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Framework for Optimal Production and Transmission of Electricity

Finn R. Førsund

Stiftelsen Frischsenteret for samfunnsøkonomisk forskning
Ragnar Frisch Centre for Economic Research

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Nøkkelord: Electricity modelling; Hydropower; Intermittent power; Nodal prices; Thermal power; Transmission

Contact: finn.forsund@econ.uio.no, www.frisch.uio.no

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Framework for Optimal Production

and Transmission of Electricity

by

Finn R. Førsund,

Department of Economics, University of Oslo and the Frisch Centre

Work Package 1 “Framework for optimal production and transmission”

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Abstract

The methodological approach is to set up a rather aggregated model designed to give qualitative insights into optimality properties. The model encompasses five technologies; hydro generation based on reservoirs, thermal generation based on fossil fuels, and intermittent generation encompassing run-of-river, wind and solar. Assuming limited generation capacity introduces the manoeuvrability as important for managing reservoirs. Furthermore, using hydropower based on reservoirs requires a dynamic analysis. The presence of uncertainty makes finding optimal use of reservoirs rather demanding. Some qualitative results are offered. Using Scandinavian hydropower to act as a battery for European intermittent generation is analysed using a two country model with one country having hydropower with reservoir, and the other country having intermittent generation and also thermal capacity to make the model more realistic. The main result is that optimal electricity prices will be the same in both countries if transmission is not restricting trade. The country having intermittent generation will typically export thermal generation and not necessarily intermittent. The consumers are aggregated to just two in the trade model. Introducing a transmission network that has nodes of generation of the five technologies and consumer nodes, energy losses and congestion have a decisive impact on the structure of optimal prices when alternating current is used and the network has loop flows. There are system-wide fundamental electricity external effects. All consumer nodes may have different prices determined by the externalities, and production nodes different costs. Short-run nodal prices may give a guide to investment in transmission and production capacities. However, future uncertainties of distribution of demand over nodes and weather patterns, long life of structures, capacities of the networks, and generation of electricity make optimal investments difficult to determine.

JEL classification: C61, Q41, Q42

Keywords: Electricity modelling; Hydropower; Intermittent power; Nodal prices; Thermal power; Transmission

1. Introduction

The purpose of the paper is to set out a basic model framework for production of electricity using five technologies; hydro with reservoirs, thermal[1], and three intermittent or renewable (the terms are used interchangeably in the paper) technologies; run-of-river, wind and solar. Some basic properties of optimal solutions for electricity prices and produced quantities are explored. Optimal location and capacity of new intermittent power, and optimal investments in network infrastructure with a degree of uncertainty inherent in a market with a sizeable share of intermittent power is explored. The methodological approach is to set up a rather aggregated model designed to give qualitative insights into optimality properties[2]. Key questions are if the short-run uncertainty of intermittent power change the optimal capacity of production and transmission, and if Scandinavian hydropower should act as a battery for a European market increasing significantly its intermittent power.

The first-best framework for optimal production and transmission in an electricity market with a large share of intermittent power could be characterised with references to costs and benefits independently of actual markets. This framework may then serve as benchmark for feasible market designs that may not be first-best. System efficiency in the short run and optimal investment incentives for generation and transmission are issues both at the national level, for Nord Pool countries, and for potential European trading partners.

It has been suggested that the hydropower of Norway and Sweden can serve as a battery for Europe, akin to the night-time – daytime exchange with Denmark before Nord Pool was founded (von der Fehr and Sandsbråten, 1997). When the international transmission capacity is constrained, import and export prices will not be equal (disregarding losses on the lines). If Germany wants to export electricity in a situation with abundant wind power, the Nordic countries can import up to the capacity of the interconnector. In a situation when lack of wind power creates a high price in Germany, the Nordic countries can export and receive a higher price than was paid for the import. In a situation with available reservoir capacity, it might be socially profitable to introduce pumped-storage utilising existing storage capacity. Using such

[1] Nuclear power is not included.
[2] The models are based on further development of models in Førsund (2015).

capacity is profitable if the earnings on a unit of water in a later period is greater than the cost of pumping up the water. Investment decisions must also consider the capital costs. For long-term management of hydro reservoirs, uncertainty about inflows will play a distinct role for the price formation since it will be optimal to process less water when inflows fall short of expectations, resulting in an optimal price increase, and vice versa if inflows are above expectations (Førsund (2007); (2015); Brekke et al 2013). When considering that also the intermittent resources - run-of-river, wind and solar - are stochastic, the optimal investments in transmission should be increased. Førsund (2015) has developed a simple theoretical model of pricing with intermittent power in a hydro-dominated system that will be extended. In general, there will be much more variability in supply and demand in the short term. The sources of uncertainty include supply factors such as wind patterns, sun conditions and precipitation, as well as demand uncertainties resulting from, e.g., cold or hot weather.

The paper is organised as follows. In Section 2 technologies for hydropower, thermal generation and three types of intermittent generation, run-of-river, windmills and solar, are introduced. In Section 3 a dynamic optimisation model is set up containing these technologies assuming perfect foresight. Necessary conditions for optimality are interpreted regarding optimal prices, quantities of electricity, and impact of renewables. Due to computational difficulties in handling the many stochastic variables in the optimisation model of Subsection 3.2, explicit modelling is not done in Subsection 3.4 addressing uncertainty. However, some qualitative conjectures are put forward. In Section 4 the role of hydro as a storage of energy, or a battery, is investigated using a simple two-period model. Trade between a hydro country with reservoirs and a country using wind power and thermal generation is analysed. Nodal pricing is presented in Section 5. Investments in transmission and production are addressed in Section 6. Conclusions are summed up in Section 7.

2. The production relations

We will specify three different technologies producing electricity; hydro, thermal and renewables (intermittent). Time is assumed to be discrete, and can vary from, e.g., one hour to days, weeks, months or years depending on the purpose of the modelling. Periods are written *t* and the horizon is *T*, i.e., $t = 1$ to $t = T$.

2.1 Hydro-power with reservoirs

Hydropower is the main technology for producing electricity in Norway. Over 1600 plants with over 1000 reservoirs had 94.3 % of the power capacity per 1 January 2018. Wind power was generating electricity from 33 turbines with 3.4 % of the capacity, and various thermal plants had 2.3 % of the capacity. The climate in Norway has cold winters and not so warm summers, so the use of electricity is largely for space heating of buildings. Norway does not have so warm summers that residential buildings need cooling. The reservoir capacity is crucial for the utilisation of water for electricity production during the wintertime, accumulating water during the spring and summer period and using most of it during wintertime. The reservoirs can hold about 70 % of the inflow depending on the yearly precipitation including accumulation of snow. Industrial use of electricity is about 20 % (production of aluminium is the largest user).

Fig. 2.1 shows the inflow of water and the production and consumption in Norway of electricity

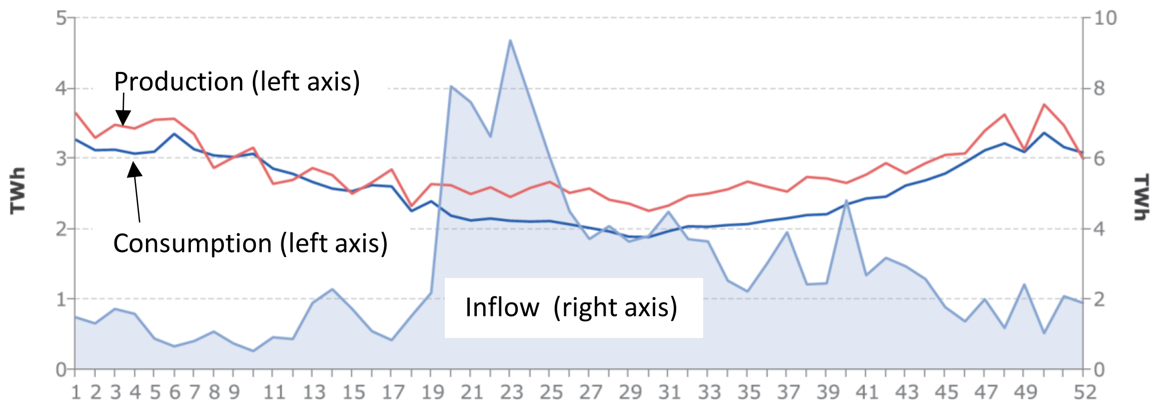

Figure 2.1. *Production, consumption and inflow during 52 weeks year 2017*
Source: Krafproduksjon - Energifakta Norge [Power generation - Energy facts Norway]

in 2017. The difference between production and consumption reflects imports and exports. In 2017 there were some very few weeks with imports in winter months; exports dominate trade. We see that the main inflow occurs in spring - summer (May, June and July). This is mainly due to the melting of snow up in the mountains. It is also raining more than in other months in September – October. The water coming from snow melting constitutes the surplus of water distributed on the other month due to having reservoir capacity. The winter periods have the lowest inflows and these are considerably smaller than the consumption, so storage of water is quite essential for the hydro-based electricity. About 70 % of average yearly inflows can be stored. There are also hydropower plants without significant reservoirs which are run-of-river plants constituting about 30 % of the hydropower production.

Hydro production relations

Electricity is produced by releasing water on to turbines. We will focus on a situation where water is released from a reservoir above the location of the turbines. The height from the reservoir to the turbines is called the head. In the first equation in (2.1) electricity e_{jt}^{H} from a unit j in period t is a function of the unit requirement of water a_j for producing one unit of electricity (measured in MWh) and the amount of water r_{jt} released (measured in m^3 of water). The coefficient a_j shown in the first equality relation in (2.1) incorporates any loss of water energy from the flow of water to the turbine and energy loss running the turbines. The unit requirement coefficient is in the model a function of the fixed head, decreasing with increase in the height of the head when comparing hydro units. We will assume that the head is not influenced by the degree of filling of the reservoir. This is a good approximation for Norway where the average head is about 200 meter.

Water is the only current variable, and it is free of charge. Capital is given as a fixed variable in our management view of production of electricity. Labour as a current input is neglected because the cost of labour is a very small fraction of the cost of capital invested, and, furthermore, labour does not follow variations in current production, but have more of a role of managing and overseeing the production. The size of the labour force is a function of total capital invested, or the capacity output. The hydro model for N^H units is:

$$
\begin{aligned}
& e_{jt}^{H} = \frac{1}{a_j} r_{jt} \\
& e_{jt}^{H} \le \bar{e}_j^{H} \\
& R_{jt} \le R_{j,t-1} + w_{jt} - e_{jt}^{H} \\
& R_{jt} \le \bar{R}_j \\
& x_t = \sum_{j=1}^{N^H} e_{jt}^{H} \\
& R_{jt}, x_t, e_{jt}^{H}, r_{jt} \ge 0 \ \text{(endogenous variables)} \\
& w_{jt}, R_{jo}, \bar{R}_j, \bar{e}_j^{H}, a_j \ \text{(exogenous variables)}, \\
& j = 1,..,N^H,\ t = 1,..,T
\end{aligned} \tag{2.1}
$$

The production of electricity is limited to the capacity $\bar{e}_j^{H}$ to produce as shown in the first inequality in (2.1). The capacity is usually measured in the power unit MW, i.e., the instantaneous flow of energy, but for our modelling it is simpler and more convenient to measure capacity as the maximal output of electricity that can be produced during the period *t*. The ratio of production and the capacity measure, $e_{jt}^{H} / \bar{e}_j^{H}$, can then be termed the capacity coefficient $a_{jt}^{H} \in [0,1]$. We then have an alternative production function formulation $e_{jt}^{H} = a_{jt}^{H} \bar{e}_j^{H}$. The capacity coefficient is period dependent.

We assume that each unit of the hydro plant has one reservoir each with independent catchment areas. The amount of water in the reservoir for unit *j* at the end of period *t* is R_{jt}. In order to simplify, water variables (except the water flow variable *r*) are all converted to energy units; R_{jt} is measured in MWh and so is the inflow w_{jt} during period *t*. Units of water are converted to MWh by dividing with the unit requirement coefficient a_j. Notice that the production of electricity is equal to the release of water r_{jt} in the production function measured in MWh. We still use the terms water and electricity although all units are measured in MWh.

In the second inequality we see that there is a connection between the reservoir levels of period *t* and *t*-1. The water in the reservoir (in MWh) at the end of period *t* is determined by the reservoir level at end of period *t*-1, the inflow of water, and the use of water for electricity production during period *t*. If we have a strict inequality this means that there is an overflow of water from the reservoir. This means that the plant functions as a run-of-river plant. From the third inequality we have that overflow implies that the water will run over the dam unless an amount equal to the inflow of water (i.e. run-of river) is processed and the reservoir filling is constant. However, this balancing is only possible if the production capacity is large enough.

The amount of water that a reservoir can room is restricted to $\bar{R}_j$, as seen in the third inequality. The reservoir may have both and upper limit and a lower one. The latter is usually based on environmental considerations imposed by the regulating authority. The maximal amount of water is the difference between the physical maximum and the regulated minimum. For simplicity it is assumed that the lower limit is never breached and therefore set to zero.

2.2 Thermal power

Thermal plants using primary energy in the form of oil, coal, gas and wood can generate electricity. The primary energy is used to heat water and generate steam that that drives turbines generating electricity. (Nuclear power as primary energy is also used to produce steam that is driving turbines generating electricity and thus a type of thermal generator. However, nuclear energy is a typical must-run form of producing electricity, and will not be treated here.)

Variable inputs in the production of electricity by thermal plant are primary energy and labour. However, the amount of labour is not usually following the variations of production, but have more a character of a fixed input determined by the maximum capacity of electricity that the plant can produce. Primary energy use, however, follows closely the actual production profile of electricity and is a variable input, especially if the primary energy is measured in heat units like British thermal units (Btu), and not in weight or volume. Taking capital as sunk cost focussing on the current management of the plant a cost function in current output of electricity is representing a short-term production function:

$$\begin{aligned} &c_{it} = c_i(e_{it}^{Th}),\ c_i' > 0,\ c_i'' \geq 0, \\ &e_{it}^{Th} \leq \bar{e}_{it}^{Th},\ i = 1,\ldots,N^{Th} \end{aligned} \tag{2.2}$$

The technology is for simplicity assumed the same for the time periods we consider (but introducing different technologies is straightforward adding t as a subindex), and e_{it}^{Th} is the production of electricity in period t by the thermal plant i. Superscript *Th* is used for the thermal plants, and the electricity produced during period t is $e_{it}^{Th} \leq \bar{e}_{it}^{Th}$, where the last term is the power capacity. The limit of turbine capacity is measured in the maximal possible amount of electricity in period t when the turbine capacity is fully utilised. Due to the cost of closing and start-up of a thermal plant, the output is not often regulated down to zero. There may also be somewhat different technologies or primary energy types used for base load plants and peak load plants. We will for simplicity assume a constant price of primary energy and labour over all periods.

2.3 Intermittent energy

The three main forms of renewable energy or intermittent energy are hydropower without reservoir, i.e. run-of-river plants, windmills, and solar energy.

Run-of-river

A run-of river plant will usually have a dam with a storage capacity too small to be regulated like a plant with a sizeable reservoir, and usually a short fall from the dam down to the turbines, making the unit requirement coefficient much higher than for plants with reservoirs.[3] There is a capacity restriction $\bar{e}_r^{Rr}$ for the turbines. The production relations for a run-of-river plant are:

$$\begin{aligned} e_{rt}^{Rr} &= \frac{1}{a_r^R} r_{rt}^{Rr} \\ e_{rt}^{Rr} &\le \bar{e}_r^{Rr},\ r = 1,...N^{Rr} \end{aligned} \tag{2.3}$$

The river flow is an exogenous variable, and this determines the electricity produced up to the upper limit.

Superscript *Rr* is used for the run-of-river plants, and the river flow during period *t* is r_{rt}^{Rr}. The limit of turbine capacity is measured as the maximal possible amount of electricity in period *t* when the turbine capacity is fully utilised. We then have $\bar{e}_r^{Rr} \le r_{rt}^{Rr}$, i.e. water may be running past the turbine.

Windmills

Windmills are driven by the prevailing wind. For the windmills to produce electricity there must be some wind (around 4 m/s), and there is a maximal wind that can be tolerated around 25 m/s (the windmill rotors are positioned to catch as little wind as possible and the turbine is not running). Use of labour after the investment is quite limited and engaged in maintenance, and thus not being regarded as a variable factor in the current production function.

[3] However, some run-of-river plants may also have high falls.

The production relation is usually expressed by using a *capacity factor* a_{wt}^{Wi} for one year and also giving the power of the turbine in MW, however, here we measure in MWh:

$$\begin{aligned} &e_{wt}^{Wi} = a_{wt}^{Wi}\bar{e}_{w}^{Wi},\ a_{wt}^{Wi} = \frac{e_{wt}^{Wi}}{\bar{e}_{w}^{Wi}} \in [0,1] \\ &e_{wt}^{Wi} \le \bar{e}_{w}^{Wi},\ w = 1,\ldots N^{Wi} \end{aligned} \tag{2.4}$$

The observed production (in MWh) of unit *w* observed in period *t* is e_{wt}^{Wi}, and the maximal capacity production is $\bar{e}_{w}^{Wi}$ This will be the total production for period *t* if windmill *w* had a maximal wind blowing (≈ 25 m/s) during all hours in period *t*. The capacity factor depends both on the type of windmill *w* and on its production in period *t* and must then be calculated for each period. However, the capacity factor may not be so different for different types of windmills. The maximal production is more decisive. This is the reason large windmills will be more productive than smaller ones for the same pattern of wind blowing during the period.

Solar power

There are two types of solar power plants in use; photovoltaic technology and using directionally adjustable mirrors concentrating the sunrays to shine on a tower heating up water inside the tower generating steam driving the electricity generator. Both technologies can be used at a large scale, but the photovoltaic technology can also be used on a much smaller scale being put onto roofs of houses or suitable buildings in general.

Solar energy can only be captured when the sun is up (cloud cover does not reduce the capture of sun much, but volcanic eruptions would reduce the output and sand storms destroy the surface of mirrors.)

The production relations can be formulated in the same way as for windmills using a capacity factor. For solar power plant *s* we have:

$$\begin{aligned} &e_{st}^{S} = a_{st}^{S}\bar{e}_{s}^{S},\ a_{st}^{S} = \frac{e_{st}^{S}}{\bar{e}_{s}^{S}} \in [0,1] \\ &e_{st}^{S} \le \bar{e}_{s}^{S},\ s = 1,\ldots N^{S} \end{aligned} \tag{2.5}$$

The largest influence on the production of a unit *s* is as for solar power the maximal capacity $\bar{e}_{s}^{S}$. For solar power, the geographic location influences the production due to the detrimental seasonal changes of sunshine at locations sufficiently away from the equator.

Use of labour after the investment follows the pattern for wind power above. We assume that labour use after investment has been done is quite limited and mainly engaged in maintenance and process control, thus not being regarded as a variable factor in the current production function.

3. The optimisation problem

3.1 The objective function

In order to find optimal prices and quantities an objective function has to be specified. In economics a standard objective function in empirical studies is to maximise consumer plus producer surplus with the consumed quantities of electricity (equal to the produced in an optimal solution) as *endogenous* variables. This is a partial equilibrium approach because no interaction with the rest of the economy is modelled.

The consumer inverse demand function is on the price form with price as a function of consumption of electricity. A technical assumption needed on the demand functions is that there is a finite choke price yielding zero demand. Otherwise, demand is assumed to decrease in price in the standard way in economics. These assumptions are all standard when employing the consumer surplus concept.

In the case of hydropower and renewables with zero operating costs, the social surplus is simplified to the area under the consumer demand function ((since the consumers price is equal to producers' prices).). For simplicity the production units are represented by vectors. The objective function is:[4]

$$\sum_{t=1}^{T} \int_{\xi=0}^{(e_t^H + e_t^{Ri} + e_t^{Wi} + e_t^S)} p_t(\xi) d\xi \tag{3.1a}$$

We assume that the demand function varies over the periods, e.g. night-time demand is less than daytime demand, summer periods have smaller demand than winter periods considering

[4] It is assumed that consumption is equal to production for all t; therefore, production is used instead in the demand function.

countries with cold winters and pleasant summer temperatures not requiring too much air condition of buildings. Renewable power is exogenously given, so it is only hydropower with reservoir that is endogenous in the objective function (3.1a).

Adding thermal power, the production side is expressed by costs of producing electricity. The objective function is extended with the thermal cost function subtracted demand; surplus is maximised and costs minimised. For simplicity the thermal units and their cost functions are represented as vectors, as in (3.1a) for the other technologies:

$$\sum_{t=1}^{T}[\int_{\xi=0}^{(e_t^H+e_t^{Th}+e_t^{Ri}+e_t^{Wi}+e_t^S)} p_t(\xi)d\xi - c(e_t^{Th})] \tag{3.1b}$$

3.2 The model framework

The production of thermal power (equal to the consumed quantity) is an endogenous variable. Discounting is not introduced since the horizon T is usually so short that the effect will be negligible even for some late yearly periods, but it will be straightforward to include.

The production of electricity from the three types of renewables is exogenous and therefore we aggregate their production to e_t^I (superscript I standing for intermittent):

$$\sum_{r=1}^{N^{Ri}} e_{rt}^{Ri} + \sum_{w=1}^{N^{Wi}} e_{wt}^{Wi} + \sum_{s=1}^{N^S} e_{st}^S \equiv e_t^I \leq \bar{e}_t^I \tag{3.2}$$

The aggregate capacity of renewables is $\bar{e}_t^I$. Subscript t is used in case of expanding the capacities over time.

We do not identify individual consumers located in different places, but treat the consumer side as if aggregated to a single consumer. The energy balance of total consumption x_t equal to production from the five technologies is:

$$x_t = \sum_{j=1}^{N^H} e_{jt}^H + \sum_{i=1}^{N^{Th}} e_{it}^{Th} + e_t^I \tag{3.3}$$

The formal optimisation problem is to maximise the sum of benefit functions and minimise thermal costs for all periods from t = 1 to t = T given the constraints for all types of electricity generation:

$$
\begin{aligned}
&Max_{e_t^H, e_t^{Th}} \sum_{t=1}^{T} [\int_{\xi=0}^{x_t} p_t(\xi) d\xi - \sum_{i=1}^{N^{Th}} c_{it}(e_{it}^{Th})] \\
&\text{subject to} \\
&x_t = \sum_{j=1}^{N^H} e_{jt}^H + \sum_{i=1}^{N^{Th}} e_{it}^{Th} + e_t^I \\
&e_{jt}^H \le \bar{e}_{jt}^H \\
&R_{jt} \le R_{j,t-1} + w_{jt} - e_{jt}^H, \; j = 1,...,N^H \\
&R_{jt} \le \bar{R}_{jt} \\
&e_{it}^{Th} \le \bar{e}_{it}^{Th}, \; i = 1,...,N^{Th}
\end{aligned}
\tag{3.4}
$$

The hydro and thermal capacities also have subscript t in case capacity is increased over time, and the cost function may change over time due to technological progress. The optimisation problem (3.4) is a discrete time dynamic programming problem. The dynamic feature is due to the reservoir accumulation relation. Special solution procedures have been developed for this class of problems (Bellman, 1957; Sydsæter et al., 2005). The variables in (3.4) may be divided into *state* variables and *control* variables. The former corresponds to the level of water in the reservoir, R_t, and the latter to the hydro production e_t^H and the thermal generation e_t^{Th}. The objective function (3.1b) inserted the optimal solution is called the *value function*, $V(R)$, that can be written as a function of the state variables. The state variable in problem (3.4) is a function of the control variable due to the water accumulation equation in (3.4), thus the value function can be expressed as a function only of the optimal R. The idea of the solution procedure in dynamic programming is to decompose the problem into sub-problems that are easier to solve. Consider a time period u as one of the periods $1,\ldots,T$ - 1. Then Bellman's principle of optimality states that the problem of finding the value function for u can be written as the sum of the optimal solution for period u and the objective function inserted the optimal solutions for the rest of the periods from u +1 to T.

The latter function is then the value function for period u + 1, yielding the dynamic programming equations (the name Bellman Equation is usually reserved for a problem with infinite horizon):

$$
V_u(R) = \max_{e^H, e^{Th}} [\int_{\xi=0}^{e_u^H + e_u^{Th} + e_u^I} p_u(\xi)\, d\xi - c(e_u^{Th}) + \sum_{t=u+1}^{T} (\int_{\xi=0}^{e_u^H + e_u^{Th} + e_u^I} p_u(\xi)\, d\xi - c_t(e_u^{Th}))] \tag{3.5}
$$

In addition, the restrictions in (3.4) have to be satisfied. (The vectors of production units are used.) Of course, the cost- and demand functions, $c_t(.)$ and $p_t(.)$, must also be known. However, because of the special structure of the problem we shall treat it as a standard nonlinear programming problem and use the Kuhn-Tucker conditions for discussing qualitative characterisations of the optimal solution.

Using the energy balance (3.3) to substitute production for consumption, the Lagrangian function for the problem (3.4) is:

$$\begin{aligned}
L = & \sum_{t=1}^{T} [\int_{\xi=0}^{(\sum_{j=1}^{N^H} e_{jt}^H + \sum_{i=1}^{N^{Th}} e_{it}^{Th} + e_t^I)} p_t(\xi) d\xi - \sum_{i=1}^{N^{Th}} c_{it}(e_{it}^{Th})] \\
& - \sum_{t=1}^{T} \sum_{j=1}^{N^H} \lambda_{jt} (R_{jt} - R_{j,t-1} - w_{jt} + e_{jt}^H) \\
& - \sum_{t=1}^{T} \sum_{j=1}^{N^H} \gamma_{jt} (R_{jt} - \bar{R}_j) \\
& - \sum_{t=1}^{T} \sum_{j=1}^{N^H} \rho_{jt} (e_{jt}^H - \bar{e}_{jt}^H) \\
& - \sum_{t=1}^{T} \sum_{i=1}^{N^{Th}} \theta_{it} (e_{it}^{Th} - \bar{e}_{it}^{Th})
\end{aligned} \tag{3.6}$$

The shadow price λ_{jt} in the second line is the shadow price on the water in the reservoir, termed *water value* for short. It shows in general the change in the value of the objective function, evaluated at an optimal solution, of a marginal change in the constraint. In our case, the water value in period *t* shows the value in terms of an increase in consumer surplus of a marginal increase either in the transfer of water from period $t - 1$ or an increase in the inflow in period *t*, assuming the optimal thermal costs being constant.

Using the envelope theorem, we have

$$\frac{\partial [\sum_{t=1}^{T} \int_{\xi=0}^{x_t} p_t(\xi) d\xi - \sum_{i=1}^{N^{Th}} c_{it}(e_{it}^{Th})]}{\partial w_{jt}} = \frac{\partial L}{\partial w_{jt}} = \lambda_{jt}, \; j = 1, ..., N^H \tag{3.7}$$

The water value λ_{jt} is an opportunity cost for using water to produce electricity in period *t*.

The necessary first-order conditions for the three types of endogenous variables hydro production, reservoir filling and thermal generation are:

$$\frac{\partial L}{\partial e_{jt}^{H}} = p_t(\sum_{j=1}^{N^H} e_{jt}^{H} + \sum_{i=1}^{N^{Th}} e_{it}^{Th} + e_t^I) - \lambda_{jt} - \rho_{jt} \leq 0 \ (= 0 \text{ for } e_{jt}^{H} > 0)$$
$$\frac{\partial L}{\partial R_{jt}} = -\lambda_{jt} + \lambda_{j,t+1} - \gamma_{jt} \leq 0 \ (= 0 \text{ for } R_{jt} > 0)$$
$$\frac{\partial L}{\partial e_{it}^{Th}} = p_t(\sum_{j=1}^{N^H} e_{jt}^{H} + \sum_{i=1}^{N^{Th}} e_{it}^{Th} + e_t^I) - c'_{it}(e_{it}^{Th}) - \theta_{it} \leq 0 \ (= 0 \text{ for } e_{it}^{Th} > 0)$$
$$\lambda_{jt} \geq 0 \ (= 0 \text{ for } R_{jt} < R_{j,t-1} + w_{jt} - e_{jt}^{H}) \tag{3.8}$$
$$\gamma_{jt} \geq 0 \ (= 0 \text{ for } R_{jt} < \bar{R}_j)$$
$$\rho_{jt} \geq 0 \ (= 0 \text{ for } e_{jt}^{H} < \bar{e}_{jt}^{H})$$
$$\theta_{it} \geq 0 \ (= 0 \text{ for } e_{it}^{Th} < \bar{e}_{it}^{Th})$$

Notice that the reservoir filling for all reservoir N^H appears in two equations; the reservoir condition for period t and also for period $t+1$ not shown explicitly in (3.8)[5].

3.3 Qualitative interpretations of results

Optimal prices should be the same for all consumers of electricity; it should not matter for consumers which type of plants that have generated the electricity, implying that hydro-, thermal- and intermittent producers face the same price[6]. The renewable generation is exogenous and therefore optimal prices for them is not part of the solution when solving the optimisation problem (3.4).

It may be the case that renewable production is more than enough to satisfy demand, e.g., at night-time in the summer in Nordic countries. Then the period price follows from the demand function based only on renewable production, $p_t = p_t(e_t^I)$ and the optimal price may even be zero.

The production constraint for a hydro unit will have an impact on the *manoeuvrability* of the stored water of the unit. If all the water in the reservoir can be emptied within a period t, then

[5] When we look at period $t+1$ in the second constraint in (3.6) we will have that the constraint for period $t+1$ is: $-\lambda_{j,t+1}(R_{j,t+1} - R_{jt} - w_{j,t+1} + e_{j,t+1}^{H})$.

[6] Some consumers may be willing to pay higher prices for intermittent energy; this is disregarded here because when electricity is transferred within a meshed transmission system it is very difficult to verify that the producer delivering the energy is exclusively an intermittent one.

the unit has maximal manoeuvrability.[7] The degree of manoeuvrability can be defined as the minimum number of periods, t^o, it takes to empty the reservoir;

$$t^o = \min t \text{ such that } t\bar{e}^H_{jt} \geq \bar{R},\ j = 1,...,N^H \tag{3.9}$$

where t^o and t are integers. The higher t^o is the less manoeuvrability, t^o=1 is maximal manoeuvrability.

Preventing overflow has to be planned for several periods before the actual threat of overflow if inflows are higher than the production capacity for some periods before the threat of overflow. The management task is to create enough space in the reservoir to contain the inflows without spilling water. The ability to run down the reservoir level is present only for periods when maximal production can exceed inflow; $\bar{e}^H_{jt} > w_{jt}$.

In order for the hydro units reaching the capacity constraint not to lose water, the plants are used as a run-of-river plant to keep the reservoir constant at capacity level (or close) until periods when demand is greater. Hydro plants with large reservoirs may not be used continuously, but produce zero in some periods, and pick high-price future periods to produce.[8]

A certain combination of inflow patterns and production restrictions may lead to a *locking-in* of water. This may happen if overflow is physically inevitable, as is the case if, starting with an empty reservoir, the inflows are so great that the reservoir flows over in a later period although full production capacity has been used in all periods.

In Førsund (2015) a seminal paper of Hveding (1968) is quoted stating:

> … no single reservoir is overflowing before all reservoirs are filled up, and … no single reservoir is empty before all are empty (Hveding, 1968, p. 131).

This property leads to an aggregation of all hydro plants and reservoirs to a single plant and reservoir that simplifies considerably finding optimal prices. The following statement is introduced as *Hveding's conjecture* in Førsund (2015, p. 81):

> In the case of many independent hydropower plants with one limited reservoir each, assuming perfect manoeuvrability of reservoirs, but plant-specific inflows, the plants can be regarded as a single aggregate plant and the reservoirs can be regarded as a single aggregate reservoir when finding the optimal solution for operating the hydropower system.

[7] Typical period length may be a week or a month.

[8] The largest reservoirs in Norway were constructed in order to alleviate dry years. However, today with transmission connections to other countries, such large reservoirs would probably not have been constructed.

However, an aggregation is not accurate if the manoeuvrability of reservoirs is not perfect, as will be the typical case in model (3.4), and in reality.

In the situation with some hydro plants facing full reservoirs it is possible that the unconstrained plants can increase their production enough to the make the previously constrained units to become unconstrained by a very small margin. (A central authority can do this reallocation of production.) Then the water values become equal for all units, or we can say that the shadow price on a previously constrained unit becomes zero. However, constraints on hydro plant production capacity of electricity will make Hveding's conjecture fail.

The constraints in (3.4) on capacity of hydro electricity generation, thermal generation and intermittent production play a crucial role in determining optimal consumer- and producer prices. We have from the first-order conditions in (3.8):

$$p_t(\sum_{j=1}^{N^H} e_{jt}^H + \sum_{i=1}^{N^{Th}} e_{it}^{Th} + e_t^I) = \lambda_{jt} + \rho_{jt} = c_{it}'(e_{it}^{Th}) + \theta_{it} \text{ for } e_{jt}^H > 0, e_{it}^{Th} > 0 \tag{3.10}$$

for all units of hydro units $j = 1,\ldots, N^H$ and thermal units $i = 1,\ldots, N^{Th}$ having positive production in period t. The optimal period price is common for all hydro-, thermal- and renewable units.

The use of thermal plants follows the principle of merit order according to marginal costs; units not producing are assumed to have a marginal cost higher than the optimal price; $p_t \le c_i'(e_{it}^{Th})$, where plant i now is the first unit not to be fully used when ordering the thermal units according to merit order.[9] Plant i +1 and the rest of the units on the upper part of the merit order ranking are not used. In the short term, no thermal plant will be used if the marginal cost at the minimum production of the most expensive thermal plant is greater than the price.

To find optimal prices and hydro- and thermal production quantities is a rather complex task. Commenting on the first-order conditions above can only give some qualitative insights. To find a full optimal solution for all periods a problem like (3.4) must be solved.

We see that hydro units with a positive shadow price on the production capacity have a smaller water value than units with lower capacity utilisation, the former are "punished" for having a limited capacity. However, this is information saying that expanding the power capacity may be beneficial, and this is interesting information for plans of investment.

[9] As mentioned in Subsection 2.2 on thermal power, the typical minimum level of positive production is higher than zero, and may be around 30-40 % of the production capacity.

The shadow price λ on the energy stored in the reservoir plays an important role for the dynamics of the model. The change in the reservoir from period to period is the only dynamic part of the model. We see from line two in (3.8) that if the shadow price γ_{jt} on the reservoir capacity is zero, then the water value is the same for both periods, $\lambda_{jt} = \lambda_{j,t+1}$. This implies that all producers with non-binding reservoir constraints are facing the same water value in the two periods, and are indifferent between using water in the two periods. Going from a period with non-binding hydro constraint to a period with binding constraints implies that the price will increase due to the increase in demand assuming that the thermal- and renewable production are constant.[10] The opposite happens going from a period with active capacity constraints of all units to a period without the constraints being active; the price decrease due to reduced demand, again assuming constant thermal and intermittent production.

There are two events that are crucial for hydro plants: reservoirs can become empty and water can flow over. Threat of both events may trigger price changes. Threats of overflow can be controlled if the production capacity of a plant exceeds the inflow of water, and the plant can be utilised as a run-of-river plant. In our model, the situation that the reservoirs of hydro plants are in between empty and full may cause the solution for plant-specific use of water is indeterminate for model (3.4). It is only the total use of water that can be determined using this compact model.

Without renewable generation, if no capacity constraints are binding in any period, then the water values will all be equal for all periods and equal to the marginal costs of thermal units, and the price will be uniform. However, this is, of course, not realistic, and we have renewable production that varies between periods due to exogenous precipitation, wind and sunshine. An increase in renewable production for a period may crowd out some hydro- and thermal generation but reduce price, and a decrease in renewables may imply more use of hydro and thermal production and increased price.

Let us assume that for some limited periods $t \in J$ within the T periods the price stays constant. The hydro plants will be the swing producers, absorbing both demand change and change in intermittent energy[11] (assuming for simplicity that capacity constraints for hydro is not reached) :

[10] We disregard the fact that when the price increases the thermal production will be increased, putting a brake on the extent of the increase in price.

[11] Hydro will also be a swing producer if prices change, but it will be a more complicated way of showing the swing.

$$\underbrace{(\sum_{j=1}^{N^H} e_{j,t+1}^H - \sum_{j=1}^{N^H} e_{j,t}^H)}_{\text{hydro swing}} = \underbrace{(x_{t+1} - x_t)}_{\text{demand change}} - \underbrace{(e_{t+1}^I - e_t^I)}_{\text{intermittent change}} \ , t, t+1 \in J \tag{3.11}$$

Because the price is assumed constant for J periods, the use of thermal generation stays the same. For a given demand change the maximal negative (positive) swing is when all intermittent energy is maximal (zero) in period t +1 and zero (maximal) in period t with no water running in run-of-river plants, either too strong wind or no wind at all, and no sunshine at daytime (or the opposite movements in intermittent energy). There are limits for how large swing hydro plants can accommodate. However, the maximal change of all three types of intermittent plants happening in the same period has a very small probability. Also, even for realistic changes in intermittent energy, the assumption of fixed prices may not be attainable.

It is possible that no hydro plants will be used if intermittent energy is abundant enough relative to total demand. The condition is that the optimal price is smaller than the water values: $p_t(\sum_{i=1}^{N^{Th}} e_{it}^{Th} + e_t^I) < \lambda_{jt}$ for all $j = 1,..., N^H$. However, at the end of period t there may be hydro units that have a threat of reservoir overflow implying that their water values are zero (see line four in (3.8)). These units have to produce in period t at least so much that overflow does not happen. Loss of water cannot be optimal if it is possible to avoid it. The price can drop substantially also in this case, even if some thermal capacity is used (see above in the first paragraph after Eq. (3.10) for the rule of no use of thermal plants).

3.4 Uncertainty

In Subsection 3.2 it was assumed that perfect predictions could be done for all variables that change from period to period. However, a decision about use of water in reservoirs to produce electricity in the current period and transferring water to the next period has to be made in the current period, while the inflows of the future periods up to the horizon are known only by their predictions. Temperature conditions influence demand; inflows to hydro reservoirs vary with precipitation and to the period when the melting of snow starts. Run-of-river production vary with precipitation and snow melting, wind energy varies with strength of wind, sunshine varies in intensity and over seasons, if located sufficiently outside equator, and cost of primary energy and labour for thermal plants are all stochastic variables in future periods.

However, it is rather complicated to use a Bellman approach of backwards induction to get qualitative insights. A deterministic model as set up in Subsection 3.3 cannot be used to find qualitative insights when future key variables are stochastic. The best we can do in the current period is to have a plan for all periods by maximising the *expectation* of the sum of consumer plus producer surpluses. Using the expectation of the benefit function (3.1b), we have:

$$\begin{aligned}
&Max_{e_t^H, e_t^{Th}} E[\sum_{t=1}^{T} \int_{\xi=0}^{x_t} (p_t(\xi) d\xi - \sum_{i=1}^{N^{Th}} c_{it}(e_{it}^{Th}))] \\
&\text{subject to} \\
&x_t = x_t^H + x_t^{Th} + x_t^I = \sum_{j=1}^{N^H} e_{jt}^H + \sum_{i=1}^{N^{Th}} e_{it}^{Th} + e_t^I \\
&e_{jt}^H \le \bar{e}_{jt}^H \\
&R_{jt} \le R_{j,t-1} + w_{jt} - e_{jt}^H ,\ j = 1,..., N^H \\
&R_{jt} \le \bar{R}_j \\
&e_{it}^{Th} \le \bar{e}_{it}^{Th},\ i = 1,..., N^{Th} \\
&x_t^I = e_t^I
\end{aligned} \tag{3.12}$$

However, all the variables that can change with period t (including renewables) are stochastic, so model (3.12) is not the way to find solutions when endogenous variables are stochastic. The most common possibility is to formulate a discrete-time stochastic dynamic model and use the Bellman principle of backward solution, starting with the terminal period T using numerical methods (Pereira 1989; Kall and Wallace 1994; Wallace and Fleten 2002). Another approach is to use numerical stochastic scenario aggregating scenarios, where each scenario is one possible future realisation of the value of the stochastic variables (Brekke et al. 2013). There are empirical distributions building on many years for precipitation, snow cover, time periods of the melting of snow, and temperature that can be used estimating the distributions.

However, these approaches are rather complicated to solve, and numerical techniques have to be applied. We will just offer some qualitative conjectures about the impact of uncertainty on optimal prices and quantities.

The degree of manoeuvrability is the key to avoid overflow. In the face of uncertainty, a conjecture is that more hydropower capacity would be installed at the time of investment than would have been installed under perfect foresight. The same holds for thermal power capacity.

Uncertainty is also a driver to invest in transmission to neighbouring countries, like Norway's grid connection to Sweden and underwater cables to Denmark, Netherlands and Germany in order to secure supply of electricity in periods of draught in Norway.[12]

In order to avoid shutting down electricity to consumers, variable prices can instead be introduced to reduce demand according to predictions of supply. In dry periods, high prices can keep all consumers with electricity instead of shutting them down if consumers react sufficiently to price signals to reduce their consumption.

In addition to emptying the reservoir and entering a situation with threat of overflow, uncertainty about future inflows will independently create price variations in the optimal planning solution. Although the model (3.4) we set up in Subsection 3.2 is quite simple, we saw that to obtain solutions may be a complex task, and has to be done numerically for real-life applications.

We saw in Subsection 3.3, Eq. (3.10), that although the optimal prices are the same for all plants for each period, the manoeuvring to avoid overflow is an individual plant task and will now involve the plant-specific uncertainty about inflows. The individual manoeuvring plans must be based on expectations about the future inflows and the optimal prices, but moving forward in real time not only creates a deviation between the real-time price and the expected one, but also implies that each individual plan based on expectations will be subject to adjustments as time evolves. The individual changes then give feedback to the actual price formation within the social planning context.

Facing uncertainty, it would be reasonable to assume that some overflow would occasionally occur. Manoeuvring such that overflow never occurs has a cost that must be weighed against the loss of water when overflow happens. Naturally, *ex ante* the probability of overflow must come into consideration. Morlat (1964) formulated the planning problem under uncertainty analogously to the Hveding conjecture in Subsection 3.3 about manoeuvring of individual plants that may be termed Morlat's conjecture:

> **Morlat's conjecture:** *Individual reservoirs should be manoeuvred in such a way that the probability of overflow is the same for all reservoirs* (Morlat, 1964, p. 172).

[12] However, there is also a profit motive here for Norway to export electricity when electricity is sufficiently cheaper in Norway than in countries with transmission connections with Norway. More on this in Subsection 4.2.

Morlat did not address the situation of emptying the reservoirs, but since the situation is symmetric, it is tempting to suggest that the continuation of Morlat's conjecture would be to state that the manoeuvrings of the reservoirs should also lead to the reservoirs having the same probability of being emptied. However, we will leave this complicated topic here and not attempt to develop a formal analysis.

4. Hydropower as a battery

In Subsection 3.3 we have seen that if all intermittent production is to be used then hydropower must act as a swing producer together with thermal if period prices vary. If there is a possibility that intermittent energy can be stored the necessity of having swing producers will be much smaller. There are several technical options, like batteries, compressed air, producing hydrogen, heat and mechanical storage using flywheels, and lifting weights up a hill that can be dropped down creating electricity. However, these options all have energy losses.

4.1 Pumped storage hydropower

An old way of smoothing the use of electricity over short-term (daily) cycles is *pumped storage*. The standard pumped storage consists of a source of water (river, lake) at the location of the generator and a purpose-built reservoir at a higher altitude without any natural inflow.[13] Water can be pumped up to the reservoir and then released on to the turbines to generate electricity. The world-wide capacity installed so far is rather limited and mostly made for supply adjustment of the daily cycle. However, equipping existing hydropower plants with turbines that can be converted to pumps sending water back to a reservoir, means that huge reservoirs already in place can be used, and seasonal demand cycles can then also be met (Warland et al. 2011).

The topic of pumped-storage hydroelectricity is traditionally an engineering one, with numerous papers in technical journals on the topic. However, because less energy is created

[13] In Japan there is a storage where sea water is pumped up to a reservoir.

than the energy it takes to pump up water, there is an economic problem at the heart of pumped storage. The fundamental requirement for pumped storage to be an economic proposition is that there must be a price difference between periods of sufficient magnitude so that the loss is overcome by the difference in price, and in addition there is the cost of the investment in pumped storage to be covered.

Crampes and Moreaux (2010) study the use of pumped storage together with thermal electricity generation within a region (country) without external links using a two-period model. A two-period model is also used here. To extend the analysis to multiple periods is not so straightforward. The reasons for this will be commented upon later (see footnote 14). However, pumped storage is used together with intermittent energy.

The case of only having intermittent energy (especially wind and solar power) may be realistic for isolated regions like islands where links to the central grid of the country in question are too expensive (Bueno and Carta, 2006). For simplicity, we use only one pumped facility and one unit providing intermittent energy. The model is:

$$\begin{aligned}
&Max \sum_{t=1}^{2} [\int_{\xi=0}^{x_t} p_t(\xi) d\xi] \\
&\text{subject to} \\
&x_1 = e_1^I - e_1^P \\
&x_2 = e_2^I + e_2^P \\
&e_1^P = \mu e_2^P \,, \mu > 1 \\
&e_2^P \le \bar{e}^P \\
&x_t, e_t^I, e_t^P \ge 0 \,, t = 1, 2 \\
&\bar{e}^P, \bar{e}^I > 0
\end{aligned} \tag{4.1}$$

The modelling of intermittent energy follows Subsection 2.3. It is assumed that there are no variable costs producing the intermittent energy e_t^I. Furthermore, we assume that available production is always used, i.e., demand is always great enough. Substituting from the energy balances for consumption we are left with only one energy decision variable; the amount of electricity to produce by pumped storage in the second period. The first two conditions state the energy balances. The electricity used for pumping in the first period is e_1^P and the hydroelectricity generated using the water that is pumped up is e_2^P. The conditions have to hold with equality since there must be balance between supply and demand in continuous time for a well-behaved electricity system. Due to the nature of renewable resources as an exogenous energy resource, it is assumed that there is a positive amount of renewables in both periods, and that the energy is split between consumption and pumped storage in the first period if the latter is undertaken. The third condition links the amount of electricity used for pumping in the first period to the amount of hydro electricity generated in the second period. Due to having only two periods we only have one period when water can be released after pumping up; all water if there is any pumping-up in period 1, will be produced in a single period; period 2.[14] Pumped storage consumes more electricity than it generates, as indicated by the restriction on the parameter; $\mu > 1$. A value of the *round-trip efficiency* of 0.87-0.77 gives a μ between 1.15-1.30.[15] The pumping operation faces three constraints; the capacity of the pump itself, the capacity of the pipe for the water transport up to the reservoir, and the capacity of the reservoir of the system. We will assume that only one constraint can cover these possibilities and constrain the water (in energy units) to be stored by the upper limit $\bar{e}^P$. The amount of water pumped up is e_1^P/μ and the water to be stored is e_2^P and these are equal. Then we have the non-negativity conditions. (The upper limit on the renewables play no role in the story.) The availability of the pumped storage facility makes the optimisation problem (4.1) in general a dynamic problem. Prices and quantities for both periods must be solved simultaneously.

In order to simplify the derivation of the first-order conditions we substitute from the energy balances inserting the expressions for the consumption variables, and eliminate the electricity for pumping as a separate variable when forming the Lagrangian for the optimisation problem (4.1):

$$\begin{aligned} L = & \int_{\xi=0}^{e_1^I - \mu e_2^P} p_1(\xi) d\xi + \int_{\xi=0}^{e_2^I + e_2^P} p_2(\xi) d\xi \\ & -\gamma^P (e_2^P - \bar{e}^P) \end{aligned} \tag{4.2}$$

The necessary first-order condition is:

$$\begin{aligned} & \frac{\partial L}{\partial e_2^P} = -\mu p_1(e_1^I - \mu e_2^P) + p_2(e_2^I + e_2^P) - \gamma^P \le 0 \ (= 0 \text{ for } e_2^H > 0) \\ & \gamma^P \ge 0 (= 0 \text{ for } e_2^P < \bar{e}^P) \end{aligned} \tag{4.3}$$

[14] In a multi-period setting the economic point is, of course, to find the period when to release the pumped water going for the highest difference in price. However, there are so many possible combinations of periods. If it is optimal with pumping in a period t it must also be optimal with producing in a later period $t + j$. We may enter these periods as pumping and production periods, respectively. Many other combinations may be optimal, but to see the qualitative nature of a necessary condition for pumping it should be enough to study two periods.

[15] [See the Electricity Storage Association, http:// energystorage.org/.]

We will make the reasonable assumption that electricity is produced in both periods. Assuming no satiation of demand implies then that prices are positive. The expression μp_1 is the price in period 1 marked up with the factor showing the amount of electricity needed in period 1 to produce a unit of electricity in period 2. We will call this expression the loss-corrected price.

The condition in (4.3) for use of the pumped storage facility tells us:

i) When the price in period 2 is strictly less than the loss-corrected price in period 1 then pumped storage is not used: $p_2 < \mu p_1 \Rightarrow e_2^P = 0$

The general condition for not using the pumped storage facility by pumping up water in period 1 and producing hydroelectricity in period 2 is that $\mu p_1 - p_2 \geq 0$ (pumping up water in period 1 and not using it in period 2 obviously cannot be part of an optimal solution). According to the complementary slackness condition for the Lagrangian parameter we have that $\gamma^P = 0$ since we do not generate hydroelectricity in period 2, but use only the intermittent energy. The typical condition is that the loss-corrected price in period 1 is greater than the price in period 2. The optimal price difference, $p_2 - p_1$, is not big enough to warrant using the pumped storage facility.

ii) When the price in period 2 is equal to the loss-corrected price in period 1 we have that the pumped storage facility typically will be used to some extent; we have an interior solution $p_2 = \mu p_1 \Rightarrow 0 < e_2^P < \bar{e}^P$.

We have that $\gamma^P = 0$ because the capacity is not constrained.

iii) When the price in period 2 is greater than the loss-corrected price in period 1 we have that the pumped storage facility is used to its full capacity:

$$p_2 = \mu p_1 + \gamma^P \Rightarrow p_2 > \mu p_1$$

We have typically $\gamma^P > 0$ when the capacity constraint is binding. The price in period 2 is lower with the use of pumped storage than without.

Notice that without using the pumped storage facility there is no connection between the periods. The optimal solution for each period is found solving static optimisation problems for each period separately.

In the case that the reservoir is constrained a shadow price will be switched on in the first-order condition in (4.3). This implies a greater gap between the prices of the two periods than in the unconstrained case, as also shown in point iii) above:

$$p_2 - p_1 = p_1(\mu - 1) + \gamma^P \tag{4.4}$$

If more storage capacity would have been available more water would be pumped up into the reservoir in the first period and more hydro would be produced in the second period thus reducing the price gap due to an increased price in the first period and a reduced price in the second.

4.2 Hydropower in Norway and Sweden as a battery for Europe

An idea especially suitable for large-scale storage that has been floated in European media is that the reservoirs of hydropower plants in Norway and Sweden can serve as “battery” storage for Europe. The idea is that surplus wind- and solar power can be absorbed by the hydro system simply by reducing the current use of stored water, and then exporting back when wind power is scarce. Utilising the difference between daytime and nighttime prices in Denmark for trade was already an old arrangement (before Nord Pool was established in 1996) between Norway and Denmark when thermal generation was about 90 % in Denmark (von der Fehr and Sandsbråten 1997). Exporting cheap thermal power to Norway saved the cost of Danish generators having to reduce production and then start up production again to satisfy daytime demand. Selling daytime hydroelectricity to Denmark to a higher price than the domestic one was profitable for Norway, but also for Danish customers getting a lower daytime price than without trade. Now wind power in Denmark and Germany has the same problem with low price when wind and solar is abundant driving down the price. Norwegian reservoirs can import cheap electricity at all intervals of abundance, and save water for export to Europe in high-price periods. The conditions for such a trade to benefit both hydro countries and intermittent ones are that increase in the low price in intermittent countries when export due to abundance is more than compensated by a reduction in the high price when hydro countries export. As to the hydro countries it is obvious that buying cheap and selling expensive is favourable. However, in periods of drought in hydro countries importing from intermittent countries may be expensive, but less so than an autarky price.

The decision to close down nuclear plants in Germany has led to an increased emphasis on ambitious plans for investing in renewables like wind and solar in Germany. These plans have been tied not only to a German interest in Scandinavian reservoirs, but also to introducing pumped storage in hydro-rich countries like Norway (SRU, 2010). Pumped storage, using reversible turbines that functions also as pumps and using existing reservoirs, increases the amount of stored water over a yearly period, and hence increases the ability of hydro reservoirs to serve as a battery for countries producing a high share of intermittent energy. The use of pumped storage at a limited number of hydro plants will extend the amount of stored water on a yearly basis. The model in Subsection 4.1 also applies on a larger scale. However, running pumped storage at the same time as other plants produce electricity necessitates a more complex modelling.

We will set up a skeleton model for interactions between two countries with aggregated generating units for T periods. One country, Hydro, uses only hydropower to generate electricity and the other, Intermittent, uses intermittent energy and thermal. We may think about Norway as the hydropower country and Denmark or Germany as the intermittent country. The benefit function is based on the consumer plus producer surplus in both countries, as may be agreed upon. The imports and exports in money terms then cancel out in the summation of objective functions. It is assumed that no income-distributional issues are linked to the trade in electricity in the model.

The capacity of the interconnection between Hydro and Intermittent (mainly under-water cables) determines the maximal volume of trade. The Hydro country cannot only use imported electricity and accumulate water in its reservoirs if the interconnection capacity is limited to be below the amount of electricity used in autarky. If this is the case, then the price level in Hydro will increase in the low price period. The remaining space in reservoirs also set a limit on the import to Hydro in a specific import period. When the reservoirs are full then the hydro system becomes a run-of-river case if maximal reservoirs during import periods are maintained.

We will us a trade model without introducing pumping, but analyse the option of accumulating water and use imported electricity instead of water for use in Hydro. To be more realistic we add thermal capacity to the intermittent energy in the country Intermittent. The variables for the countries are marked with super- and subscripts H and I,Th, respectively. Export- and import variables are denoted e^{XI} with country subscripts as mentioned above. The "cooperative" optimisation problem is shown in Eq. (4.5):

$$
\begin{aligned}
&\max \ \sum_{t=1}^{T}[\int_{\xi=0}^{x_t^H} p_t^H(\xi)\,d\xi + \int_{\xi=0}^{x_t^{I,Th}} p_t^{I,Th}(\xi)\,d\xi - c(e_t^{Th})] \\
&\text{subject to} \\
&x_t^H = e_t^H + (e_{I,t}^{XI} + e_{Th,t}^{XI}) - e_{H,t}^{XI} \\
&x_t^{I,Th} = (e_t^I + e_t^{Th}) - (e_{I,t}^{XI} + e_{Th,t}^{XI}) + e_{H,t}^{XI} \\
&e_{I,t}^{XI} + e_{Th,t}^{XI} \equiv e_{I,Th,t}^{XI} \\
&R_t \le R_{t-1} + w_t - e_t^H \\
&R_t \le \bar{R} \\
&e_t^H \le \bar{e}^H, e_t^{Th} \le \bar{e}^{Th}, e_t^I \le \bar{e}^I \\
&e_{H,t}^{XI} \le \bar{e}^{XI}, e_{I,Th,t}^{XI} \le \bar{e}^{XI} \\
&x_t^H, x_t^{I,Th}, e_t^H, e_t^{Th}, e_{I,t}^{XI}, e_{Th,t}^{XI}, e_{H,t}^{XI}, R_t \ge 0 \\
&T, w_t, R_o, \bar{R}, e_t^I, \bar{e}^H, \bar{e}^{Th}, \bar{e}^I, \bar{e}^{XI} \ge 0 \text{ given} \\
&R_T \text{ free }, t = 1,..,T
\end{aligned}
\tag{4.5}
$$

Because one country's export is the other country's import we only need to consider export variables *from* the two countries in the model. The consumption in Hydro in the first line is own hydro production plus import from Intermittent of both intermittent energy and thermal, and subtracted export from Hydro. The first two equality constraints in (4.5) are the energy balances for the hydro and intermittent country, respectively. The hydro generation with water storage and upper limit on water storage is covered by the next two constraints. The three constraints for the production capacities then follow. Lastly, the two upper constraints on the export variables due to the interconnector between the two countries are specified. The interconnector capacity $\bar{e}^{XI}$ is, of course, equal for export from the two countries. All variables are non-negative, and variables that are given are shown in the next row (R_o is the initial amount in the reservoir for t =1). The specific functioning of the interconnector concerning loss is not taken into account (this will be treated in Section 5).

Simplifying by substituting production for consumption in the two countries using (4.5), the Lagrangian for the optimisation problem is:

$$L = \sum_{t=1}^{T} [\int_{\xi=0}^{e_t^H + (e_{I,t}^{XI} + e_{Th,t}^{XI}) - e_{H,t}^{XI}} p_t^H(\xi)\, d\xi +$$
$$+ \int_{\xi=0}^{(e_t^I + e_t^{Th}) - (e_{I,t}^{XI} + e_{Th,t}^{XI}) + e_{H,t}^{XI}} p_t^{I,Th}(\xi)\, d\xi - c(e_t^{Th})]$$
$$- \sum_{t=1}^{T} \lambda_t (R_t - R_{t-1} - w_t + e_t^H)$$
$$- \sum_{t=1}^{T} \gamma_t (R_t - \bar{R}) \tag{4.6}$$
$$- \sum_{t=1}^{T} \rho_t^H (e_t^H - \bar{e}^H)$$
$$- \sum_{t=1}^{T} \theta_t^{Th} (e_t^{Th} - \bar{e}^{Th})$$
$$- \sum_{t=1}^{T} \alpha_t^H (e_{H,t}^{XI} - \bar{e}^{XI})$$
$$- \sum_{t=1}^{T} \alpha_t^{I,Th} (e_{I,t}^{XI} + e_{Th,t}^{XI} - \bar{e}^{XI})$$

Intermittent energy is assumed to be given exogenously and not subject to optimisation. However, the amounts to be exported are endogenous.

The first-order conditions are:

$$\frac{\partial L}{\partial e_t^H} = p_t^H(x_t^H) - \lambda_t - \rho_t^H \le 0 \;\; (= 0 \text{ for } e_t^H > 0)$$
$$\frac{\partial L}{\partial R_t} = -\lambda_t + \lambda_{t+1} - \gamma_t \le 0 \; (= 0 \text{ for } R_t > 0)$$
$$\frac{\partial L}{\partial e_t^{Th}} = p_t^{I,Th}(x_t^{I,Th}) - c'(e_t^{Th}) - \theta_t^{Th} \le 0 \; (= 0 \text{ for } e_t^{Th} > 0)$$
$$\frac{\partial L}{\partial e_{H,t}^{XI}} = -p_t^H(x_t^H) + p_t^{I,Th}(x_t^{I,Th}) - \alpha_t^H \le 0 \; (= 0 \text{ for } e_{H,t}^{XI} > 0)$$
$$\frac{\partial L}{\partial e_{I,Th,t}^{XI}} = p_t^H(x_t^H) - p_t^{I,Th}(x_t^{I,Th}) - \alpha_t^{I,Th} \le 0 \; (= 0 \text{ for } e_{I,Th,t}^{XI} > 0)$$
$$\lambda_t \ge 0 \; (= 0 \text{ for } R_t < R_{t-1} + w_t - e_t^H) \tag{4.7}$$
$$\gamma_t \ge 0 \; (= 0 \text{ for } R_t < \bar{R})$$
$$\rho_t^H \ge 0 \; (= 0 \text{ for } e_t^H < \bar{e}^H)$$
$$\theta_t^{Th} \ge 0 \; (= 0 \text{ for } e_t^{Th} < \bar{e}^{Th})$$
$$\alpha_t^H \ge 0 \; (= 0 \text{ for } e_{H,t}^{XI} < \bar{e}^{XI})$$
$$\alpha_t^{I,Th} \ge 0 \; (= 0 \text{ for } e_{I,Th,t}^{XI} < \bar{e}^{XI}),\; t = 1,..,T$$

Notice that e_t^I is an exogenous variable, but that $e_{I,t}^{XI}$ is endogenous. The last two first-order conditions show the situation when Hydro exports, and then the situation when Intermittent exports. Intermittent import the same volume, and vice versa when Intermittent exports. When Hydro exports the price in Intermittent is found by inserting the import volume in the demand function together with the solution for the endogenous thermal production. When Intermittent exports the amount of endogenous export is deducted from the consumption in Intermittent.

We have that if the interconnector constraint is not binding, the pair of period prices when there is trade become equal. However, if the reservoir capacity is constrained the shadow price on capacity is typically positive and the pair of prices may differ for the years.

The question to investigate is whether trade should take place between the two countries. To answer the question, the autarky situation for the two countries can be established. Removing the relations for Intermittent in the model, the two first conditions in (4.7) together with the first three complementary slackness conditions for the Lagrangian parameters, define the autarky solution for Hydro.

As in the Subsection 3.2 we assume that intermittent energy is just accepted as Nature provides it. The third first-order condition gives then the solution for the thermal output. The period price in autarky are then determined by insertion in the inverse demand function; $p_t^{I,Th} = p_t^{I,Th}(x_t^{I,Th}),\ t = 1,...,T.$

If the period autarky prices are the same for the two countries, there is no benefit doing trade. If a country has a higher (lower) price for a period than the other country, it will have an incentive to import (export) when trade possibilities open up. The general feature when doing trade is that the price in the exporting country will go up and that in the importing country the price will go down compared with autarky prices.

The share of intermittent electricity output and thermal output going to export will be determined by three conditions; (i) the interconnection capacity allows this volume, (ii) this amount of export does not exceed the reservoir limit in country Hydro, and (iii) the demand in country Intermittent requires positive production. Intermittent energy is by assumption produced without any variable cost. However, it may not be realistic that only intermittent energy is used at home to a zero price. It is more realistic with a positive price implying some use of thermal. Then only thermal output will be exported given that the two first conditions still hold. This depends on the intersection point between the demand curve and the marginal cost curve giving a price lower than the autarky price in Hydro.

We see from the last two first-order conditions in (4.7) how the cooperative trade model works. Export will not take place from Hydro to Intermittent if $p_t^H > p_t^{I,Th}$ and Intermittent will not export to Hydro if $p_t^H < p_t^{I,Th}$. Without binding interconnector constraints, the condition for trade is equality of the prices between countries for each period. Trade with binding interconnector constraints implies that the price in the exporting country typically is less than the price in the importing country.

We will interpret the first-order conditions further and only use t =1, 2 in order to illustrate the situation. Further simplifications are that trade is not constrained by interconnector capacity in the figure, and that the capacity constraint in Hydro is not reached (constraining transport of electricity will be discussed below). From the first condition in (4.7) we have when hydro is not used in Period 1 but in Period 2, and the shadow prices on production capacity and reservoir are zero:

$$p_1^H \le \lambda_1 \,, p_2^H = \lambda_2 = \lambda_1 \tag{4.8}$$

The relationship between the prices in the two countries Hydro and Intermittent follows from the two last first-order conditions in (4.7) assuming no binding interconnector constraint. In the case of the interconnector constraint being binding in both periods for the country that is exporting the constraint holds with equality, and for the country that is importing the constraint holds with an inequality. The impact of a constrained interconnector is of special interest. This means that, because export from Hydro in period 1 is zero, the active condition is:

$$p_1^H - p_1^{I,Th} - \alpha_1^{I,Th} = 0 \Rightarrow p_1^H \ge p_1^{I,Th} \tag{4.9}$$

The shadow price on Hydro's export constraint is zero. The price in Hydro is typically greater than the price in Intermittent in period 1 when the export from Intermittent to Hydro is constrained.

In period 2 the situation is reversed and we have:

$$-p_2^H + p_2^{I,Th} - \alpha_2^{I,Th} = 0 \Rightarrow p_2^H \le p_2^{I,Th} \tag{4.10}$$

The shadow price on the binding export constraint is typically positive. The price in Hydro is typically lower than the price in Intermittent because the export from Hydro is now constrained. Using (4.7) and the results above we have that

$$p_2^H \geq p_1^H \Rightarrow p_2^H > p_1^{I,Th} \text{ and } p_1^{I,Th} < p_2^{I,Th}, \tag{4.11}$$

the last two strict inequalities being typical results.

If import to Hydro is constrained in the first period, this means that the price in Hydro will increase compared with the unconstrained case; Hydro wants to import more than what is feasible. There will still be a price difference in Hydro between the two periods if no use of water in period 1 remains optimal, but it will be smaller than in a situation with unconstrained import. If import to Hydro is not constrained in period 1 but export from Hydro is constrained in the second period, then the price in Hydro in period 2 will be lower than in the unconstrained case; Hydro would have liked to exported more, but now more electricity has to be consumed at home instead. The price difference will become smaller between the two periods. If the interconnector is constrained in both periods, then the price difference shrinks "at both ends" compared with the unconstrained case.

In Fig. 4.1 the trade between Hydro and Intermittent for two periods is illustrated. The

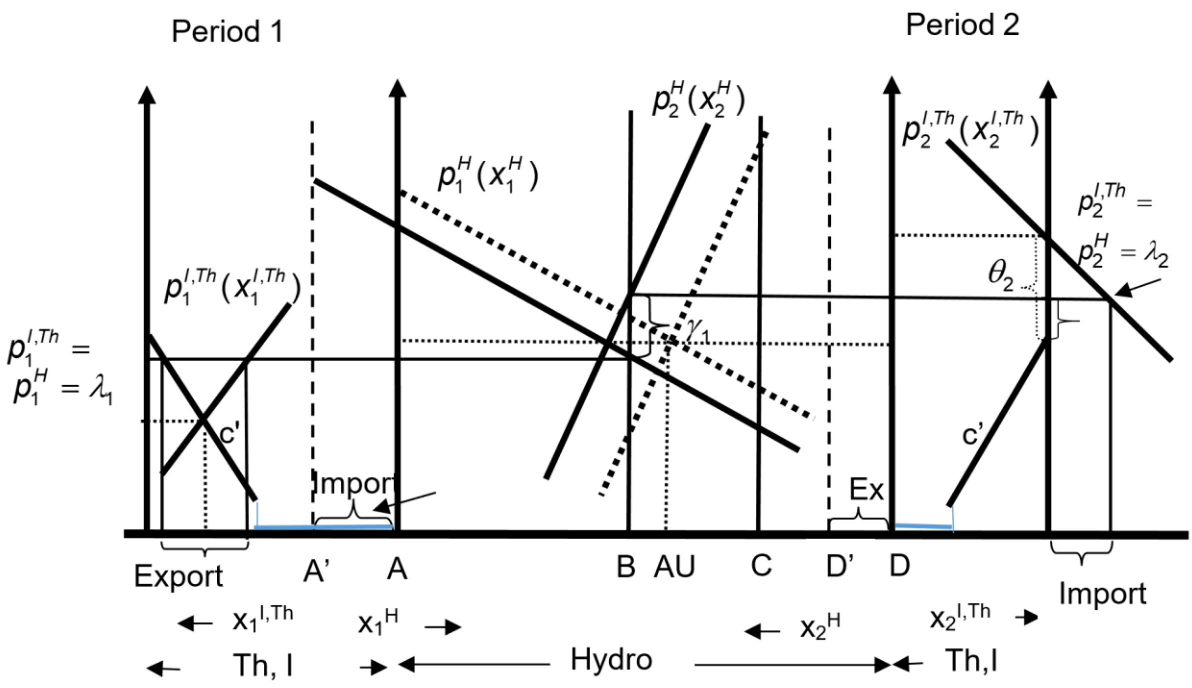

Figure 4.1. *The optimal solutions i autarky (dotted lines) and trade (solid lines)*

production possibilities for Hydro for the two periods assuming autarky is shown in the middle with a vertical axis with an arrow tip up from A on the left, and a vertical axis with an arrow tip on the right from D. The quantities of electricity consumed in Hydro are measured by the line A to D along the horizontal axis. The line B to C shows the reservoir capacity. Inflow in period 1 is A to C and in the second period C to D. The production possibilities of Intermittent for period 1 is shown to the left of Hydro, and period 2 is shown on the right of Hydro with intermittent measured along the horizontal axis (blue lines), and (identical) thermal marginal cost curves starting with a small positive cost and ending at the same level at the outer vertical axes with arrow tips.

The Hydro demand curves for the two periods in autarky are shown as the broken lines (linearisation is done for ease of representation) with prices measured along the vertical axis for Hydro up from A for the first period and up from D in the second period. The demand curve is coming down from the left in period 1 and from the right in period 2. The optimal price for the two periods in autarky is shown by the intersection of the demand curves, and read off on the price axes by the horizontal thin dotted line. The allocation of the use of water on the two periods is given at point AU on the horizontal quantity axis. The autarky prices are equal. The part AU to C of water saved in period 1 is used in period 2 to equalise the prices. This is the optimal autarky solution.

The production possibilities of thermal plants and intermittent in country Intermittent are not connected over time. The optimal solutions for the autarky situation can be found for each year separately. In period 1 the autarky solution is shown by the intersection of the marginal cost function down from the outer left vertical axis with the demand function coming down from the right-hand axis up from A. The exogenous production of intermittent is shown by the blue line along the horizontal floor from A. All intermittent production is used, but a part of thermal capacity remains unused. The thin dotted lines show the price and the quantity of electricity used from thermal and intermittent generation on the vertical and horizontal axes, respectively.

In period 2 the demand outstrips maximal supply of electricity. The whole capacity is used, and the price is the marginal cost of the most expensive capacity plus a shadow price θ_2 on capacity necessary in order to have equilibrium in the market, as shown by the dotted line from the outer right-hand axis to the axis up from D.

Opening up for trade Hydro will import in period 1 and export in period 2 and vice versa for Intermittent (following the rules stated in Eq. (4.11)). Hydro will save the whole reservoir BC in period 1 to be used in period 2. The consumption of energy in Hydro in period 1 is higher than in autarky in spite of not using any water from the full reservoir handed over to period 2 due to the import from Intermittent being higher than the water in the reservoir accumulated in period 1. In order to see this, the line up from A is shifted to the left to the broken line up from A' to mark the scope of energy consumption due to trade. The broken autarky demand curve is given a parallel shift to the left equal to the import, and made a solid line. The import to Hydro is A'A, and the export from Intermittent is the identical quantity illustrated by the length of the price line delimited by the intersection point between the marginal cost function and the demand curve of intermittent.

The optimal price in period 1 is determined by the intersection of the solid demand curve for Hydro with the vertical line from B that is the left limit of the reservoir, and the intersection with the demand curve for Intermittent yielding the same price equal to the price in Hydro. The result is that the electricity price in Hydro in period 1 is lower than in autarky, but the opposite is the case in Intermittent. The prices are equal, and equal to the water value and the marginal cost of thermal generation.

Notice that demand in Intermittent is larger than the production of intermittent. In order to export Intermittent must also produce thermal power using the most efficient thermal units. The production of thermal energy in Intermittent goes up compared with autarky, and increases capacity utilisation, but the use of the energy in Intermittent is reduced due to its export and causing the increase in the price. The statement that countries with intermittent energy can export intermittent electricity to hydro-rich countries when they have too much intermittent is not supported by the model results. Notice that one cannot distinguish which type of electricity is exported as long as both thermal and intermittent energy are produced. No country has yet only intermittent energy, so our model may be realistic.

If intermittent energy is to be exported in the model the amount of intermittent energy must be greater than the demand for electricity at zero price (remember that variable costs for intermittent is by assumption set to zero). However, with a trading possibility it will hardly be optimal to just export the surplus intermittent energy and hold on to a zero price at home. For the two trading partners it will be optimal with a common period price. This means a lower use of intermittent in Intermittent and a higher price, but an export income that more than compensate the higher domestic price. Because of the positive trade price, it will become profitable to start up some part of thermal if the price is higher than the minimum thermal cost.

In period 2 Hydro will export to Intermittent. In autarky the price in Intermittent is higher than in Hydro. The export reduces the consumption of electricity in Hydro and increases the price, but the price in Intermittent is reduced. As for period 1, the optimal price is equal in both countries. The capacity of thermal is still fully utilised, and a shadow price must still be added to reach the equilibrium price. However, the shadow price indicated by the right brace from the end point of the marginal cost curve up to the optimal price is much smaller than in autarky.

The consumption of electricity is expanded with the export from Hydro. The size of the import is measured by the length of the line on the horizontal axis from the end of the production possibilities to the intersection of the demand curve and the price line outside the production possibilities of Intermittent. The import is identical to the export from Hydro measured by D'D.

Notice that from a climate change point of view increasing capacity utilisation in Intermittent in period 1 increases CO_2 emissions from thermal generation because the thermal capacity in Intermittent is fully utilised in period 2 even with import from Hydro.

The export in period 2 reduces the consumption possibilities of Hydro. The right-hand axis is moved from the vertical solid line from D to the broken line up from D'. The autarky demand curve is moved to the left with the amount D'D and made a solid line. The intersection of the demand curve and the vertical line from B representing the reservoir limit gives the price of period 2. The difference in the period prices is the shadow price γ_1 on the reservoir constraint according to (4.6). Reaching the reservoir limit in period 1 results in the price being higher in period 2.

Summing up, the exporting country Intermittent gets a higher price and consumes less electricity than in autarky in period 1, but imports from Hydro in period 2 and gets reduced price and consumes more electricity compared with autarky. Hydro imports in period 1 and gets a lower price and consumes more electricity than in autarky, and in period 2 Hydro exports and get a higher price and a lower consumption than in autarky. We cannot directly see that this result is beneficial for both countries, but it is the optimal result using consumer- and producer benefit functions that yields the results. However, we have formulated the sum of the benefit functions without constraining the solution to benefit both countries, or more precisely, to constrain the results to be equal or better than the autarky solutions.

5. Transmission and nodal pricing

In Sections 2 and 3 all the consumers of electricity are aggregated to a single consumer, and there are no transmission lines in the models. In Section 4 we have two consumers – two countries – and a single transmission line making trade possible. A capacity limit was specified, but no loss of energy was modelled. We will now focus on one country (or region). To approach reality, it is necessary to introduce multiple consumers and transmission lines connecting producers and consumers. Both producers and consumers are located in what are termed *nodes*. In a country like Norway having hydro and intermittent producers it may be a single producer located in each node. Depending on the geographical size of a node there may be more than one producer in a node. If this the case, we aggregate the production units with the same technology to a single producer. Electricity is delivered to individual firms and households. However, we will aggregate all consumers within a node to a single consumer. In a country with most of the electricity generated by hydro or intermittent it will seldom be the case that production units are located in the same node as consumers, so this is our assumption in the following. However, possible co-location depends on the size of the area of the node.

There are usually three types of transmission lines in a country; a central grid with a high voltage, regional grids with a lower voltage located closer to consumer nodes, and finally low voltage distribution networks serving final consumers. We will focus on having a high voltage central grid with connections to low voltage utility networks. A key feature of transmission networks is that due to resistance there are energy losses. For a given power flow (power is the rate at which energy is flowing), the loss is inversely proportional to the square of the voltage, therefore it is rational to have high voltage in the central grid. The heat produced by resistance may cause lines to sag and eventually break. Therefore, there are limits on how much energy that can be transported. To preserve quality to consumers the voltage must also be maintained within strict limits.

National grids use *alternating current* (AC). A difficulty is that if there are two or more ways for electricity to flow from a generation node to a consumer node, then it is necessary to find out how the flow of energy is distributed on transmission lines feeding consumer nodes. In the case of *direct current* (DC) the flows are relatively easy to calculate. Therefore, DC is often assumed to be the case as an approximation when building theoretical and applied models (see e.g. Bohn et al (1984); Schweppe et al (1988); Green (2007).

The key questions are how to find optimal period quantities and prices in order to maximise producer and consumer surplus, as formulated in the objective function in Eq. (3.4) in Subsection 3.2.

5.1 A general transmission externality model

A transmission network can have a high number of devices like transformers and various switches and gears. A large network model for pan European high-voltage electricity network has more than 2000 lines and more than 4000 nodes (Leuthold et al 2012)[16]. It is claimed that the model is one of the largest empirical models developed to date. It is illustrated in Fig. 5.1.

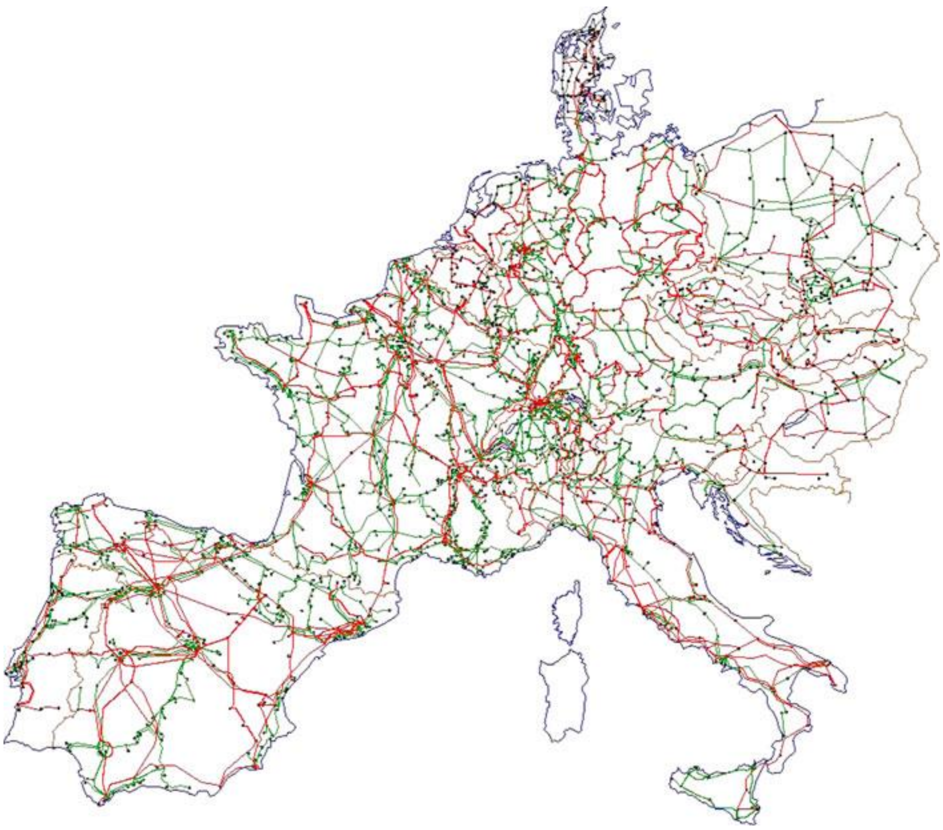

Figure 5.1. *The data on lines and nodes used by the DC load flow model ELMOD*
Source: Leuthold et al. (2012)

[16] This information presented in the Introduction (p. 76) is in conflict with the abstract stating over 2000 nodes and over 3000 lines. We adopt the information in the Introduction.

"Various forms of spatial price discrimination can be implemented, such as locational marginal pricing ("nodal pricing"), or zonal pricing", according to Leuthold et al (2012, p.75).

Our model ambition is very modest, but made to catch the fundamental externalities of the central grid. We will expand the model (3.4) in Subsection 3.2 to encompass producer- and consumer nodes and networks connecting them. The number of all types of nodes are kept constant over time for convenience. We assume for simplicity that there is no production in consumer nodes.[17] We look at aggregated demand for each consumption node, and production at nodes for five types of technology, aggregating production for each technology if there are more than one production unit at the node. Loss is generated in each line. Ideally, we would have liked to specify functions that accurately reflect the underlying physical and engineering properties of electricity based on which lines connect producer- and consumer nodes. A complete characterisation of the network in a period t requires that we know the flows and losses along each line, and the net injections or withdrawals at each node. However, this task is complex and will take us too far outside a traditional economic approach to electricity issues. The purpose of the modelling effort here is to maintain a model structure familiar to economists, but still reflecting main features of physical and engineering properties. It will not be shown explicitly how the various links within the transmission network are connected. The network connecting nodes *implicitly* behind the scene is in general exhibiting loop-flows, implying that it is not possible to direct electricity along specific lines if there are two or more path between a production node and a consumer node. We will capture the physical network implicitly through the generation of losses on each line and possible congestions. These losses on lines are related to generation at all production nodes and consumption at all consumption nodes. The transmission system has pervasive externalities when AC is used on meshed transmission network exhibiting loop-flows; everything depends on everything else.

The transmission externalities can be formalised by establishing a function for net flow on a line, and introducing a connection between the level of net line flows and losses and congestion. Keeping our variables in energy units MWh[18] we define the net flow, b_{nt}, on a line n in period t. We then assume that the generation at each node for five technologies and the consumption

[17] This reflects the general situation in Norway, although there are some river-based hydro plants within consumer nodes. However, co-location will not be a problem for the aggregated level of our analysis. In Green (2007) there are 13 nodes based on zones with both production and consumption in all nodes (except one or two; it is difficult to see the exact number from Table 1, p. 133), and 21 lines connecting the nodes.

[18] Engineers will use instantaneous power MW, but for our type of aggregate modelling MWh is used for all other energy variables and it is more convenient also to use the same measuring unit for all energy variables.

at each consumer node will influence net flow on lines, using the same total nodes as plants in Section 3 for simplification:[19]

$$\begin{aligned}&b_{nt} = b_{nt}(x_t, e_t^H, e_t^{Th}, e_t^{Hr}, e_t^{Wi}, e_t^S),\ n=1,..,N,\ t=1,..,T,\\&x_t = (x_{1t},..,x_{Mt}), e_t^H = (e_{1t}^H,..,e_{Ht}^H), e_t^{Th} = (e_{1t}^{Th},..,e_{Ft}^{Th}),\\&e_t^{Hr} = (e_{1t}^{Hr},..,e_{Rt}^{Hr}), e_t^{Wi} = (e_{1t}^{Wi},..,e_{Wt}^{Wi}), e_t^S = (e_{1t}^S,..,e_{St}^S)\end{aligned} \tag{5.1}$$

For convenience we introduce a vector x_t for consumption nodes and vectors for the five different technologies of generation, e_t^H, e_t^{Th}, e_t^{Hr}, e_t^{Wi} and e_t^S. There are N lines, M consumption nodes, N^H hydro nodes, N^{Th} thermal power nodes, N^{Hr} run-of-river nodes, N^{Wi} wind nodes, and N^S solar technology nodes. Because we use discreet time periods the measuring unit for energy has been MWh, and this is also most conveniently used for energy flows along the lines.

The partial derivatives of this flow relationship may be both positive and negative, and, of course, zero. The equation captures the pervasive electric externalities in a general network. We see how "everything depends on everything else" by the variables in the function (5.1). The losses are then created on each line as a function of the net flow on the line (increasing proportional to the square according to Ohm's law), and the given length of the line l_n[20]:

$$e_{nt}^L = e_{nt}^L(b_{nt}; l_n),\ \frac{\partial e_{nt}^L(b_{nt}; l_n)}{\partial b_{nt}} > 0,\ \frac{\partial^2 e_{nt}^L(b_{nt}; l_n)}{\partial b_{nt}^2} > 0,\ n=1,..,N, t=1,..,T \tag{5.2}$$

Loss is increasing in line flow. It would be fine if the network could be modelled in a point-to-point way expressing how much electricity is lost in the transport of electricity from a generating node j to a consumer node, i. However, loss incurred may be quite impractical to calculate in such a way and also difficult in the case of AC and a meshed grid. We therefore stick to a general way of capturing the loss incurred on each line by injections and withdrawals.

In principle electrical equilibrium at all nodes could be modelled: consumption at each node must be equal to the net flow coming in, but this will be a major undertaking. Instead we will study total network loss and congestion by a marginal increase in consumption at one node at a time. Which production nodes that will be activated by the marginal increase in consumption will not be found. Then system loss and congestion will be studied by a marginal increase in production at each production node. Increase in consumption at each node can then not be

[19] The number of nodes will be less if there is aggregation of technologies at nodes.

[20] The length of line l_n is constant over time, but there may be new lines constructed within the horizon adding to N. This last aspect is not pursued.

found. The characterisation of power flows over each line is instead implicitly embedded in the energy-balance equation for the total system.

Loss and congestion are pervasive phenomenon in a network model. A congested line somewhere may create repercussions throughout the total network. This may be brought out in the simplest possible illustration of a loop-flow possibility using the popular three nodes example; two generation nodes and one consumption node.

The ubiquitous (or infamous) example of three nodes; two producer nodes and one consumer node, is shown in Fig. 5.2. The current can either flow directly from a generation node to the

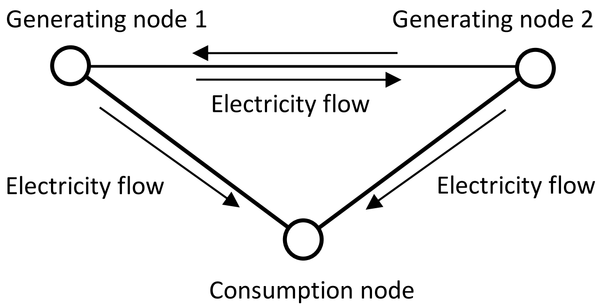

Figure 5.2. *Two generation nodes and one consumer node with loop-flows*

consumption node, or flow the other way through the other generation node to the consumption node. The loop-flows are created by the possibility of the flows from the generator nodes to take two different ways to the consumption node. Kirchhoff's laws tell us that the power between any two nodes is necessarily distributed across all parallel paths. The distribution on the loops is proportional to the inverse of relative resistance on the lines. The size of the flow in Fig. 5.2 going directly from a generation node to the consumer node, compared with the flow going the other way through the other generation node, is in proportion to the resistances on these loop lines. But the really intriguing paradoxical consequence of the physical laws is that if a flow restriction on a line is reached, then this will determine the maximal flows on the other loop-lines, too, even if the latter lines have a greater thermal capacity. Consider an upper limit on the line between the two generators in Fig. 5.2. Then the relative resistances multiplied with the capacity on the link between the generators - even though the capacity on the direct link may be larger - will determine the maximal flow on the direct link between the generation node and the consumption node. However, it will take us too far into electrical engineering to try to capture loop-flow externalities.

We will introduce the externalities by assuming that not only the flows on the lines are influenced, but also the capacities on the lines. All injections and all withdrawals influence both flows and the upper limits:[21]

$$\begin{aligned} & b_{nt} = b_{nt}(x_t, e_t^H, e_t^{Th}, e_t^{Hr}, e_t^{Wi}, e_t^S) \le \bar{b}_{nt}(x_t, e_t^H, e_t^{Th}, e_t^{Hr}, e_t^{Wi}, e_t^S) \\ & n = 1,..,N, t = 1,..,T \end{aligned} \tag{5.3}$$

In addition to the injections of energy, the capacity levels are influenced by ambient temperature and wind, and production of *reactive power* that alternates between positive and negative power, averaging to zero and cannot do any useful work, but it does contribute to heating and then losses. The lower the temperature the lower is the resistance on the lines, and the more generation of reactive power the more heat will influence the capacity negatively. Another source of transmission constraint in addition to the thermal aspect is the voltage. Reactive power occurs on a sinusoidal alternating current network also creating voltage stability problems. A complete analysis of the network requires modelling both real and reactive power. However, we will not attempt to include such issues here.

5.2 Nodal pricing

According to Hogan et al (1996, p. 209), "Caramanis et al (1982) were the first to derive *spot prices* which vary across space to reflect the marginal cost of losses and the costs of network congestion, and across time in response to changing demand and generator availability, with each generator or consumer simply selling or buying energy at the local spot price." However, Caramanis et al (1982, p. 3239) point out that "It is not feasible to have a distinct spot price for each point in the network." (Cf. Fig. 5.1.) They also point out that it is not desirable from a metering cost point of view to have too short time periods, as, e.g., five minutes[22]. In the book Schweppe et al (1988) the idea of spot pricing is extended and a DC-type flow model is introduced for empirical implementation.

The term *nodal pricing* is more often in use now. *Locational pricing* is also used, and Americans often use the term *bus* instead of node. (Textbooks covering nodal pricing are e.g. Stoft (2002); Biggar and Hesamzadeh (2014); Léautier (2018), and informative papers are e.g. Bohn et al (1984); Hsu (1997).)

[21] All the arguments in the functions are vectors, as set out in Eq. (5.1).

[22] PJM Interconnection, originally encompassing Pennsylvania, New Jersey and Maryland (more parts of Eastern states have now joined) has experimented with changing prices every five minutes.

The general social planning problem with a transmission network using an extended version of model (3.4) can then be formulated:

$$
\begin{aligned}
&Max[\sum_{t=1}^{T}\sum_{m=1}^{M}\int_{\xi=0}^{x_{mt}} p_{mt}(\xi)d\xi - \sum_{f=1}^{N^{Th}} c_{ft}(e_{ft}^{Th})]\\
&\text{subject to}\\
&\sum_{m=1}^{M} x_{mt} + \sum_{n=1}^{N} e_{nt}^{L} = \sum_{h=1}^{N^H} e_{ht}^{H} + \sum_{f=1}^{N^{Th}} e_{ft}^{Th}\\
&+(\sum_{r=1}^{N^{Hr}} e_{rt}^{Hr} + \sum_{w=1}^{N^{Wi}} e_{wt}^{Wi} + \sum_{s=1}^{N^{S}} e_{st}^{S})\\
&e_{ht}^{H} \le \bar{e}_{ht}^{H}\\
&R_{ht} \le R_{h,t-1} + w_{ht} - e_{ht}^{H}\\
&R_{ht} \le \bar{R}_{h}\,,\quad h = 1,...,N^{H}\\
&e_{ft}^{Th} \le \bar{e}_{ft}^{Th},\ f = 1,...,N^{Th}\\
&b_{nt} = b_{nt}(x_t, e_t^H, e_t^{Th}, e_t^{Hr}, e_t^{Wi}, e_t^{S})\\
&b_{nt}(.) \le \bar{b}_{nt}(x_t, e_t^H, e_t^{Th}, e_t^{Hr}, e_t^{Wi}, e_t^{S})\\
&e_{nt}^{L} = e_{nt}^{L}(b_{nt}; l_n),\ n = 1,..,N\\
&T, w_{ht}, R_{ho}, \bar{R}_h, l_n, e_t^{Hr}, e_t^{Wi}, e_t^{S} \text{ given}
\end{aligned}
\tag{5.4}
$$

The pervasive externalities are captured by the energy flow functions and the energy loss functions and congestion due to line capacity restrictions. The model formulation gives us the consumer node prices, but does not give the producer prices. The first equality constraint covers the energy balance for the whole transmission system of consumption, production, losses and congestions on the lines. However, the model does not give us the energy balance for each consumption node. This implies that we cannot substitute consumption with production as in the Lagrangian (3.6) for the optimisation model (3.4).

As in Subsection 3.2 the subindex *t* on the thermal cost function opens up for technological change over time by improving existing technology and invest in more modern capacity, and the subindex *t* on the hydro capacity allows for technical upgrading of water feeding tunnels and turbines, as well as investing in more turbines. It may also be possible to expand the water inflow to an existing generation station by casting a larger net regarding water sources.

Inserting the aggregate energy balance in the fifth constraint the Lagrangian is:[23]

[23] There is no summation over line length l_n in (5.5); l_n may here be regarded as a vector.

$$
\begin{aligned}
L = [& \sum_{t=1}^{T}\sum_{m=1}^{M}\int_{z=0}^{x_{mt}} p_{mt}(z)dz \; -\sum_{f=1}^{N^{Th}} c_{ft}(e_{ft}^{Th})] \\
& -\sum_{t=1}^{T}\sum_{h=1}^{N^H} \lambda_{ht}(R_{ht} - R_{h,t-1} - w_{ht} + e_{ht}^{H}) \\
& -\sum_{t=1}^{T}\sum_{h=1}^{N^H} \gamma_{ht}(R_{ht} - \bar{R}_h) \\
& -\sum_{t=1}^{T}\sum_{h=1}^{N^H} \rho_{ht}(e_{ht}^{H} - \bar{e}_{ht}^{H}) \\
& -\sum_{t=1}^{T}\sum_{f=1}^{N^{Th}} \theta_{ft}(e_{ft}^{Th} - \bar{e}_{ft}^{Th}) \\
& -\sum_{t=1}^{T}\tau_t[\sum_{m=1}^{M} x_{mt} + \sum_{n=1}^{N} e_{nt}^{L}(b_{nt}(x_t, e_t^{H}, e_t^{Th}, e_t^{Hr}, e_t^{Wi}, e_t^{S}); l_n) \\
& -(\sum_{h=1}^{N^H} e_{ht}^{H} + \sum_{f=1}^{N^{Th}} e_{ft}^{Th} + (\sum_{r=1}^{N^{Hr}} e_{rt}^{Hr} + \sum_{w=1}^{N^{Wi}} e_{wt}^{Wi} + \sum_{s=1}^{N^S} e_{st}^{S}))] \\
& -\sum_{t=1}^{T}\sum_{n=1}^{N} \mu_{nt}[b_{nt}(x_t, e_t^{H}, e_t^{Th}, e_t^{Hr}, e_t^{Wi}, e_t^{S}) - \bar{b}_{nt}(x_t, e_t^{H}, e_t^{Th}, e_t^{Hr}, e_t^{Wi}, e_t^{S})]
\end{aligned}
\tag{5.5}
$$

The output from the three intermittent technologies are given and thus not variables to be determined.[24]

The necessary first-order conditions are:

[24] Because the three types of intermittent technologies are assumed located at separate nodes the intermittent generation is not aggregated as in Section 3. Furthermore, due to the given levels of the intermittent generations a marginal change in the intermittent generation is not performed. However, when intermittent generation changes from one time period to another, consumer prices, water values and marginal cost of thermal generation will be influenced.

$$\begin{aligned}
&\frac{\partial L}{\partial x_{mt}} = p_{mt}(x_{mt}) - \tau_t - \tau_t \sum_{n=1}^{N} \frac{\partial e_{nt}^L}{\partial b_{nt}} \frac{\partial b_{nt}}{\partial x_{mt}} \\
&-\sum_{n=1}^{N} \mu_{nt} \left(\frac{\partial b_{nt}}{\partial x_{mt}} - \frac{\partial \overline{b}_{nt}}{\partial x_{mt}}\right) \le 0 \ (= 0 \text{ for } x_{mt} > 0),\ m = 1,..,M \\
&\frac{\partial L}{\partial e_{ht}^H} = -\lambda_{ht} - \rho_{ht} + \tau_t - \tau_t \sum_{n=1}^{N} \frac{\partial e_{nt}^L}{\partial b_{nt}} \frac{\partial b_{nt}}{\partial e_{ht}^H} \\
&-\sum_{n=1}^{N} \mu_{nt} \left(\frac{\partial b_{nt}}{\partial e_{ht}^H} - \frac{\partial \overline{b}_{nt}}{\partial e_{ht}^H}\right) \le 0 \ (= 0 \text{ for } e_{ht}^H > 0) \\
&\frac{\partial L}{\partial R_{ht}} = -\lambda_{ht} + \lambda_{h,t+1} - \gamma_{ht} \le 0 (= 0 \text{ for } R_{ht} > 0) \\
&\frac{\partial L}{\partial e_{ft}^{Th}} = -c_{ft}'(e_{ft}^{Th}) - \theta_{ft} + \tau_t - \tau_t \sum_{n=1}^{N} \frac{\partial e_{nt}^L}{\partial b_{nt}} \frac{\partial b_{nt}}{\partial e_{ft}^{Th}} \\
&-\sum_{n=1}^{N} \mu_{nt} \left(\frac{\partial b_{nt}}{\partial e_{ft}^{Th}} - \frac{\partial \overline{b}_{nt}}{\partial e_{ft}^{Th}}\right) \le 0 \ (= 0 \text{ for } e_{ft}^{Th} > 0) \\
&\rho_{ht} \ge 0 (= 0 \text{ for } e_{ht}^H < \overline{e}_{ht}^H) \\
&\lambda_{ht} \ge 0 (= 0 \text{ for } R_{ht} < R_{h,t-1} + w_{ht} - e_{ht}^H) \\
&\gamma_{ht} \ge 0 (= 0 \text{ for } R_{ht} < \overline{R}_h),\ h = 1,..,N^H \\
&\theta_{ft} \ge 0 (= 0 \text{ for } e_{ft}^{Th} < \overline{e}_{ft}^{Th}),\ f = 1,..,N^{Th} \\
&\mu_{nt} \ge 0 (= 0 \text{ for } b_{nt} < \overline{b}_{nt}),\ n = 1,..,N \\
&t = 1,..,T
\end{aligned} \tag{5.6}$$

The shadow price τ_t on the energy balance is free in sign since the balance is an equality constraint, but must be positive the way the parameter is entered in the problem. Looking at the number of endogenous variables and equations, the endogenous variables may in principle be determined, but due to the somewhat unclear properties of the line-flow functions the sufficiency conditions may be violated, indicating that there may be problems with attaining a unique optimum.

We will assume that there is positive consumption at each consumer node, implying that the first condition in (5.6) holds with equality. The optimal consumer price is the only explicit price in the solution of the model and can then be expressed at node m as:

$$p_{mt}(x_{mt}) = \tau_t + \tau_t \sum_{n=1}^{N} \frac{\partial e_{nt}^L}{\partial b_{nt}} \frac{\partial b_{nt}}{\partial x_{mt}} + \sum_{n=1}^{N} \mu_{nt} \left(\frac{\partial b_{nt}}{\partial x_{mt}} - \frac{\partial \overline{b}_{nt}}{\partial x_{mt}}\right),\ m = 1,..,M,\ t = 1,..,T \tag{5.7}$$

The first term on the right-hand side is the shadow price on the energy balance that is free in sign since the energy balance is an equality constraint. This is the opportunity cost of the unit increase in consumption at node m and must be positive the way we have set up the problem.

The second term on the right-hand side is expressing the marginal losses on all the N lines created due to the marginal increase in consumption at node m evaluated using the shadow price on the energy balance. However, the path of energy flows from production units to consumer node m may not involve all lines. Given an increase of the flow on line n the loss is increasing, but flows on lines may go up as well as down due to a change in paths connecting production at nodes when consumption at node m increases marginally. Therefore, the total expression for loss may be positive as well as negative. This is also the case for the expression for congestion consisting of two terms; change in line flows and change in the thermal limits. However, the congestion term cannot be negative for all consumer nodes if one of the constraints is binding. The change in the thermal limit is influenced by the change in reactive power; the derivative is negative (positive) if reactive power increases (decreases). We would expect as a normal result that the majority of the two main expressions are positive. Furthermore, we will expect that congested lines will be rather few, but will tend to increase if thermal limits are lowered due to increasing reactive energy. Remember that congestion only occurs when the energy flow is equal to the limit of the line, if the flow is less the shadow price is zero.

However, one must be careful not to confuse a characterisation of the optimal solution with some line constraints being binding. A consumer node located in, e.g., a locked-in export region may have a negative congestion term, but the shadow price on a congested link out of the region may still remain positive. The consumption in the export region will increase compared with an unconstrained case due to a lower consumer price, and the congestion is thereby not relieved to the extent that the shadow price on the link becomes zero.

If the loss decreases more than the unit increase in consumption at a node, then it might seem possible that the optimal price becomes negative if the loss term outweighs the sum of the shadow price on the energy balance and the congestion and thermal limit terms. The consumers at the node would then be paid to use more electricity. However, since the shadow price on the energy balance is common for all consumer and generating nodes it seems rather impossible that all the nodes are characterised by having negative losses. We will therefore stick to the assumption that the shadow price on the energy-balance constraint is positive. It is still possible for a consumption node to have a negative optimal price.

In the case of no losses being created and with no binding line capacity constraints, the optimal consumer price equals the shadow price on the energy balance constraint corresponding to the model in Section 3.

Consumers located at a node generating higher losses and congestion at the margin in the transmission network should get incentives to scale back consumption. The general case is that all optimal consumer prices are different. The optimal prices between pairs of consumption nodes will only become equal if the loss and congestion effects at the margin are identical.

The second first-order condition in (5.6) show that the value of hydroelectricity is reduced when electricity is transmitted through the network. The water value at hydro node *h* is greater than the value of the hydroelectricity reaching consumer nodes. The value of generated electricity at the margin is adjusted downwards by the externalities due to loss and congestion in the transmission system. Assuming that there is positive hydro generation at node *h* we have:

$$\lambda_{ht} = \tau_t - \rho_{ht} - \tau_t \sum_{n=1}^{N} \frac{\partial e_{nt}^L}{\partial b_{nt}} \frac{\partial b_{nt}}{\partial e_{ht}^H} - \sum_{n=1}^{N} \mu_{nt} \left(\frac{\partial b_{nt}}{\partial e_{ht}^H} - \frac{\partial \overline{b}_{nt}}{\partial e_{ht}^H} \right),\ h = 1,..,H,\ t = 1,..,T \tag{5.8}$$

The value of the hydroelectricity generated at node *h* is reduced by the shadow price on the production capacity (if reached), the system losses created at the margin due to the injection of the production unit at node *h* - valued at the shadow price of the energy balance - and the shadow-valued congestion costs. We see that if the water value at production node *h* is less than the difference between the shadow price on the energy balance and the sum of loss and congestion terms for all feasible values of production, then production is set to zero in this period. If node *h* is close to a large consumer node the value of the electricity will be closer to the water value at the production node. The congestion term may also be negative contributing to an increase in the value higher than the water value. This may be the case for a generating unit within an import-restricted region.

Due to losses and congestion on the lines the utilisation pattern of hydro generation will change from the pattern discussed in Section 3. Hydro units closer to major consumption nodes generating less loss will be utilised more intensively. However, the dynamic time frame for planning utilisation of hydro generators implies relatively few cases of maximal utilisation at time *t*. Remote and smaller consumption nodes will tend to experience higher nodal prices caused by reallocating the use of hydro over space, according to Kirchhoff's laws.

The water value applying to a generator node represents the opportunity cost of hydropower. However, this cost is no longer uniquely decisive for the production. The losses and congestion will play a role in the use of hydroelectricity. What is special for hydropower is the dynamics of the shadow prices of water and reservoir limits as revealed in the third condition in (5.6). As long as reservoir levels stay in between empty and full the water value for a hydro unit remains constant for the time periods in question. The three elements shadow price on the energy balance, value of total marginal losses, and congestion may change from period to period, but the water value at the production node remains the same.

The spatial distribution of dispatch of hydro generators within a period determined by the Transmission System Operator (TSO) must take losses into consideration, created simultaneously by the spatial distribution of demand. The utilisation profile of reservoirs over time will be influenced by spatial variation in losses. When consideration of overflow necessitates a specific manoeuvring of a reservoir, the creation of loss connected to the utilisation profile will also enter the picture. Less water will be used in high-demand periods due to the increased losses incurred. Differential losses on lines will also influence the relative use of hydropower plants connected to consumer nodes with different line losses, e.g., due to different geographical distances. Reservoirs connected through lines with less loss will be used relatively more intensively in high-demand periods than low-demand periods, and vice versa for reservoirs connected through lines that have relative higher loss.

If losses and congestion are zero the water value becomes equal to the shadow price on the energy balance, implying as in the models of Section 3 that the water values are all the same and equal to the common water value of active generators.

The marginal cost of thermal generator(s) at a node *f* is

$$c'_{ft}(e^{Th}_{ft}) = \tau_t - \theta_{ft} - \tau_t \sum_{n=1}^{N} \frac{\partial e^L_{nt}}{\partial b_{nt}} \frac{\partial b_{nt}}{\partial e^{Th}_{ft}} - \sum_{n=1}^{N} \mu_{nt} \left(\frac{\partial b_{nt}}{\partial e^{Th}_{ft}} - \frac{\partial \overline{b}_{nt}}{\partial e^{Th}_{ft}} \right),\ f = 1,..,N^{Th} \tag{5.9}$$

The difference in marginal cost between thermal nodes will be influenced by the shadow price on the generating capacity (if the thermal unit's capacity is fully utilised), the total marginal loss on lines and total marginal congestion costs on lines. However, the marginal cost cannot be less than when running at full capacity, so the loss and congestion terms may drive down the marginal cost implying that $\theta_{ft} = 0$. The loss and congestion terms may eventually drive down production to zero. Merit order of nodes will not be used now due to the externalities in the transmission system.

A marginal change in the renewable outputs will also generate marginal losses and congestion similar to hydro- and thermal generation. Varying levels of renewable energy will give repercussions in the transmission network and influence nodal consumer prices.

Comparing the nature of the optimal solution with transmission to the one without in Section 3 we no longer obtain uniform marginal cost in the system, but-node specific water values and marginal costs of thermal generators.[25] Furthermore, the prices at consumer nodes become in general different. The differences in consumer prices all stem from the way losses and congestion are incurred in the system. Congestion is even more important than we have modelled due to loop-flow effects.

The spatial distribution of dispatch of generators within a period determined by the TSO must take losses and congestion into consideration, created simultaneously by the spatial distribution of production and demand. The utilisation profile of reservoirs over time will be influenced by spatial variation in losses. When consideration of overflow necessitates a specific manoeuvring of a reservoir, the creation of loss connected to the utilisation profile will also enter the picture.

However, it should be evident from our analysis and the physical electrical realities, that, even for a social planner, it would be quite an involved operation in practice in real time to mirror the physical system completely by fully implementing the spatial structure of optimal prices at consumer nodes, and individual water values at generating nodes that takes incurred losses and congestion fully into consideration. The transaction costs in the form of gathering information, processing it, and sending instructions to generators may involve costs that are higher than the social benefit of spatial pricing. The way the energy flow from one generating node is distributed on consumption nodes varies continuously over time and with the changes in the configurations of consumption and generation, thus creating "electrical externalities" of losses and congestion involving loop-flow effects in the network system. It may be impractical, or too costly, to internalise the full extent of externalities.

However, the externality type of model used will not reveal optimal prices of production units as in the models in Section 3 without a transmission system. As stated before the energy balance at each node is not modelled, and the model cannot tell where the electricity from the five types of production units ends up.

[25] By the nature of intermittent generation, we have assumed that there is no variable cost supplying electricity, therefore there is no revenue generated; all costs are sunk when investing. We are neglecting that there are owners of intermittent generation that take profit from the generation.

A common market system for determining prices is day-ahead prices delivered by producers and utilities distributing electricity to consumers. In principle each node should then represent a market. However, the problem of where the electricity ends up is still there. Large-scale energy flow models are used in practice to approximate the flow pattern.

5.3 Separation into zones

Congestion may lead to separation of a system covered by a grid into zones that become independent as to price formation (Bjørndal et al (2012); Bye et al (2010); Green (2007); Pettersen et al (2011); An example is provided in Fig. 5.3. The generating nodes are indicated

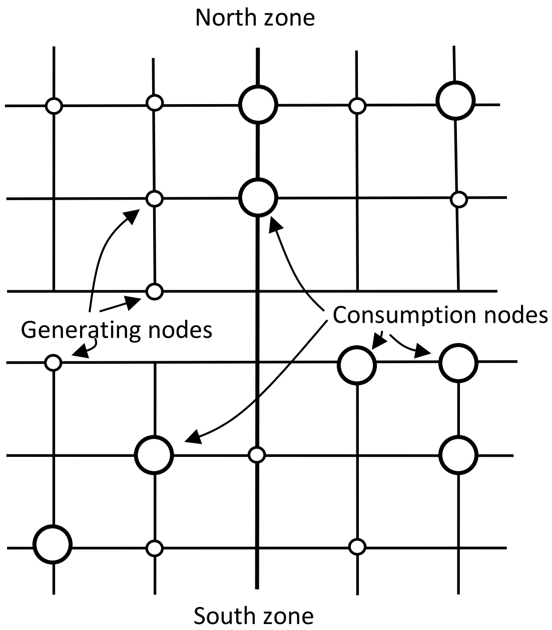

Figure 5.3 *A general transmission network. Generating nodes are represented by small circles, consumption nodes are represented by large circles. Bottleneck in the middle*
Source: Førsund (2015)

with small circles and the consumption nodes with large circles. The meshed grid pattern just indicates that there are several ways for the electricity to flow from production nodes to consumption nodes, i.e., loop flows may occur. Size of generation and demand, or capacities of links are not indicated in the figure. The network falls in two parts; the southern and the northern parts, and there is only a single link that may be a bottleneck between these two parts. This is the case between the North and the South Island of New Zealand, and almost the case between North and South Norway. This connecting link may be congested for certain configurations of supply and demand in the total network. Usually there are critical links with restricted capacity that cause congestion. But these links may change with demand and supply configurations, as discussed in Subsection 5.2. Notice that with loop-flows we may have congestion occurring without resulting in separate zones. Such separation was assumed in Section 4 when looking at two countries, and separate prices resulted when the link between the countries was congested with the importing country having the highest prices.

When a grid is separated by congestion the determination of optimal consumer prices, water values, and other shadow prices will take place insulated from events in the other parts. However, the drawback with a zone, more accurately called a price zone, is that all consumers within the zone will pay the same price. Producers will also get the same price. A day-ahead-scheme will establish the prices within the zone according to demand and supply in the zone. All producers in the zone and all consumers will in reality be aggregated. Different nodes within the networks in the zone will not be treated as separate nodes and therefore the real externalities as described above will not be used to set prices according to nodal prices discussed in the previous subsection.

6. Investment in transmission and production[26]

Investment in transmission will in general be positively correlated with investment in generating electricity[27]. This is especially the case if new generation of small-scale hydro plants and windmill parks are located in areas needing new network lines to the central grid. However, transmission investment may be undertaken for constant production capacity in order to reduce losses upgrading the voltage, and relieve congested links.

6.1 Investment in transmission

We will mainly be concerned with the high-voltage central grid and not with regional grids of lower voltage and the distribution network involving consumers at low voltage, although the border lines between central grid, regional grids and local grids are not always clear-cut. The focus will be on economic issues involved, avoiding going into technical or engineering details more than necessary. The purpose is to illuminate

[26] This section is based on Førsund (2007).

[27] See Skjeret (2008) for a general review of both generation and transmission investment issues and extensive references to points raised in the paper.

fundamental questions faced when undertaking transmission investments. There are several models as to organising the management of a transmission system and investments. We will concentrate on a transmission systems operator (TSO) owned by the public sector and responsible both for operating the network and undertaking investments. Private ("merchant") investments in transmission will not be considered.[28]

The fundamental role of a transmission network in a spatial setting is to secure that consumers at their locations are supplied with power produced by generators, typically located in a different geographical pattern. The main demand on a transmission system is that power should be delivered reliably. This means that the network should have the capacity to satisfy fluctuating demand. However, demand is in general a function of price, so this criterion is not purely technical.

The reorganisation of the electricity sector seen in many countries the last three decades of unbundling generation and transmission of electricity, previously carried out by a vertically integrated utility, has separated investment decisions in generating capacity and in transmission capacity. Markets for power have been organised implying competition between generators, and decentralising investment decisions for generating capacity to the generators. There are signs that transmission networks in several countries are experiencing increasing levels of stress (Joskow, 2006), resulting in a higher frequency of service failure and even blackouts.

Changes in consumer demand is influenced by changing demographic patterns and residential and commercial building activity as regards general consumption, and change in demand from large energy-intensive industrial customers, like aluminium, ferro alloys, refineries, pulp and paper factories, off-shore oil and gas platforms, etc., following economic development of exit and entry of firms. Many of these changes are uncoordinated with investments both in generating and transmission capacity. At the present state of transmission technology there is no room for more than a single network, so the network as a cost-based natural monopoly is generally operated under one management. In many countries the public sector is the owner of the central grid. In

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[28] Merchant (private) investment in transmission has been a popular issue, especially in American literature. But following the critique in Joskow (2006) we disregard this possibility. However, for direct current (DC) interconnector between areas or countries private investments may be fitted in together with a publicly owned TSO. The DC sea cable between Norway and the Netherlands (NorNed) was a joint project between the publicly owned Norwegian company Statnett and the Dutch central net company Tenne T.

European countries it is also common that the network operator is the system operator, i.e., we have the Transmission System Operator (TSO) construct.

The network operator is responsible for the investments in the network. However, due to the new organisation of the electricity sector there are fundamental coordination problems on the investment side, remembering also the spatial aspects involved. Changes in patterns of demand and patterns of generation must be coordinated in some way with changes in transmission capacities when we have generators and consumers making independent investment decisions.

The regulation of networks usually demands that the cost of the activities of a network including investments should be covered by charges levied on the users. But some charges have the role of making the current use of the network efficient. If charges, e.g., reflect losses on lines and congestion, then increasing transmission capacity will decrease such efficiency based charges. In order to finance investments pure connection charges independent of energy flows must also be used.

There is a new role for the transmission system in countries running competitive short-term wholesale markets. Transmission should also facilitate competition by reducing the potential for use of market power[29]. Within a constrained transmission system import-restricted sub regions may be created where market power can be exercised. Increasing the capacity of the transmission system will reduce the physical potential for using market power. This may imply that if a well-functioning competitive market is judged to be desirable on its own, then a certain amount of excess investment from a technical point of view in transmission may be called for.

Reliability criterions

As pointed out in the Introduction reliability is a fundamental demand on the transmission system. The transmission system serves as a hedge against unplanned generator outages. The system should be redundant enough to avoid service interruptions in the face of various system contingencies. However, reliability can involve several aspects. In Blumsack et al. (2007) the following criterions are suggested as common reliability

[29] This is actually explicitly stated as an objective of the Norwegian TSO, Statnett.

metrics:

1. The N - k criterion; if k out of N pieces of equipment is lost, damaged or disconnected from the network, then the system should continue to provide uninterrupted service (k may commonly be set to 1).
2. The Loss of Load Probability (LOLP); the probability that the network will fail to provide uninterrupted service.
3. The Loss of Energy Expectation (or Probability); the expected amount or proportion of demand not served (Unserved Energy Expectation or Probability).

To improve reliability according to the metrics above is obviously costly. Therefore, there is a trade-off between investment costs and operating costs on one hand and degree of reliability on the other hand. In the final decision about level of probabilities the social cost of interruptions (i.e., the willingness to pay to avoid them) have to enter the picture.

Introducing wind mills on a large scale in a region may affect the short-run use of the transmission system, since wind production is intermittent. In Holttinen (2005, p. 4) it is argued that the N - 1 criterion provide a too conservative estimate of transmission capacity in these instances. The strain on the network at maximal wind-power production has to be taken into consideration.

In addition to reliability there is also the related question of quality of service, i.e., the quality of electricity itself. Reduction in quality may be measured by deviations from target voltage and frequency levels.

Transmission investments relieving congestion

Studies by economists of transmission in the short run have often focussed on congestion of lines. Congestion on a line is usually defined with reference to the thermal capacity of the line. The line is congested when the capacity is exhausted. For systems with alternating current (AC), that is the rule, the line resistance to electricity flows is more general than Ohm's resistance, adding inductance, and the total resistance is termed impedance. The meshed nature of many grids results in loop flows complicating the definition of congestion, due to Kirchhoff’s laws distributing flows over possible loops in proportion to the impedance of each loop. Thus if one link in the loop is congested, this set an upper limit on electricity flows also on the other lines.

Congestion of lines happens in real time and necessitates actions by the TSO in order to keep the system in physical equilibrium with supply equal to demand at every node. Economists have proposed using congestion charges to improve efficiency of utilising the network in the short run. Investments in transmission capacity may be motivated by relieving congestion. What is, then, the connection between investments focussing on congestion relief, termed economic investments by Joskow (2006), and investments to maintain reliability? According to Joskow it is meaningless to distinguish between congestion-relief and reliability investments since most investments are driven by reliability concerns. However, Blumsack et al. (2007) take this critique one step further by arguing that in many cases it will be not only meaningless to draw a line between these types of transmission investments, but actually wrong. A type of network termed a 'Wheatstone Network' is used as an illustration where introducing a new line, the *Wheatstone bridge*,[30] between production nodes increases the reliability of the system, but decreases the capacity. This phenomenon is parallel to *Braess Paradox* for adding a link to a road network. The paradox is that adding a link may increase traffic congestion. The paradox appears in meshed electricity networks due to loop-flows governed by Kirchhoff's laws. The system must have embedded Wheatstone sub-networks for the paradox to appear. Such subsystems allowing lines to be introduced between generator nodes are commonly found in meshed networks.

In meshed networks with pervasive loop flows it is not so easy to pinpoint the congested line which capacity should be increased. A change in one place of the system may give system- wide repercussions leading to quite another line being congested. Therefore, a system-based approach of simulating the new flow patterns, following an investment in transmission capacity, must be performed to create enough information to make the best optimal choice. It is not enough just to monitor the system and report where congestion appears in the existing system.

Types of transmission investment

Transmission investments can either represent incremental reinforcements and upgrades

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[30] A Wheatstone bridge is an electrical circuit used to measure an unknown electrical resistance.

of existing facilities, or be investments in new lines. Investments in new lines may be a consequence of new generation capacity coming on stream, or substantial demand growth, e.g., new energy-intensive plants. The generator has to be connected to the net, and new demand has to be served, and then there is a question whether the existing net has to be reinforced due to the generating investment or increased demand. In Joskow (2006) the following list of upgrading investments is found:

a) new relays and switches
b) new remote monitoring and control equipment
c) transformer upgrades
d) substation facilities
e) capacitator additions
f) reconductoring of existing links
g) increasing the voltage of specific sets of transmission links

There may be significant technical progress taking place concerning the different technologies represented in the list above. During the last decades the bulk of transmission investment in Norway has been of these types, excluding interconnector capacity investments to Sweden, Finland, Denmark and recently the Netherlands. Investing in new lines may use existing corridors, or set a new "geographical footprint." The problem with new corridors in Norway is mainly environmental concerns since hydro power development is usually taking (or has usually taken) place in remote areas of natural beauty. But reactions against new corridors are widespread, as expressed by the term NIMBY (Not In My Back Yard).

Criterions for investment

The classical investment criterion is to follow a procedure of minimising social costs of transmission investments and loss in the network, given a number of physical constraints expressing present and planned levels of demand and generation, and given reliability standards. The question of how far one must look ahead is related to lead times from identifying relevant projects and to when capacities are installed, and related to the degree of lumpiness of transmission investments. These factors vary according to the type of transmission investment, as described above. For small enough improvements of transmission system improvements, the cost minimising mode with system characteristics as exogenous variables will do. The usual criterion of non-negative net present value applies. New lines, especially along new corridors, would be at the other end of the scale as being significantly lumpy or indivisible. For such investments the question of which variables to regard as exogenous and which ones to regard as endogenous will be crucial for the investments proposed. Stoft (2006) illustrates a case where - when taking several periods into account - privately profitable investments should not be undertaken. The value of postponing investments may be positive.

The planning benchmark

However, the investment criterion set out above is too limited. The dimensioning of the electricity system should be seen in a social welfare perspective. The most common approach followed in this paper is to formulate a partial model and maximise the sum of consumer and producer surpluses, without modelling the links of the electricity system to the rest of the economy. The ultimate benchmark for transmission investment will be integrated resource management planning, coordinating both generation and transmission investment within a spatial setting. In addition to assuming that the network is centrally planned it is also required that the TSO accurately works out the optimal sequence of generation and network investments into the future. Even in a setting with integrated planning, in order to realise perfect coordination, short-run utilisation of existing generating- and transmission capacities incorporating the spatial structure, must also be in place. Such pricing schemes are necessary for giving the right incentives for efficient utilisation of existing capacities. This means extensive use of nodal pricing and correspondingly differentiated locational energy charges as the benchmark. Without such schemes realising short-term efficient utilisation in place, the investment signals will be distorted. As commented in Sauma and Oren (2006) such a benchmark planning procedure may never have been in place in any country, even before unbundling of generation and transmission. In order to be realistic one then has to look at second-best approaches as benchmarks. Turvey (2006) discusses the long-run and short-run incentives for optimal location of generation capacity. He does so with a particular focus on the British system, and its use of geographically differentiated system charges.

Proactive planning

The restructuring of the electricity sector, as in Norway, means that investment decisions in new generation are taken by independent companies, publicly or privately owned. It is reasonable to assume that investment decisions are based on standard profitability considerations, i.e., based on maximising net present value. Investors will make their location and plant choices based on the present value of selling at future prices and future charges related to the use of the network. The expectation about prices, and in a transmission context the expectations about transmission charges in terms of connection charges and production- related charges, will directly influence investment decisions. It is therefore important for transmission investments that the reactions of generation investments to transmission investments are considered when planning transmission investments. Kirby and Hirst (1999) and Wu et al. (2006) point out that transmission line investments take longer to complete than what most generation investment does. It is reasonable to believe that generating investments and location of such investments will react to expectations about changes in charges due to new transmission investments. Sauma and Oren (2006) call transmission investment planning, taking the reaction of generating investments into consideration, for *proactive planning.*

An alternative model is *reactive planning,* where planning is based on given generation capacities and the purpose is to minimise transmission costs observing reliability constraints (see also Hirst and Kirby, 2002). The reaction of generating companies to proposed transmission investments is then not considered. Reactive planning means reacting to generation investments when in place without influencing the scale or timing of the investment.

In order to formulate an efficient proactive planning procedure, it is important to be clear about which variables are exogenous and which variables are endogenous. The change in demand and its spatial distribution are often treated as exogenous. This may be in order for consumer nodes with slow and organic change in demand for electricity due to demographic changes and expansion in residential and office buildings, and development of light industry and services. But a political decision on space heating technology, e.g., going from electricity to natural gas, may be important enough to warrant coordination with transmission investment. There is a question mark about how to treat large demand changes due to exit and entry of energy-intensive industries. Examples from Norway are supply of electricity to oil platforms from the mainland, restructuring of aluminium production between two different locations in Western Norway, and closing down of a pulp and paper mill. It is a question whether such consumption changes should be taken as given when planning transmission investments, or whether transmission considerations should also influence such large changes in demand. Different spatial configurations of energy intensive industries may have large differences in transmission costs.

Coordination of generation and transmission investments is also needed because there may be a substitution between generation and transmission. Investing in transmission will always substitute for increased generation if losses are reduced because of transmission investment. Generation can substitute for transmission by relieving congestion by counterflows, e.g., investing in generation in an import region is a substitute for increasing the transmission capacity on a congested line from an export region to an import region (Wilson, 2002).

When following a proactive planning approach, it is important to recognise the fundamental externality of system reliability, making reliability a public good. One cannot therefore look at efficient short-run prices and charges only in order to deduce when new investments in transmission should be done. A wholesale market for electricity, mimicking a competitive solution due to efficient prices being applied, will still not be a relevant guide for investment in a public good. The prices and charges reflect short-run efficiency and will not necessarily serve as socially correct incentives for transmission investments. Considering energy charges reflecting marginal short-run losses in the system before investments, the physics of loss generation will change when investments are carried out, and then also the new efficient levels of loss charges will be different from old levels. In general, many energy-related charges will decrease. The power transfer distribution factors used in studies of flow and losses in a network will change due to investments. Given the lumpiness of transmission investment large deviations of current nodal prices from their values after transmission investments have been carried out can be expected.

The key feature of the proactive planning approach of Sauma and Oren (2006) is the ability of the TSO to predict the reactions of generator investors and load demand when transmission investment is carried out. In order for prediction to be of high quality, the TSO needs, in principle, as much information about the investment procedures of the investors as in the central planning benchmark case. If such information is not available, it will be difficult to design investment incentives in such a way that the optimal response to transmission investments is forthcoming.

Towards a practical planning procedure

We assume that the net operator (TO) of the high-voltage central grid is also responsible for carrying out transmission investments, as in the Norwegian model. The TSO in Norway, Statnett, is responsible for carrying out transmission investments based a general social-economic criterion. More specifically, Statnett is responsible for reliable delivery of electricity and to promote a well-functioning market for electricity. California ISO (2004) gives an example of one method to plan transmission investments. Other examples of planning approached can be found in Wu et al. (2006) and Hirst and Kirby (2002). The state is the owner of the net in Norway, but this is not so important. The importance lies with the instructions about how to carry out transmission investments. The general objective is to achieve the best social solution, but the question of endogenous and exogenous variables is not so clearly specified. As to financing of operations and investments the net operator is in principle to be self-financed. The net operator has the power to introduce connection charges, independent of energy flows, and energy charges that may be related to marginal losses. The regulatory setting for the electricity sector is dominated by unbundling. Markets for energy and reserves are unbundled, while there is open access to transmission within NordPool for agents connected to the net. There is no independent market as such for transmission services, and the TSO deals with congestion through arranging counter flows and splitting Norway up in different price areas. New investment in generation is proposed by generating companies (private or mostly publicly owned; municipal and state ownership) and subjected to concessions given by the Norwegian electricity regulator (NVE) that may consult the transmission operator.

Identifying needs for transmission investment

The first stage in planning transmission investments is to identify investment needs. Monitoring continuously the network, the TSO may observe increased stress within the net in the form of increasing marginal losses, increasing frequency of periods with congestion problems and increasing episodes with struggling to maintain the reliability standards set for the system. However, in a social efficiency perspective, investment plans should not be made unless current charges are adapted and used to obtain short-run efficiency in using the system. Peak load episodes may be alleviated by increasing the energy-related charge during such episodes, etc.

Uncertainty about occurrence and severity of stresses on the transmission network may make it optimal to invest more in transmission capacity, considering the magnitude of the cost of net failures like blackouts compared with additional investment costs.

There is a system externality effect when considering reliability, making calculating for one piece of equipment at a time a faulty procedure. Various options for relieving system constraints concerning reliability and congestion must in general be considered in order to develop a suitable package of micro investments that it is socially profitable to carry out.

If a zonal pricing scheme is used to deal with congestion it may not lead to optimal investment in transmission, following a policy of equalising prices as advocated in Brunekreeft et al. (2005). Optimal nodal prices differ both before and after transmission investments, so equalisation of zonal prices is a too limited objective to pursue. The general objective is maximisation of consumer plus producer surplus subject to reliability constraints (reliability may also be considered baked into the surplus measures).

As mentioned earlier a new objective introduced by using market-based systems is to mitigate market power. In electricity networks the creation of importing and exporting zones may give opportunities for exercising market power even for firms without a large market share within the total grid area. Over-investing in transmission will reduce the possibility of taking advantage of zones created temporarily.

The role of network charges

A transmission net may function within given reliability criterions at different levels of total costs. A natural purpose of investments is to seek to minimise these costs. However, the problem is that it is not straightforward under what conditions such a minimisation should be carried out. At a micro level an investment opportunity, of the type of system improvements described above, may be found by comparing cost savings with investments costs as a regular activity with a certain frequency over time, and the investment activity carried out when the net present value is non-negative, using the relevant rate of discount (usually set by the owner). However, such an identification procedure is not as straightforward as it appears if the overall objective with transmission investments is to maximise the social value of the investments within a standard objective of maximising the sum of consumer and producer surplus. First, the existing transmission system must be utilised in an efficient way in the short run. Without any element of charging for current transmission services provided by the network the calculation of the profitability of the investment project may easily be biased. If there is a charge for transmission services, then this charge must be set correctly. The general reference is nodal prices. These prices change in principle over continuous time, and may be complicated to calculate in meshed networks with loop-flows. As pointed out in Brunekreeft et al (2005) there is a question whether nodal pricing sets efficient long-term investment signals to generator and load, or whether additional locational differentiation of grid charges is necessary. The structure of network charges will have a potentially significant impact on network use, and influence location of new generation and load as well as influencing bids in wholesale spot markets.

According to Brunekreeft et al (2005, p. 75) the structure of network charges should encourage:

- The efficient short-run use of the network (dispatch order and congestion management)
- Efficient investment in expanding the network
- Efficient signals to guide investment decisions by generation and load
- Fairness and political feasibility
- Cost recovery.

The theoretical notion of nodal pricing as continuously changing prices in real time is not implemented in any system. In US the PJM has established a scheme with important nodal pricing features. In Norway there is zonal pricing with a small number of price zones (five) formed endogenously by the TSO for shorter or longer periods. In view of transaction costs, cost of information gathering, etc. the more interesting question in practice seems to be what kind of most efficient second-best system to aim for. Transaction costs of running perfect pricing schemes may easily become excessive, so a more realistic second-best approach of pricing policies may be followed with corresponding effects on optimal transmission investments.

6.2 Investing in generating capacity

The general incentive for investing in generation is the standard profit opportunity looking at the net expected price of electricity and the investment costs. Since the spatial aspect is crucial for the coordination of generating and transmission investments, the net price received by the generator has to be spatially differentiated. This can be done by applying an energy charge based on location-specific marginal loss.

The requirement of self-financing of the TSO will typically lead to connection charges (of a postage-stamp nature independent of transacted energy volume), collected per period over time, also having to be used to generate enough income. Such a charge has no locational signal. The most powerful locational signal can be given by using a connection charge varying according to location to reflect the cost of extending and upgrading the network, in addition to generator-specific shallow connections, caused by the generator investment. This charge is by nature a lump sum capital charge, but may be annuitised (Turvey, 2006).

As stated in Brunekreeft et al (2005), wind power in particular creates new flow patterns across grids. In Norway many wind-mill investments have been planned along the coast. The location of many of these planned projects is such that there is some distance to the existing network, but there may also be a need for large upgrading investments of existing lines if wind power is to be phased in. As reported in Brunekreeft et al (2005) grid reinforcement costs to handle modest amounts of extra wind power in Scotland might be 75 - 80% of the total cost of all wind power. Excessive costs of net upgrading in order to phase in wind power in Finnmark are also documented in Statnett (2003). The question of wind power has therefore accentuated the discussion of what part of the grid investment to regard as a part of the generating investment to be covered by that investor, and what part should be socialised and covered by the general use of the net. This corresponds to the terminology of "shallow connection charges" (initial charge for being connected to an existing grid) and "deep connection charges" (also paying for extending the grid) found in the literature (Joskow 2006). Using shallow connection charges only may promote new entry, which may be attractive to those wishing to promote renewables.

The general problem is how to charge new generators where their entry requires expanding and/or upgrading the grid, both in the sense of a shallow connection and in the sense of a deep connection. When line investment and other network investments are indivisible it may be optimal to increase capacities by more than what is needed just to phase in a single generator investment (Turvey 2006). If upgrades take account of such indivisibilities to over-build ahead of future demand, what fraction of the costs is attributable to the present connection? Is it reasonable to charge the security benefit to a new entrant?

In principle the additional net investments that have to be done due to the new generation, should be regarded as a part of the generating investment project if the transmission investment is of no additional benefit to any other existing or potential future generator. This should also hold for wind power projects. There may be willingness to subsidise wind power, e.g., due to environmental consideration of zero emission of CO_2, but the extent of the subsidy needed to make the project economically viable (without payment for emission reductions) should be made explicit. Charges applied to wind generators must properly reflect the load patterns of wind compared to conventional generation,

Grid reinforcement may, however, contribute to reliability, which is a public good, implying that a part, even all, of this investment should be socialised. In addition, there is the question of lumpiness of grid investment. It may be that the minimum scale of the grid reinforcement is considerably larger than what is strictly necessary to accommodate the single wind power project in question. It may also be the case that there is significant economies of scale in investing even more. This means that there is room for future growth in generation without additional grid reinforcement investments. The optimal strategy for the net operator may be to give incentives to more projects than just one in order to utilise these scale effects and investment in a public good. The first project to be accepted cannot then be charged with the total cost of reinforcing the net. The investment cost is to be shared with future growth in generation and the general system. See for instance Turvey (2006) for a discussion in relation to the British charging regime.

In order to dimension the development of transmission capacity in a socially optimal way the net operator must also predict the growth in generating investment. Potential hydro power projects, new development or upgrading of existing power plants, have given locations. Wind power will be located along the coast, so although a specific wind mill has a fixed location, ex ante, before commitment to investment, the wind mills can be regarded as footloose along the coast. The planning task of the net operator is to give incentives to the location creating the highest net social value. In view of the lumpiness of the transmission upgrades and externalities, one way to proceed can be to consider larger areas for development and not single wind mills. Based on information about wind conditions and availability of sites, taking environmental cost considerations duly into account, model simulations should point to areas to start with that have the highest social value. Providing transmission capacity first to one or more of such areas will then offer firm commitment as investment incentive.

The expectation of future gross prices and level of user charges plays a crucial role in the generation investment calculations. In order to facilitate coordination of generation and transmission investment the net operator should see to it that the potential investors have as much information as possible to form expectations about future prices and charges. The net operator may be in a better position to run model analyses yielding price predictions. It is certainly best placed to give predictions about the development of connection charges and energy charges. It has been suggested in the literature that the net operator should give investors contracts for charge levels for a considerable number of future periods in order to reduce risk for investors. At least the net operator should provide predictions based on model simulations for future periods incorporating new generation and transmission investment and the time path for charges based on the present rules for setting these charges.

Simulation scenarios

It is difficult to think that a unique modelling approach will be feasible, yielding only a single unique solution. The more tentative nature of the proactive planning approach envisaged here calls for a number of alternative investment plans in transmission to be investigated. These alternatives should reflect the lumpiness of especially investments in new lines and in lines in new corridors. As to wind power investments and small-scale hydro sufficient cost and production efficiency data must be collected, and both the need for shallow connection investments and deep investments in line upgrades and reinforcements explored. It is important for the economic analysis of deep connection transmission investments that the wind power investment does not only consist of a single wind power project, but covers a number of wind park sites over a larger area or region. It should then be possible to work out a list of a limited number of prioritised locations for wind power according to values of the social objective function used for the model simulations. The simulations will also give information about the optimal sequence of phasing in capacity from the prioritised location of wind power.

The next step is to tailor-make investment incentives to match the prioritised investment areas. The simulations will also give information about the level of charges to apply to satisfy the requirement of cost recovery, setting the postage stamp charge residually. It then has to be checked whether these price signals will lead generators to locate efficiently. The investors must be given a sufficient degree of certainty both about energy independent charges and energy-dependent charges for a number of future years. But such incentives may not be enough to actually realise investments. It should then be observed that is not the role of the TSO to subsidise wind power investments, unless explicitly instructed to do so. A better approach seems to be that political organs determine the degree of subsidisation. Direct investment support then seems most attractive (in order to avoid underinvestment). The role of the model simulations of the TSO should serve the need of identifying both location and corresponding volume of wind power (or other renewables) that minimise the level of investment subsidies necessary to realise a total volume of investment in renewables formulated as a policy goal by politicians.

As part of the simulations the reliability of the system also has to be tested by applying the most relevant version of the N - k criterion, and to investigate, e.g., the consequences of choosing different probability levels for outages.

The production capacities of generators, capacities of reservoirs, and capacity of the transmission network have all been assumed constant for the dynamic management problems we have addressed. Analysing optimal investment in capacities of various types is a huge task outside the scope of this paper. But calculation of the shadow prices corresponding to the given capacities will give an indication of at least the direction of desirable investment.

The shadow price on a reservoir constraint tells us the increase in the objective function of marginally increasing the reservoir capacity (see Subsection 3.2). This may be possible by either better utilisation of the present amount of water by reducing friction inside tunnels, increasing the size of the reservoir, or by increasing the catchments of water into previously untouched sources. The costs of such investments can be calculated. The point is now that the benefit side of a marginal investment is the sum of the positive shadow prices within the horizon. It may not be feasible to carry out a marginal investment, but this simple cost-benefit calculation gives an indication of whether it is interesting to carry out investment analyses. In a system characterised by optimal amount of capacity there should be equality between benefit and costs at the margin, provided sufficient flexibility of dimensioning the investment project. Whether production capacity should be increased can be investigated by a similar comparison of the sum of positive shadow prices and the cost of investment. If the turbine capacity is the limiting factor the investment project is not so large, but if the water-feeding capacity through tunnels from reservoirs has to be increased, this is a more major undertaking.

Increasing the catchment area has environmental costs. There are also significant environmental costs investing in small-scale hydro plants defined by EU regulation as providing renewable electricity.[31] Such investments can be undertaken by private investors and also foreign investors, in contrast to the public ownership (municipal and state) of the lion's share of pre-existing hydro plants. The small-scale plants are usually of the type run-of-river.

The increasing investment in wind turbines in Norway represents the largest environmental costs. This is due to the location in areas that have high scenic value and user value as recreational areas. Windmill parks with height increasing much above the plans accepted many years before has become a controversial issue in Norway. Investors (mainly private and foreigners) state that the windmills are now profitable, but forget that this is private profit and not social profit; environmental costs are not considered by private investors.

The concession regulation of water flows downstream hydro plants are introduced in order to reduce environmental damage. The first type of regulation of water-flows downstream of power plants was based on the need to transport timber in the timber-floating season. This does not make much sense years after lorries have taken over such transports. There are tapping rules for reservoirs in the spawning season for trout and salmon, and staircase for salmon may have to be build. Demand for river-based recreation of various types, or willingness to pay for unspoiled ecosystems of rivers, may have increased considerably (see Johansson and Kriström, 2011). Regulations may be reviewed every 10-year period.

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[31] It is difficult to understand the logic of this classification and not to acknowledge all hydro plants as producing renewable electricity.

7. Conclusions

The purpose of the paper has been to characterise theoretically an optimal electricity system. Optimal consumer prices and quantities of electricity and marginal cost of producing electricity have been studied using rather aggregated models compared to engineering ones in order to draw qualitative conclusions. Being economic type of models, they have objective functions of consumer and producer surplus that are to be maximised. Consumer surplus is based on the demand function for electricity, and producer surplus is the profit of producers. Consumer demand is endogenous as opposed to detailed engineering models of, e.g., the MARKAL type having demand as an exogenous variable.

The first model is based on a single consumer (aggregating all consumers) and five types of production plants, encompassing a number of hydro generation of plants having one reservoir each by assumption, a number of thermal generating plants based on fossil fuels (nuclear plants are not considered), and three intermittent (or renewable) plants that are run-of-river with no or very limited reservoirs, wind mills, and solar energy plants. Discrete time is used that can vary from an hour, weak, and a month. Transmission is not introduced at this stage.

The use of hydro plants with reservoirs necessities a dynamic approach. The fundamental question is when to use how much water. The opportunity cost of water is named water value and is the shadow price on the reservoir filling. The reservoir filling in period t is linked to the filling in period t-1. Managing water use is forward looking; if the water value is higher in a future period the water is left in the reservoir conditional on the feasibility of storing water. If the reservoirs are between empty and full for a set of successive periods the optimal price becomes the same.

The thermal units are used according to merit order; i.e. according to a ranking of marginal cost. This means that all the units in use are producing to maximal capacity, but typically for the last unit it is used partially. The units used at maximal capacity has a shadow price for full capacity added to the marginal cost.

The optimal price to consumers is independent of all the types of production units that are in use, including intermittent ones, only total capacity. However, the exogenous variation of the intermittent energy has consequences for the use of hydro and thermal. Assuming a constant price for some successive periods – keeping thermal generation constant – hydro will absorb the swings in intermittent generation, if there is enough water in the reservoirs and sufficient power capacity.

Uncertainty plays an important role for managing hydro generation. We have uncertainty about demand in general, but also due to unforeseen changes in temperature. Precipitation has consequences for hydro with reservoirs and run-of-river. Wind condition and amount of sunshine causes uncertainty for wind mills and solar generation. Future price of fossil fuel and labour and reduction of capacity due to concerns about climate change, create uncertainty for thermal units. However, it is rather complicated to analyse the situation acknowledging uncertainty. An explicit modelling based on expectation of stochastic variables is not attempted. Manoeuvrability of hydro generation is important, and uncertainty will mean that this has to increase, i.e., power capacity should increase and a higher minimum filling levels of reservoirs should be introduced.

It is difficult to store intermittent generation of electricity. One old possibility at least for more modest storage is pumped storage. Based on a lake or river an artificial storage for water without a natural inflow at a higher level is used as a storage of water pumped up.

The main purpose is to even out the price difference between peak load and lower priced periods. However, round trip efficiency of pumping up and releasing the water to turbines below may be about 20 %, so there must be a sufficient price difference between the pumping-up period and the price peak load period. This situation is explored by a simple model.

At a grander scale is the European (especially German) interest in using Scandinavian hydro reservoirs as a battery. A skeleton model for trade between two countries is set up. One country, Hydro, has hydro generation based on reservoirs, and the other country, Intermittent, has intermittent energy but also thermal to make the example more realistic. There is a transmission between the two countries (e.g. a sea cable). Using a joint benefit function for the two countries and no capacity restriction on the cable and disregarding losses on the cable, the general result is that the importing country realises a lower home price, and the exporting country earns more that if both countries are in autarky. An interesting result, illustrated by a two-period figure, is that the optimal price is equal between the two countries for all trade periods.

To increase the realism of the model a transmission system is introduced connecting nodes of consumers and nodes of producers. We focus on the central high-voltage network for a country or a region. The key feature of the model is that the transmission net has losses (generated by heat) and congestion. In order to get optimality of prices and quantities these two features are crucial. The concept of nodal pricing was introduced for this purpose. The alternating current (AC) in use for national grids introduces additional problems because of reactive power generated at nodes, influencing also voltage stability. To establish a workable model is a huge and complex task. The electricity generated at one production node may be used at several consumer nodes obeying Kirchhoff's laws that energy flows according to relative resistance on lines. The approach of the paper is to model the externalities of the system by formulating a function for energy flows along lines, and a loss function for the lines. The capacity of the lines restricts flows. A special feature of AC is that reactive power reduces the capacity of lines, so this has to be reflected in the model. An AC network also exhibit the property that in loop flows the capacity of involved lines is determined by the line with the lowest capacity.

In order to establish an externality model where everything depends on everything else we introduce a flow function for a line with total levels of consumption at each node and the outputs at nodes for the five technologies hydro, thermal, run-of-river, wind and solar. Due to the technologies being located at different nodes we cannot aggregate intermittent production as in the previous model in Section 3. Then a function for losses on a line is introduced based on Ohm's law. The flow limit for each line restricts the upper flow capacity for lines. However, due to the externalities the limit is not fixed, but a function of the same arguments as the flow function.

Finding the optimal nodal price for consumers, a model with the objective function as the demand functions for all consumer nodes summed subtracted the total cost of running thermal generators is set up. Further elements are an aggregate equality for the energy balance, the restrictions on hydro- and thermal generation as used in Section 3, the energy flow equation, the upper limit on line flows, and the generation of losses.

The result for the consumer price at a node is consisting of the shadow price on the aggregate energy balance, plus the total loss on lines valued by the energy balance shadow price and any congestion on lines valued by the shadow price on maximal energy power (many lines will not be used to a specific consumer node). The shadow price on the energy balance is the price without any loss and congestion. The nodal prices are only equal if the loss and congestion terms are equal.

The type of externality-focussed model in Section 5 does not give us producer prices. The water value for hydro production is valid at the node. When reaching consumer nodes losses and congestion are generated and absorbed in the consumer price.

The marginal cost of thermal generation at a node is also generating loss and congestion before energy flows reach consumers. The implication is that merit order of a cost ranking at the node cannot be used to dispatch thermal capacity optimally.

The cost of loss and congestion will influence the optimal use of hydro- and thermal production units. This is also the case for the three types of intermittent that also generate loss and congestion, but are not so easy to control. These types are often regarded as must run. However, since marginal loss is proportional with the square of the power and may induce congestion, it may be optimal to control the amount of intermittent energy to be fed into the transmission network.

Investment in generation capacities of the five technologies may necessitate investment in transmission. The investment boom in intermittent technologies make take place in locations outside existing lines. Due to the nature of intermittent the dimensioning of their lines may be based on the expected maximal production, thus having on average overcapacity. Upgrading of existing lines to higher voltage levels have been undertaken as transmission investment for a long time. Increasing voltage will *cet par* reduce losses. Another problem is that "greenfield" lines create a resistance because of disturbing pristine natural locations valued for its recreational or scenic properties.

Investment in production capacity may be linked to demand projections. However, this is a rather passive approach to environmental concerns. Having reached a certain level of consumption of electricity it should not be a right to invest in more capacity. Environmental concerns should play an active part of a decision. An option is simply to increase the price of electricity if environmental cost makes investment in capacity socially negative.

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Frischsenteret
Gaustadalléen 21
0349 Oslo
Tlf: 22958810
frisch@frisch.uio.no
www.frisch.uio.no