is close to the analysis in Section 2. A typical phase diagram is shown in Figure 4.

**Figure 4 around here**

We keep the notation from the previous analysis so $\phi(\mu, \alpha)$ is the harvest rate that maximises the Hamiltonian, $\mu(x, \alpha)$ is the stable manifold/derivative of the value function, $\Phi(x, \alpha)$ is optimal harvest rate as a feedback control and $x_c^j$ is the critical stock level below which one should not harvest when the productivity is of type $j$.

Also with stock dependent costs there is a critical value $x_c$ determined by the lowest stock of $x$ such that the first order condition for the maximised Hamiltonian is that its derivative with respect to $h$ is zero:[3]

$$\frac{\partial H}{\partial h} = D'\left(0\right) - C_h'\left(0, x_c, \alpha\right) - \mu = 0 \tag{17}$$

We can use this to find conditions that ensure that $x_c$ is a increasing function of $\alpha$.

**Proposition 6.**

[3] The proof for this statement is much simpler than with the model without state dependent harvest cost given in Nævdal and Skonhoft (2018). Here it suffices to note that as long as marginal cost goes to infinity as $x$ goes to zero and marginal revenue $D(0)$ is a positive finite number, such a critical $x_c$ will exist, Leung and Wang (1976).

$C''_{hx}\left(0,x,\alpha\right)\leq 0$ is a sufficient, but not necessary condition for $x_c$ being an increasing function of α.

**Proof:** See online appendix A.

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Assuming that $C''_{hx}\left(0,x,\alpha\right)\leq 0$ is in line with standard parameterisations of cost functions used in fisheries. It is also reasonable to expect that the higher the stock, the lower is the marginal cost with respect to *h*. However, one can not rule out that there are exceptions to this rule. One can e.g. envision changes in technology that improves productivity for some values of *h* and *x* and reduces it for others. If that is the case a more detailed numerical investigation is warranted to establish the sign of $\partial x_c/\partial\alpha$.

**The effect of improved productivity on optimal harvest rate when $x > x_c$.**

Clark and Munro (1975) demonstrate in a model that is linear in the harvest rate that an increase in productivity will lead to lowered steady state stocks. Caputo

(1989) demonstrates in a fairly general non-linear model that the long-run effect of a productivity increase is also a lower stock level. Caputo (1989) also show and that a productivity increase may be commensurate with both a higher and a lower long term harvest rate $h_{ss}$. The initial response will however be an increase in the harvest rate. The results in Caputo (1989) are based on the response to a productivity shock if the initial stock is in steady state. However, even if the steady state response to a positive productivity shock implies that the steady state stock decreases when $\alpha$ increases it is still the case that $\Phi(x, \alpha)$ will be a decreasing function of $\alpha$ if $x$ sufficiently small as proven in Proposition 7.

**Proposition 7.**

If Proposition 6 holds, an increase productivity from $\alpha^{low}$ to $\alpha^{high}$ implies that there is an non-empty interval $\left[x_c^{low}, \hat{x}\right)$ such that $\Phi(x, \alpha^{high}) < \Phi(x, \alpha^{low})$ for all $x$ in $\left[x_c^{low}, \hat{x}\right)$.

**Proof:** See online appendix A.

■

This effect remains regardless of the size of productivity increase. An important thing to note from Figure 4 is that there is a lower bound to how low the steady state stock will be reduced by improved technology. Proposition 7 shows that below this lower bound, there will be an interval $[x_c^{low}, \hat{x})$ where the higher the productivity, the lower the harvest rate. The effect of increased productivity is summarized in Figure 5.

**Figure 5 around here**

Figure 5 shows that the results in the Clark and Munro (1975), Caputo (1989) and Squires and Vestergaard (2013) where increased productivity leads to lower steady state stocks are compatible with the results in Section 3. If increased productivity increases rents in steady state and the stock is low enough one should still decrease harvesting in order to increase stocks faster even if the desired steady state level has decreased.

Squires and Vestergaard (2013) also present results where improved technology leads to short term decreases in harvesting. However, their mechanism is that if the regulator knows there will be technology improvements in the future, then it will pay to reduce harvest rates until the new technology can be utilized. In the present paper the technology improvement is unanticipated, but the improvements are still best utilized by letting the fish stock grow faster even though instantaneous profits could increase today by fishing more.

A final technical remark is that the assumption that $C''_{h\alpha}\left(0,x,\alpha\right)=0$ is crucial for the result in Proposition 6 that $\partial x_c/\partial\alpha$ is positive for all α. It is however a sufficiency condition and not a necessary condition. If $C''_{h\alpha}\left(0,x,\alpha\right)<0$ there may be some intervals of α values where $\partial x_c/\partial\alpha < 0$. However, as may be seen from Figure 5, if $\lim_{\alpha\to\infty} C\left(h,x,\alpha\right)=0$ then $x_c$ will increase, perhaps not monotonically, and eventually converge to the stock level that solves the equation $G'\left(x\right)=\rho$. It follows that $h = \Phi(x, \alpha)$ will also be reduced as α increases, but perhaps not monotonically. Again, if there is reason to believe that $C''_{h\alpha}\left(0,x,\alpha\right)<0$ then the effect of improved productivity must be numerically investigated.

## 5. Concluding remarks

The present paper examines the basic nonlinear control variable biomass fishery model originating from Clark and Munro (1975), and demonstrates in a model of a schooling fishery the effects of improved productivity on optimal management. We build on a result in Nævdal and Skonhoft (2018) where it is shown that there is a lower bound for the fish stock below which it is optimal to set the harvest rate to zero. The results presented in this paper indicates that improved technology imply that this lower bound should increase. It is also shown in a model with harvest cost independent of stock size that if the fish stock is below the steady state of an optimally managed stock, then for any given stock size it is optimal to reduce the harvest rate. These results were slightly modified by assuming that costs depend on the stock of fish. Improved technology implies a more conservative management regime is in contrast to the outcome of an unregulated fishery where improved productivity increases harvests and may destroy rents. It follows that the better the technology, the more important is it to properly manage fisheries.

Jim Wilen (2000) bemoaned the lack of influence that resource economics has had on the practical regulation of the world's fisheries. Since then *harvest control rules* have gained traction as a management tool, Kvamsdal *et al* (2016). These rules establish a relationship between the estimates of fish stocks at a given point in time and total allowable catch and, if properly designed, harvest control rules may be seen as approximations to optimal management policy, Deroba and Bence (2008), Nævdal and Skonhoft (2018). The existence of a lower bound for fish stocks below which harvest rates should be set to zero is a feature that has been suggested as part of a harvest control rule, Engen *et al* (1997). Nævdal and Skonhoft (2018) argue that this is consistent with optimal management as such a bound, termed $x_c$ above, appears endogenously in general models of fish management. The results in the present paper show how harvest control rules that approximate optimal management policy should be changed in response to changes in technology. In general, improved technology should lead to more restrictive harvest control rules for low stock sizes. To the author's knowledge this issue has not been addressed in the literature.

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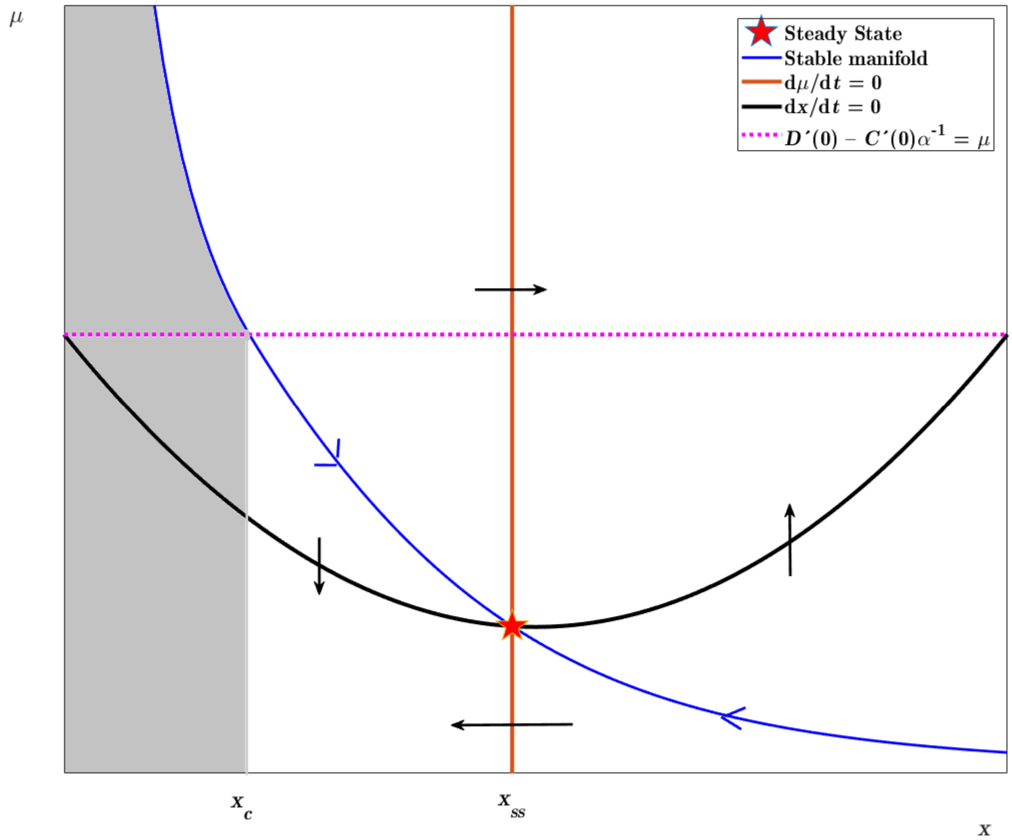


**Figure 1.** A phase portrait for an optimally managed fishery. The figure is computer generated and uses a model where $D(h) - C(h)\alpha^{-1} = ph - \frac{c}{2\alpha}h^2$ and $G(x) = rx(1 - x/K)$. The intersection between the isocline where $dx/dt = 0$ and the isocline where $d\mu/dt = 0$ defines the steady state which is a saddle point. Convergence towards the steady state is along the stable manifold that shows combinations of $x$ and $\mu$ that are compatible with optimality. There is a critical value of $x$, $x_c$, such that harvesting is zero for all $x \leq x_c$. $x_c$ is determined by the intersection of the stable manifold and the line $\mu = D'(0) - C'(0)\alpha^{-1} = D'(0)$. The stable manifold, $\mu(x, \alpha)$, is the derivative of the value function and the value of the fishery is zero when the stock is zero. This implies that the value function, $V(x)$, is given by the integral under the stable manifold. Thus $V(x_c)$, indicated by the grey area, is the value of the fishery at the stock level where it is optimal for the fishery to commence. Parameter values are given by $p = 5$, $c = 3$, $r = 1$, $K = 10$, $\rho = 0.05$ and $\alpha = 2$.

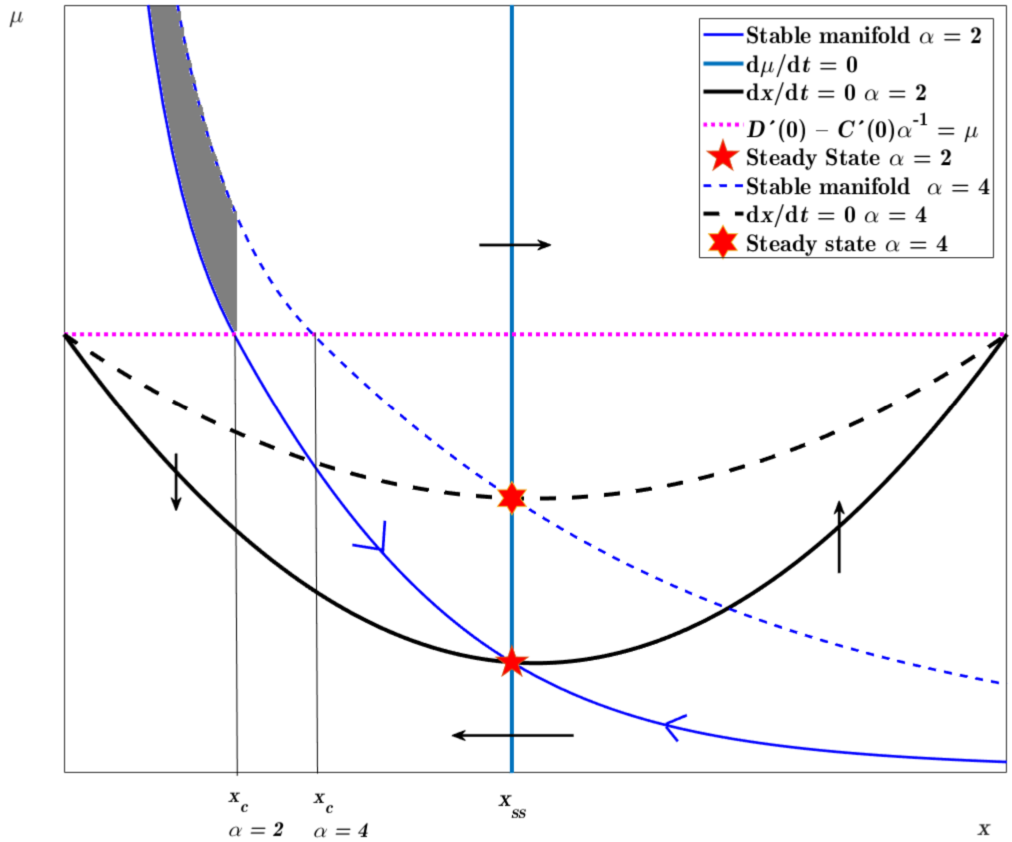


**Figure 2. The effect of technological change in a standard phase portrait. The model and parameters are as in Figure 1. The technological change consists of $\alpha$ increasing from 2 to 4. The isocline for $d\mu/dt = 0$ and the line $\mu = D'(0) - C'(0)\alpha^{-1}$ are not affected. As indicated by Proposition 1, the isocline for $dx/dt = 0$ shifts upward as indicated by the shift from the solid black line to the dashed black line. This changes the steady state from the point indicated by ★ to the point indicated by ✶. Thus the distance between the steady state and the line $\mu = D'(0) - C'(0)\alpha^{-1}$ decreases. It follows from Proposition 2 that at the stable manifolds for high and low productivity cannot intersect for any $x \in (0, x_{ss})$. These factors taken together imply Proposition 3; the stable manifold with high productivity lies above the stable manifold for low productivity for all $x \in (0, x_{ss})$. Proposition 4 follows; $x_c$ increases in response to a productivity increase. Propositions 1, 2, 3 and 4 can then be used to prove Proposition 5. The economic explanation for Propositions 3, 4 and 5 is that higher productivity increases the value of current stock growth as the value of future harvests increase. The shaded area indicates the increase in the objective function resulting from the productivity improvement if $x(0) = x_c$ for $\alpha$ low.**

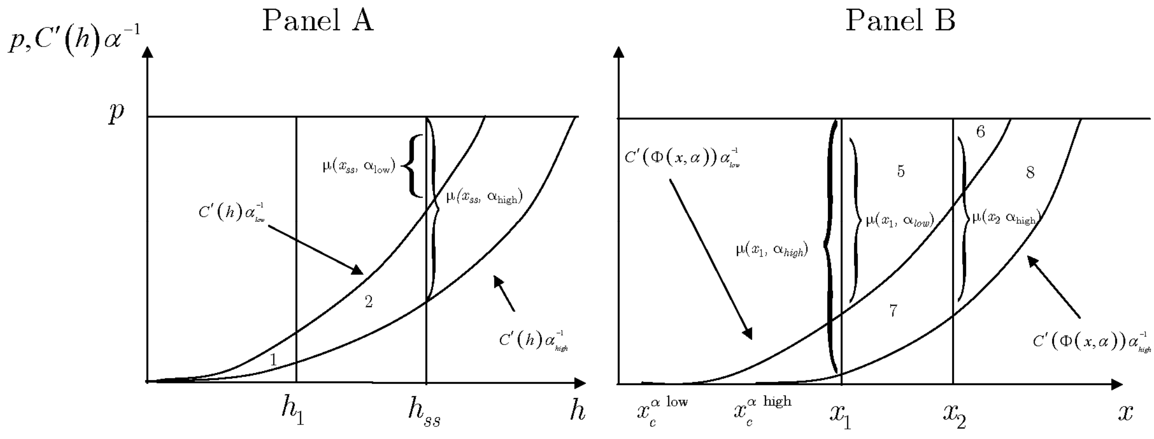


**Figure 3. Panel A shows the incentive to reduce $h$ if productivity increases. Panel B illustrates how the response to the productivity increase is implemented through a shift in $\mu(x, \alpha)$. The figure is explained in detail in the main text.**

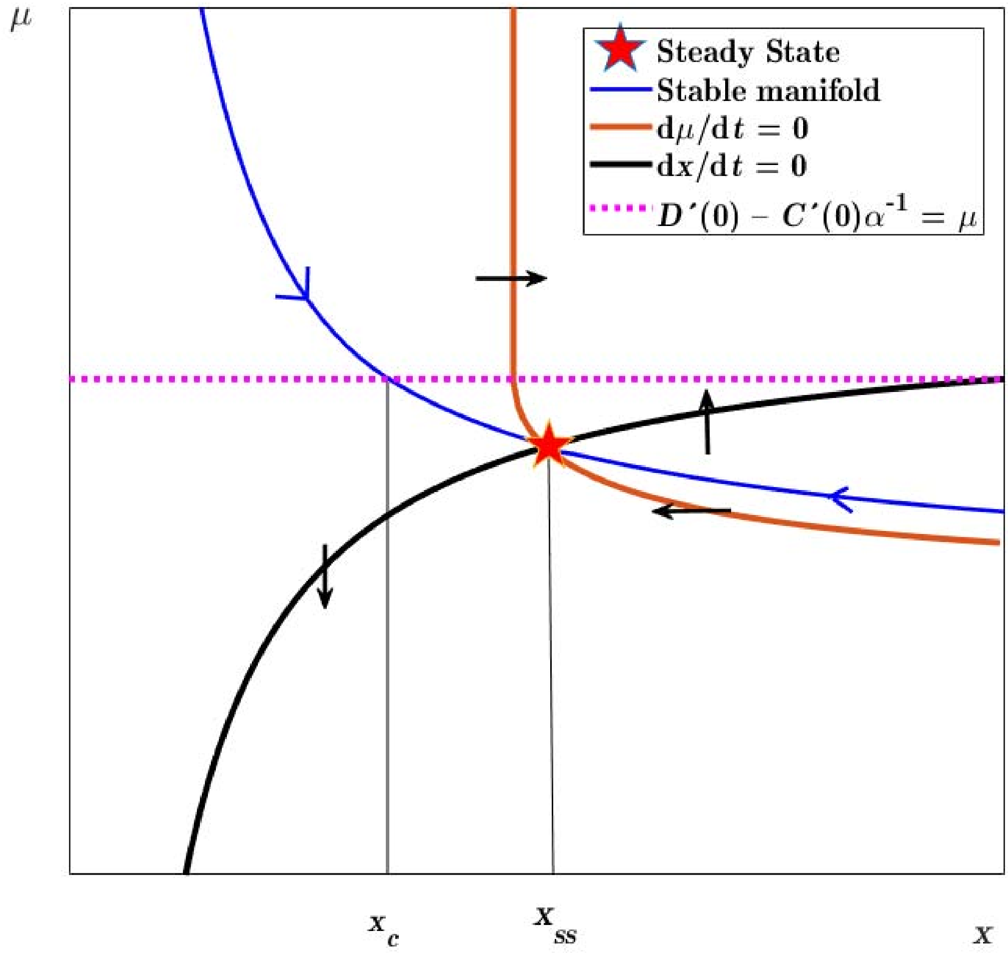


**Figure 4. Phase portrait when costs are stock dependent. The figure is computer generated and uses a model where $D(h) - C(h, x)\alpha^{-1} = ph - \frac{1}{2}c(h/x)^2\alpha^{-1}$ and $G(x) = rx(1 - x/K)$. We then have that $D'(0) - C_h'(0, x, \alpha) = p$. The major difference between this portrait and the portrait when harvesting costs are independent of stock size is the shape of the isoclines. The qualitative effect on the general shape of the stable manifold is minor. Note that for $x \leq x_c$ where $x_c$ is defined by where the stable manifold intersects the line where $\mu = p$, optimal harvest becomes zero. The value of $x$ that solves $\rho = G'(x)$ is a lower bound for the steady state stock regardless of how technology affects the cost function. Thus, there is a limit to how much stock dependent costs can affect the stable manifold at values of $x$ below this lower bound as optimal $h$ will become zero very quickly below it. Parameter values are given by $p = 5$, $c = 3$, $r = 1$, $K = 10$, $\rho = 0.05$ and $\alpha = 3/4$.**

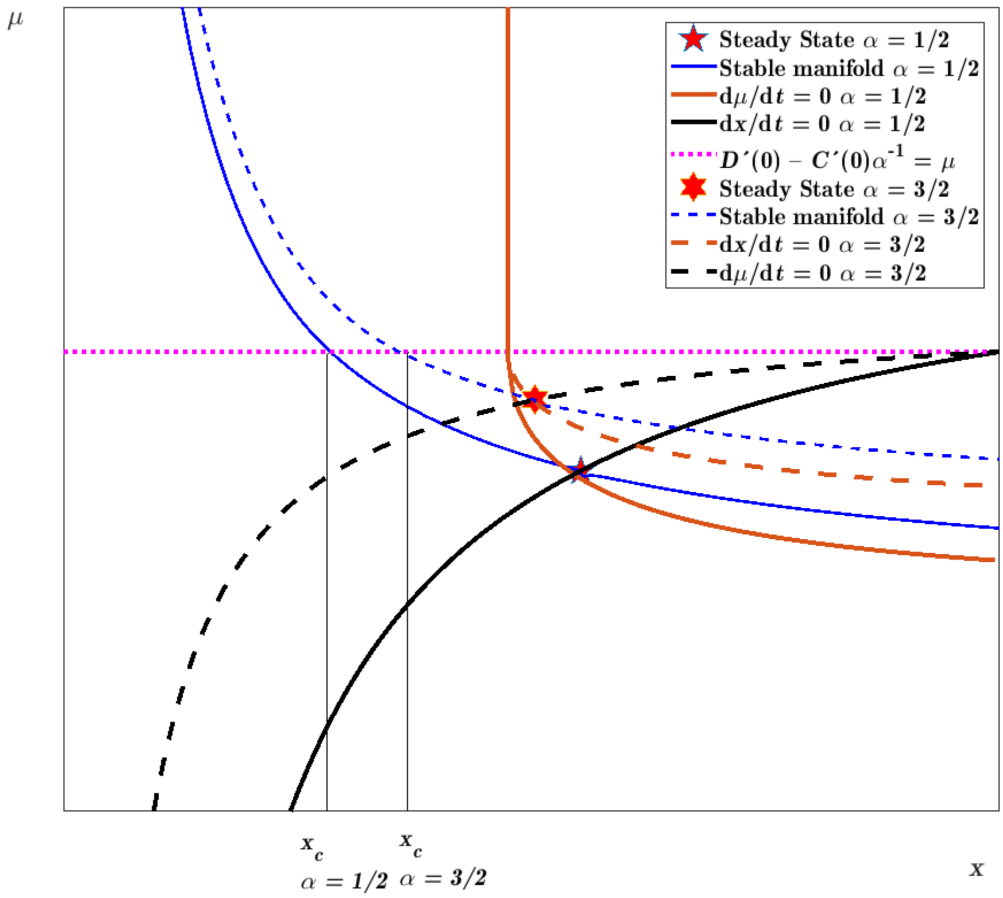


**Figure 5. The effect of a productivity increase with stock dependent costs. The figure is computer generated with the same parameterization as Figure 4 except that α is 1/2 when productivity is low and jumps to α = 3/2. In line with Propositions 6 and 7, $x_c$ increase as a function of α and reduce harvests for stock levels close to $x_c$. At the same time increased productivity decreases the steady state $x_{ss}$, so, in line with results in Caputo (1989), at least for a period it holds that higher productivity increases harvest rates if the stock is close to the steady state. At the limit, as $\alpha \to \infty$, $x_c$ and $x_{ss}$ will converge to the same point determined by $\rho = G'(x)$.**