# FAR OUT OR ALONE IN THE CROWD:
# A TAXONOMY OF PEERS IN DEA*

by

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## Abstract

A method is presented for classifying strongly efficient units in DEA as interior or exterior, and as self-evaluators or active peers. The exterior strongly efficient units are found by running the enveloping procedure "from below". There is no firm production-function evidence of the efficiency of exterior self-evaluators. Interior self-evaluators are more likely to have active peers as neighbours in more directions and may therefore represent technology. When performing a second stage regression analysis of efficiency scores, exterior self-evaluators should be removed. The proportion of exterior active peers also provides information on whether the variable specification is supported by the data.

**Keywords:** Interior and exterior peer, active peer and self-evaluator, DEA, referencing zone, nursing homes

**JEL classification:** C44, C61, D24, I19, L32

The calculation of efficiency scores for production units based on a non-parametric piecewise linear frontier production function is well established within the last two decades. Originally introduced by Farrell (1957) the method was further developed in Charnes, Cooper and Rhodes (1978), where the term the *Data envelopment analysis (DEA)* model was coined.

Given the variable specification, the assumed scale properties (e.g. variable or constant returns to scale), and the orientation of the efficiency measures (e.g. input-saving or output-increasing), the DEA method calculates an efficiency measure of 1.0 for a number of units. The motivation for the classification that we propose below is that this number does not always convey much information about the actual performance of the unit in question. Among the units with an efficiency score of 1.0, the strongly efficient units, termed *peers*, span the frontier. The classification of some of these units as efficient may not be based on the occurrence of some similar observations, but may be due to the method only. We are referring to units which are classified as being *self-evaluators* in the literature, a concept introduced by Charnes et al. (1985a, p.110), which we formalise in the input and output direction below.[1] Our main contribution is to suggest a second partitioning of the peers of a model-run into interior and exterior peers. We argue that an efficiency score of 1.0 conveys very little information on the performance of the group of peers that are both exterior and self-evaluators, given the variable, scale and orientation specification of the model under which these efficiency measures are calculated.

Our approach is one of several that are interested in classifying the observations that are measured as efficient by the method, i.e. the peers.[2] In the literature there are approaches to detect those peers that are extreme in some sense. One approach is concerned with exploring whether efficient observations have undue influence. Andersen and Petersen (1993) proposed to rank efficient units by calculating what later was called super-efficiency scores in order to discriminate between efficient units and deem some peers as more efficient than others. Wilson (1995) picked up this idea and proposed to detect outliers that may unduly influence the efficiency by weighing the super-efficiency score with the average change in efficiency scores for the remaining units when removing the efficient unit being investigated from the data on which the frontier is based. Torgersen et al. (1996) proposed a measure of influence of units that takes into account the amount of potential output increase or input saving that an efficient unit is referencing. The taxonomy we define below is not concerned with influence, but rather with classifying peers as to the information the status implies about their own efficiency.

This perspective is shared with the concept of “efficient by default” suggested by Tulkens (1993), but our method has a different classification of efficient units. The free disposal hull (FDH)- approach (Deprins et al. 1984) is based on the basic notion of dominance in the input – output space. Among those that are on the FDH – frontier, i.e. those that are undominated, a unit is called “efficient by default” if it dominates no other units. In DEA, the input or output orientation of the efficiency measures may influence the classification we will propose when the location of the inefficient units behind the frontier matters, while in FDH some measures will be invariant since dominance is based on both the output- and input-dimension simultaneously. We will return to the differences in Section 2 below.

Self-evaluators may most naturally appear at the “edges” of the technology, but it is also possible that self-evaluators appear in the interior. One contribution of this article is to distinguish between those self-evaluators that are *exterior* and those that are *interior*.[3] The motivation for this dichotomy is that much less is known about the performance of the exterior self-evaluators than the interior units. An approach often followed in actual investigations[4] is to find the influence of some variables on the level of efficiency by running regressions of efficiency scores on a set of potential explanatory variables. Using exterior self-evaluators with efficiency score of 1.0 may then distort the results, because to assign the value of 1 to these self-evaluators is arbitrary. In the case of interior self-evaluators, on the other hand, neighbouring peers may be fairly similar. Interior self evaluators need not therefore be dropped when applying the second stage regression approach.

In addition to classifying self evaluators, the remaining peers can also be characterised in a similar way that may be instructive for interpretation of results within DEA. The pattern of location of peers as being interior or exterior gives information about which parts of the frontier are relatively best supported by data, and in the case of variable returns to scale will illuminate the empirical support for scale variation.

The plan of the paper is to review the role of peers in DEA models in Section 1 and establish the new taxonomy for peers. In Section 2 the method for classifying the peers is introduced. Actual data are presented in Section 3 and the method for classifying peers is applied. Section 4 concludes.

## 1. Self-evaluators and active peers

*DEA models*

Consider a set, $J$, of production units transforming multiple inputs into multiple outputs. Let $y_{mj}$ be an output and $x_{nj}$ an input $(m \in M, n \in N, j \in J)$. As the reference for the units in efficiency analyses we want to calculate a piecewise linear frontier based on observations, fitting as closely as possible and obeying some fundamental assumptions, like free disposal, and the technology set being convex and closed, as usually entertained (Banker, Charnes and Cooper (1984), Färe and Primont, 1995). This frontier can be found by solving the following LP problem, termed the *additive model* in the DEA literature (Charnes et al., 1985b), for each unit *i*:

$$Sl_i = Max\left\{\begin{array}{l} \sum_{m\in M} s_{mi}^{+} + \sum_{n\in N} s_{ni}^{-} \text{ , } s.t. \\ \sum_{j\in J} \omega_{ij} y_{mj} - y_{mi} - s_{mi}^{+} = 0, \\ x_{ni} - \sum_{j\in J} \omega_{ij} x_{nj} - s_{ni}^{-} = 0, \\ \sum_{j\in J} \omega_{ij} = 1 \\ s_{mi}^{+}, s_{ni}^{-}, \omega_{ij} \geq 0, n\in N, m\in M \end{array}\right\} \tag{1}$$

The frontier is found by maximising the sum of the slacks, $Sl_i$, on the output constraints, $s_{mi}^{+}$, and input constraints, $s_{ni}^{-}$. The last equality constraint in (1) imposes variable returns to scale (VRS) on the frontier, while dropping this constraint imposes constant returns to scale (CRS). Our analysis will be valid for both scale assumptions. The *strongly efficient* units (using the terminology of Charnes et al., 1985b) are identified by the optimal sum of the slacks being zero and therefore all the slack variables being zero for these units.[5] The set of strongly efficient units, *P*, and the set of inefficient units, *Q*, are then defined as:

$$\begin{array}{l} P = \{i \in J : Sl_i = 0\} \\ Q = \{i \in J : \; Sl_i > 0\} \end{array} \tag{2}$$

Since the maximal value in (1) is unique, this is a partitioning of the set of units *J*, by which we mean that *P* and *Q* are disjoint sets $P \cap Q = \varnothing$, whose union is the original set of units $P \cup Q = J$ .

So far we only have slacks as measures of inefficiency. If we want only one efficiency measure for each unit, and a measure that is independent of units of measurement, the radial Farrell (1957) measure of technical inefficiency is the natural choice. The standard DEA model on primal (enveloping) form, is set up as a problem of determining the Farrell technical efficiency score, either in the input direction, $E_{1i}$, or the output direction, $E_{2i}$, for an observation, $i$. The following LP model is formulated for each observation in the case of input-orientation:

$$E_{1i} = Min \left\{ \theta_i \text{ s.t. } \sum_{j\in P} \lambda_{ij} y_{mj} \geq y_{mi},\ \theta_i x_{ni} \geq \sum_{j\in P} \lambda_{ij} x_{nj}, \sum_{j\in P} \lambda_{ij} = 1,\ \lambda_{ij} \geq 0, n \in N, m \in M \right\} \tag{3}$$

In the case of output orientation we have the following LP program:

$$E_{2i} = 1 \Big/ Max \left\{ \phi_i \text{ s.t. } \sum_{j\in P} \mu_{ij} y_{mj} \geq \phi_i y_{mi},\ x_{ni} \geq \sum_{j\in P} \mu_{ij} x_{nj}, \sum_{j\in P} \mu_{ij} = 1,\ \mu_{ij} \geq 0, n \in N, m \in M \right\} \tag{4}$$

The proportionality factors, $\theta_i$ or $\phi_i$, and the weights, $\lambda_{ij}$ or $\mu_{ij}$, are the endogenous variables in these LP-problems. For compactness (3) and (4) use inequalities instead of explicit slacks, but the implied production possibility set is the same as in (1).

The point defined by the weighted sum of inputs and outputs in (3) and (4) serves as the frontier *reference point* for unit $i$, and this point is a convex combination of strongly efficient units from the set $P$, which in this setting can be termed *peers*.[6] The peers that participate in the reference point of an inefficient unit can be identified by a positive weight, but these weights are not always unique. To ensure that any strongly efficient unit that could participate is counted as a peer, we define the maximal weights consistent with (3) and (4) by

$$\tilde{\lambda}_{ij} = Max \left\{ \lambda_{ij} \text{ s.t. } \sum_{j\in P} \lambda_{ij} y_{mj} \geq y_{mi},\ E_{1i} x_{ni} \geq \sum_{j\in P} \lambda_{ij} x_{nj}, \sum_{j\in P} \lambda_{ij} = 1,\ \lambda_{ij} \geq 0, n \in N, m \in M \right\} \tag{5}$$

$$\tilde{\mu}_{ij} = Max\left\{\mu_{ij} \text{ s.t. } \sum_{j\in P}\mu_{ij}y_{mj} \geq \frac{1}{E_{2i}}y_{mi},\ x_{ni} \geq \sum_{j\in P}\mu_{ij}x_{nj}, \sum_{j\in P}\mu_{ij} = 1,\ \mu_{ij} \geq 0, n\in N, m\in M\right\} \tag{6}$$

where the radial factors have been replaced by their optimal values (i.e. their efficiencies) from (3) and (4).

Following Edvardsen and Førsund (2003), for each peer, *p*, we can define the *Referencing set* in input and output direction[7] respectively as:

$$\begin{aligned} J_1^p &= \left\{i\in J : i \neq p, \tilde{\lambda}_{ip} > 0\right\},\ \ p\in P \\ J_2^p &= \left\{i\in J : i \neq p, \tilde{\mu}_{ip} > 0\right\},\ \ p\in P \end{aligned} \tag{7}$$

*The self-evaluators*

Each of the Referencing sets in (7) may be empty, in which case the peer unit *p* is called a self-evaluator in the respective orientation:

> *Definition 1: A strongly efficient unit* $p\in P$*, is a self-evaluator in the input direction if* $J_1^P = \varnothing$, and in the output direction if $J_2^P = \varnothing$ [8].

The set of peers may thus in each orientation be partitioned into a set of self-evaluators and a set of *active peers*, i.e. peers with non-empty referencing sets. In order to simplify the notation the four partitioned sets will be denoted $S_1$, $S_2$ and $A_1$, $A_2$ ($P = S_1 \cup A_1 = S_2 \cup A_2$) for self-evaluators and active peers respectively:

$$\begin{aligned} &\text{Input partition: } S_1 = \left\{p\in P : J_1^p = \varnothing\right\},\ A_1 = \left\{p\in P : J_1^p \neq \varnothing\right\} \\ &\text{Output partition: } S_2 = \left\{p\in P : J_2^p = \varnothing\right\},\ A_2 = \left\{p\in P : J_2^p \neq \varnothing\right\} \end{aligned} \tag{8}$$

The self-evaluators are so called *not* just because they reference themselves, but because they *only* reference themselves. The vertex points for the facets that make up the DEA frontier are strongly efficient units, and each such unit may be a vertex point for many facets. Our definition of a self-evaluator implies that there are no reference points for other units on any of its facets.

## 2. Exterior and interior peers

*Enveloping from below*

The purpose of this article is to develop a method for classification of strongly efficient units into exterior or interior active peers, and into exterior or interior self-evaluators using only the standard DEA format. The production set is by construction convex. If all inefficient units are removed from the data set, and a new run is done with only the strongly efficient units, we will find the exterior peers by *reversing* the enveloping of the data from "above" to be from "below". All that needs to be done is to reverse the inequalities in the LP program (1) by *adding* the slack variables instead of subtracting, thus for each strongly efficient unit $i \in P$ :

$$\widetilde{Sl}_i = Max \left\{ \begin{array}{l} \sum_{m\in M} s^+_{mi} + \sum_{n\in N} s^-_{ni} \text{ , } s.t. \\ \sum_{j\in P} \omega_{ij} y_{mj} - y_{mi} + s^+_{mi} = 0, \\ x_{ni} - \sum_{j\in P} \omega_{ij} x_{nj} + s^-_{ni} = 0, \\ \sum_{j\in P} \omega_{ij} = 1 \\ s^+_{mi}, s^-_{ni}, \omega_{ij} \geq 0, m \in M, n \in N \end{array} \right\} \tag{9}$$

The units that turn out as "efficient" in the solution to (9), in the sense that all slacks are zero, must be units belonging to the exterior facets[9] in the solution to the original model (1) due to convexity of the production possibility set. We will use this result to define exterior and interior strongly efficient units:

*Definition 2: A strongly efficient unit* $i \in P$ *is exterior iff* $\widetilde{Sl}_i = 0$.

This forms the basis of the partitioning of the set *P* into exterior and interior strongly efficient units:

$$\begin{aligned} E &= \left\{ i \in P : \widetilde{Sl}_i = 0 \right\} \\ I &= \left\{ i \in P : \ \widetilde{Sl}_i > 0 \right\} \end{aligned} \tag{10}$$

This partition does not depend on the orientation of the model. Combining the partition into exterior and interior strongly efficient units with the partition into self-evaluators and active peers we get several possible partitions that do depend on orientation. Often the researcher is working with a model that for some reason has a natural orientation, i.e. input-saving efficiency for cost-minimising firms, or output-increasing efficiency for a budget-constrained public service provider. In such cases one could partition the strongly efficient units into four sets for the orientation in question, i.e. the exterior or interior self-evaluators (superscript *E* and *I* respectively), and the exterior or interior active peers (superscript E and I respectively):

$$\begin{aligned} &\text{Input partition: } S_1^E = E \cap S_1,\ S_1^I = I \cap S_1,\ A_1^E = E \cap A_1,\ A_1^I = I \cap A_1. \\ &\text{Output partition: } S_2^E = E \cap S_2,\ S_2^I = I \cap S_2,\ A_2^E = E \cap A_2,\ A_2^I = I \cap A_2. \end{aligned} \tag{11}$$

It is the exterior self-evaluators in $S_1^E$ or $S_2^E$ in input or output orientation respectively that are “far out”, since they neither are peers to any other units, nor are in the interior of the input-output mixes. Each of these criteria reduces the information content[10], but in combination there is very little we can say about performance.

An exterior self-evaluator in the input direction can sometimes be an active peer in the output direction, or visa versa, though still exterior. A full partition would list the eight sets of strongly efficient units that result from using all possible combinations of the three partitionings in (8) and (10), and this would be particularly interesting in the cases where there is no natural orientation of the efficiency model. Among these, the set $S_1^E \cap S_2^E$ contains the units we know the least about. An empirical example is discussed in conjunction with table 1 below.

Figure 1 shows the two different cases of exterior and interior peers and the subdivision into

*[Figure 1: DEA taxonomy]*

active peers and self evaluators in the simplest case of two dimensions. The observations represented by points *A*, *B*, *C*, *D*, *F* and *G* are efficient, while *H* is inefficient. Considering output-orientation, the peers are *D* and *F*, and the reference point is *d*. To illustrate the referencing set of a peer, the shaded area in Figure 1 shows the *referencing zone* for the efficient unit *D* in the case of output orientation. All the inefficient units being in unit *D*'s referencing set must be located here (such inefficient units may also appear in referencing sets of other peers; e.g. unit *F*'s). If no observations are located in the referencing zone of a peer then this peer is a self-evaluator. Removal of such a self-evaluator will not change the efficiency scores for any other units. We would expect the self-evaluators to be extreme points in one or more of the mix or scale dimensions, but if the referencing zone is narrow a self-evaluator may also be interior, i.e. centrally placed within the set of observations. A narrow zone means that other peers are close to the self-evaluator.

We see that the classification of peers is dependent of the orientation of the efficiency measure. In the output direction, we have that both units *B* and *C* are interior self-evaluators, while units *A* and *G* are exterior self-evaluators, and units *D* and *F* are interior active peers.

Considering input orientation, the radial reference or projection point for unit *H* is *a*. The reference point defined by (5) coincides with the peer *A* that is the only active peer, and in addition it is exterior. We have that the units *B*, *C*, *D* and *F* are interior self-evaluators, while *G* is an exterior one. In both cases of orientation the unit *G* could have been observed anywhere between the line g' (the continuation of the line *DF*) and the line g'' (referenced by *F*), without any unit changing its estimated efficiency or its status as peer. The efficiency score of 1 assigned to unit *G* therefore contains little information. In e.g. the output oriented case we see that there is a considerable scope for output variation for a given input yielding the efficiency score of 1.

Illustrating the enveloping from below using Figure 1, we have that the new "from below frontier" will be the line from *A* to *G*, thus these units are the only ones on the "from below frontier" and therefore exterior points in *E*. This classification is independent of orientation, and they are both located on exterior facets in the original problem (1). In the case of output orientation, the self-evaluators *B* and *C*, according to the solution to problem (4), will not appear on the new frontier, and they are therefore interior according to Definition 3. The self-evaluators *A* and *G* appear on the new frontier and are therefore exterior. In the case of input orientation solving problem (4) gives *B, C, D, F* and *G* as self-evaluators, and we have that *B, C, D,* and *F* are interior self-evaluators and *G* an exterior one. While *A* is an exterior peer in input orientation, it is not a self-evaluator. In the case of CRS, facets without any reference points may also be found in the interior of the frontier surface with respect to mixes, while for VRS interior also means interior regarding scale.

Figure 2 provides another type of illustration. In a two-dimensional input space an isoquant is

*[Figure 2. Reversing the DEA program]*

shown formed by the efficient units *A, B, C* and *D*. Consider input orientation and CRS. Assuming inefficient units are only located northeast of the isoquant segment *AB* in the cone delimited by the rays going through the points *A* and *B,* we have that *C* and *D* are self-evaluators, and *A* and *B* are active peers. Running the "reverse" program (9) we will envelope the four peers from "behind" by the broken line from *A* to *D*. We then know that units *A* and *D* are exterior peers, and using the information from running the DEA model (1) we then have that unit *C* is an interior self evaluator, and unit *D* is an exterior self evaluator.

The difference between the Tulkens (1993) concept of efficient by default within the FDH-approach and the self-evaluator concept within DEA can be illustrated by considering unit C on the frontier. This unit is dominating the inefficient unit F, and thus is not efficient by default, but it is an interior self-evaluator within our DEA approach.[11]

Olesen and Petersen (1996) provide indicators of *ill-conditioned* data sets concerning using the shadow prices on output- and input constraints in the linear programming problems calculating the efficiency scores in (3) or (4) to measure substitution characteristics. For exterior facets one or more of these shadow prices will be zero, thus reducing the possibility of measuring substitution. Data sets are termed ill conditioned if a relatively large number of units are located in an area where exterior facets are used as references in the efficiency evaluation. An exterior peer must be a vertex of at least one exterior facet. The proportion of exterior peers can therefore be used as an indicator of the data set in this respect; the higher the proportion the more ill-conditioned is the data set.

*Identifying the exterior peers*

The program (9) is not the standard DEA additive formulation, since the sign of the slacks in the restrictions on inputs and outputs have been changed. However, by negating these equalities, (9) can be rewritten as:

$$\widetilde{Sl}_i = Max\left\{\begin{array}{l}\sum_{m\in M} s_{mi}^{+} + \sum_{n\in N} s_{ni}^{-}\ ,\ s.t. \\ \sum_{j\in P}\omega_{ij}x_{nj} - x_{ni} - s_{ni}^{-} = 0, \\ y_{mi} - \sum_{j\in P}\omega_{ij}y_{mj} - s_{mi}^{+} = 0, \\ \sum_{j\in P}\omega_{ij} = 1 \\ s_{mi}^{+}, s_{ni}^{-}, \omega_{ij} \geq 0, m\in M, n\in N\end{array}\right\} \tag{12}$$

Comparing (1) and (12) we see that these are identical except that inputs and outputs are exchanged. Since existing DEA software often will solve the additive model (1), we may as well for convenience find the set of exterior peers *E* by exchanging inputs and outputs and running (12) on the strongly efficient units, rather than running (9) on these units.

## 3. An empirical application

*The data*

We apply the method for determining interior and exterior peers and self-evaluators on a cross section data set of the nursing and home care sector of Norwegian municipalities with 469 municipalities as units. The data is found in Edvardsen et al. (2000). The model was first formulated in Erlandsen and Førsund (2002) based on expert advice on how to define outputs and inputs using available statistics. Resource usage is measured by financial data and number of man-years of different categories. Production data contains mainly the number of clients dealt with by institutionalised nursing, home based nursing, and practical assistance. Quality information is lacking, but the clients are split into some age groups that may be of significance for resource use.[12]

There are three inputs in the model; *Trained nurses*, *Other Employees* (both measured in man-years), and *Other expenses* (measured in 1000 NOK (Norwegian crowns)). There are 10 outputs: *Institutions age 0-66* and *Institutions age 67+* are the number of institutionalised clients in the age groups 0-66 and above 67 respectively. *Short-term stay* shows how many visits the institutions in the municipality have gotten from clients who are not residents, while *Closed ward* shows how many of the residents are in a special ward for dementia clients. *Single person room* is the number of rooms with single occupancy, and represents a quality variable. *Mentally disabled* shows how many of the clients are mentally disabled (almost all of these clients get home care). *Practical assistance 0-66* and *Practical assistance 67+* counts how many clients get practical assistance (such as cleaning and making food) in the indicated age groups, while *Home based nursing 0-66* and *Home based nursing 67+* count the same for clients getting nursing services in their own homes.

The number of variables is rather extensive compared with other DEA studies of the sector, and may contribute to the relative high incidence of self-evaluators found.

*The taxonomy of peers*

Running the output-oriented LP model (4) assuming variable returns to scale resulted in 129 of the 469 municipalities being classified as strongly efficient. Both large and small units are found among efficient units. The average efficiency is 86 percent, while the efficiency of the (arithmetic) average unit is 67 percent.

An overview of the taxonomy developed in Sections 2 and 3 for classification of units is given in Figure 3, together with the actual decomposition for the data set at hand. The figure will be read from left to right. In view of

*[Figure 3: The taxonomy of units in DEA efficiency analyses]*

the relatively large number of observations (469) it may be surprising that as many as 28 percent of the units are efficient (set *P*). This may be due to the unusually high number of dimensions, 13 variables in all. Since the efficient units span out the frontier technology it is to be expected that the number of exterior ones (set *E*) is higher than the interior ones (set *I*), 75 and 25 percent respectively. Turning to the Farrell efficiency model (4) for output-orientation at the right-hand top of the figure, the self-evaluators (set $S_2$) represent 22 percent of the efficient units[13]. As expected the relative share of exterior peers (set $S_2^E$) is larger in the group of self-evaluators than in the group of active peers, 86 versus 72 percent. The number of interior self-evaluators (set $S_2^I$) is rather small. Among the active peers the share of interior units (set $A_2^I$) is much higher, 28 percent, while the share of active exterior peers (set $A_2^E$) is still as high as 72 percent. These distributions are of importance for the empirical support of the frontier and the associated efficiency distribution. A rather similar distribution of the different categories is found for input-orientation in the lower right-hand corner of the figure.

In order to see the details of the impact of the two orientations on our classifications a detailed break-down is offered in Table 1. (The grey cells cannot contain any numbers by definition.)

*[Table 1. Impact of orientation on classification]*

The numbers in the column- and row sums in the table correspond to the right-hand column of boxes in Figure 3. We see that elements on the main diagonal dominates, i.e. if a unit is classified as an exterior or interior self-evaluator within one orientation, or an active exterior or interior peer, it tends to be so also within the other orientation. However, we have some numbers in the off-diagonal boxes, e.g. six units that are classified as exterior self-evaluators within input orientation are classified as active exterior peers within output orientation.

*Far out or alone in the crowd*

The location of the interior and exterior self evaluators can be measured by the relative distance from the total sample average unit.[14] The interior self-evaluators in our application are on both sides of the average, and one of the four units is quite close to the sample average. While the exterior self-evaluators are mostly either very large or very small, the interior self-evaluators are quite different and tend to be mid-sized on most variables. It seems appropriate to use the expression “alone in the crowd.”

The 25 exterior self-evaluators measured in output orientation are distributed with half above and half below the sample average. One unit has maximal sample values for two of the variables. There are several output variables with zero as the lower limit. The variable *Institutions age 0-66* has seven exterior units with the minimum value of zero; while for *Closed ward* there are eight exterior units with the minimum value of zero. So given that “far out” means both small and large units the exterior units deserve well this classification. The influence of extreme mixes may also be investigated, but due to all the possible comparison we leave this exercise out.

## 4. Conclusions

The units found strongly efficient in DEA studies can be divided into *self-evaluators* and *active peers*, depending on whether the peers are referencing any inefficient units or not. The contribution of the paper starts with subdividing the peers into *interior* and *exterior* ones. The exterior self-evaluators are efficient "by default"; there is no firm evidence from observations for the classification. If the set of efficient units is used for identifying "winners", or efficiency scores are used within a yardstick competition scheme, then exterior self evaluators should be excluded. Self-evaluators may most naturally appear at the "edges" of the technology, but it is also possible that self-evaluators appear in the interior. Finding the influence of some variables on the level of efficiency by running regressions of efficiency scores on a set of potential explanatory variables is an approach often followed in actual investigations. Using exterior self-evaluators with efficiency score of 1 in such a "two-stage" procedure may then distort the results, because to assign the value of 1 to these self-evaluators is arbitrary. Interior self-evaluators, on the other hand, are more likely to be fairly similar to active peers since they have neighbours in more directions. They should then not be dropped when applying the two- stage approach.[15]

A method for classifying self-evaluators based on the additive DEA model, either CRS or VRS, has been developed. The exterior strongly efficient units are found by running the enveloping procedure "from below", i.e. reversing the signs of the slack variables in the additive model (1), after removing all the inefficient units from the data set. Which of the strongly efficient units from the additive model (1) that turn out to be self-evaluators or active peers, will depend on the orientation of the efficiency analysis, i.e. whether input-or output orientation is adopted. The classification into exterior and interior peers is determined by the strongly efficient units zero or positive slacks in the "reversed" additive model (9) or the equivalent model (12).

The empirical application showed that the majority of self evaluators were exterior ones, implying that one should be careful using that part of the DEA frontier as representing the estimate of technology. But also the majority of active peers were exterior, giving rise to some concern about the quality of the estimated frontier since exterior peers are associated with at least one exterior facet that have at least one shadow price on an input-or output constraint that is not non-zero and finite. Scale- and substitution properties will then not exists for all variable combinations. As pointed out in Olesen and Petersen (1996, p.206); "insufficient variation in data may imply estimation of a frontier actually located in subspaces of lower dimension than the input output space." The result may be taken as an indication of the necessity of investigating the frontier estimate further by e.g. applying bootstrap techniques (Simar and Wilson, 1998). It will be an interesting task for future research if the proposed peer taxonomy could be related to bootstrap results for the size of confidence intervals for efficiency scores. Preliminary bootstrap runs on the dataset indicate that the efficiency estimates of exterior peers tend to have larger standard errors than the estimates for the interior peers.

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**Figures:**



**Tables:**

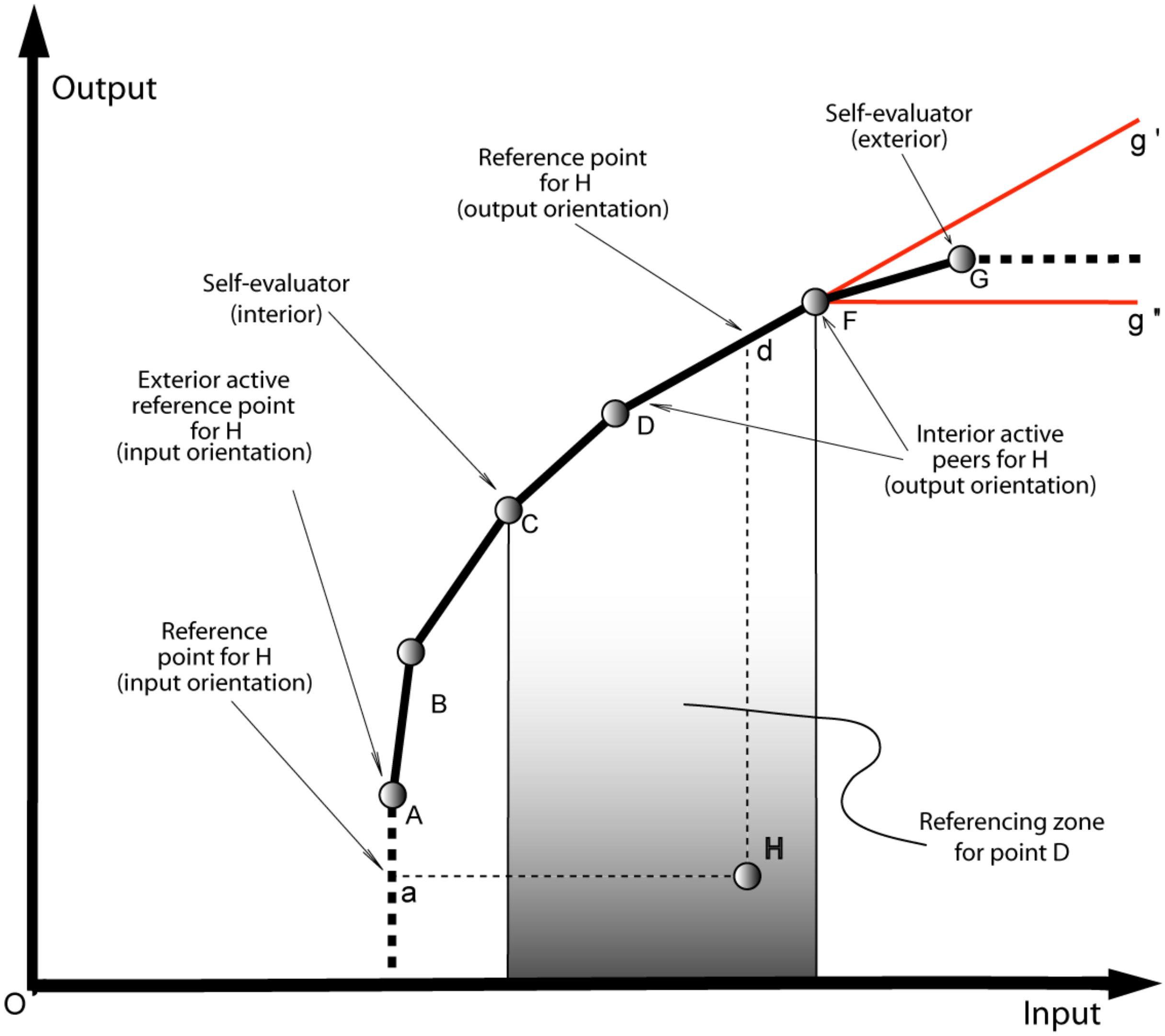


*Figure 1. DEA taxonomy*

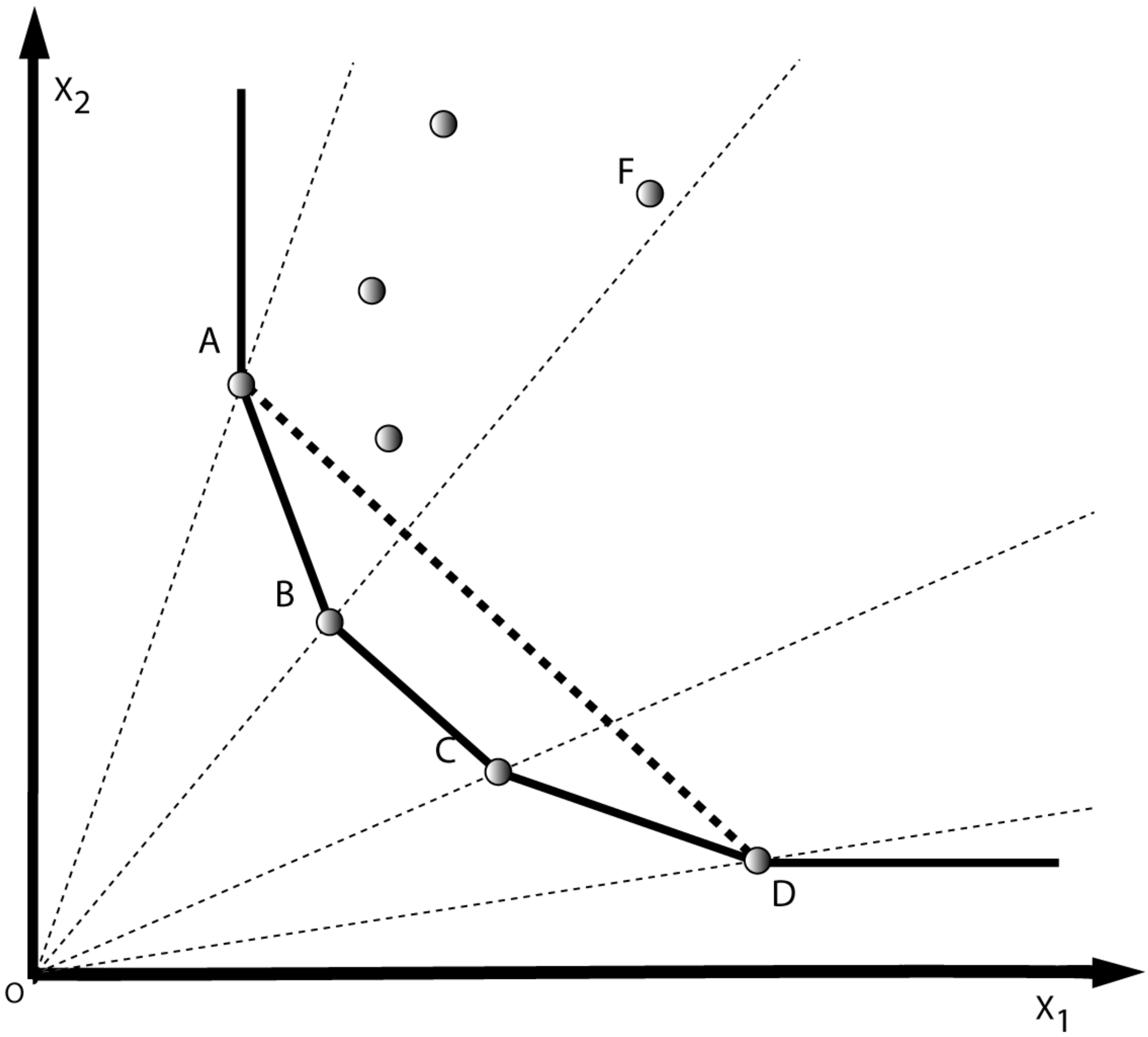


*Figure 2. Reversing the DEA program*

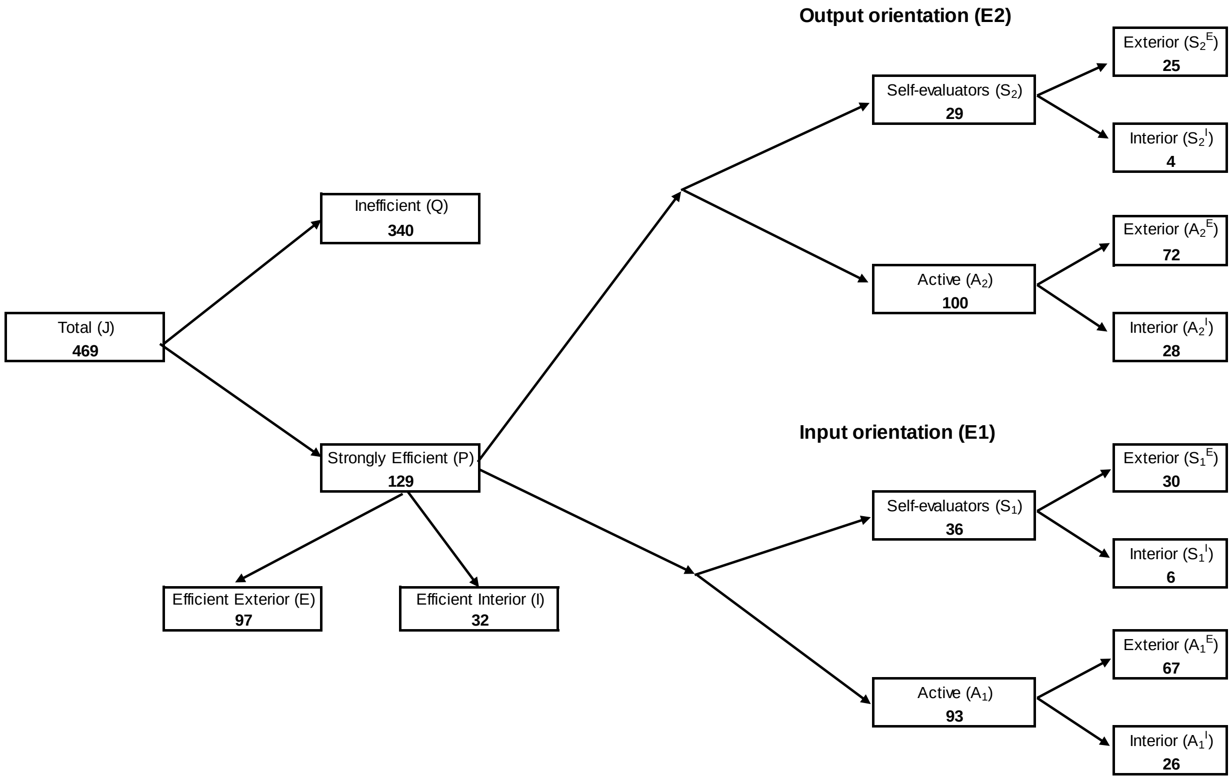


*Figure 3: The taxonomy of units in DEA efficiency analyses*

*Table 1. Impact of orientation on classification*

| | | Output orientation | | | | |
|---|---|---|---|---|---|---|
| | | $S_2^E$ | $S_2^I$ | $A_2^E$ | $A_2^I$ | ***Sum:*** |
| **Input orientation** | $S_1^E$ | **24** | | **6** | | *30* |
| | $S_1^I$ | | **4** | | **2** | *6* |
| | $A_1^E$ | **1** | | **66** | | *67* |
| | $A_1^I$ | | **-** | | **26** | *26* |
| | ***Sum:*** | *25* | *4* | *72* | *28* | *129* |

## End-Notes

[*] The paper is based on results from the Frisch Centre project “Better and Cheaper?” and written within the project “Efficiency analyses of the nursing and home care sector of Norway” at the Health Economics Research Programme at the University of Oslo (HERO) and the Frisch Centre. Both projects are financed by the Norwegian research council (NFR). We thank the editor and two anonymous referees for valuable comments and very constructive suggestions.

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[1] As far as we know such a formalization is not found in the literature. Charnes et al. (1985a, p.110) defines a self-evaluator as “A DMU which appears only in its own evaluation facet…”

[2] In Charnes et al. (1986) the set of all DMUs is partitioned into six sets, but neither active peers and self-evaluators, nor external and internal self-evaluators are covered.

[3] Charnes et al. (1985a, p.110) seemed to have only exterior self-evaluators in mind since the reason offered for a DMU being a self-evaluator was that it is “…being characterized as efficient only because of specialized properties which set it off from other DMUs.”

[4] The approach was originally introduced in Seitz (1967), inspired by Nerlove (1965); see Førsund and Sarafoglou (2002). Simar and Wilson (2007) review the approach and find it at fault in general due to serial correlation between the efficiency scores, and provide a new statistically sound procedure based on specifying explicitly the data generating process and bootstrapping to obtain confidence intervals**.**

[5] These units are strongly rather than weekly efficient since it is not possible to decrease (increase) any input (output) without changing the level of at least one other variable. This parallels the concept of Pareto efficiency, although Pareto efficiency in economics is used for comparisons of the utilities of individuals when evaluating an allocation of a good.

[6] Because our interest is in the partitioning of the set of strongly efficient units $P$, the summations of (3) and (4) use this set, instead of using $J$ which is more common in the literature. This also improves the computational efficiency, since the membership of $P$ is already determined in (2).

[7] As for the efficiency measures, we follow the Farrell tradition of using subscript 1 to denote the input direction and subscript 2 to denote the output direction.

[8] An alternate definition could be in terms of the reference shares defined in Torgersen, Førsund and Kittelsen (1996), where a self-evaluator has a reference share of zero.

[9] By an exterior facet we mean a facet with at least one shadow price that is not non-zero and finite. In two-dimensions this corresponds to the vertical or horizontal segments of the frontier in diagrams such as figures 1 and 2.

[10] As one referee rightly points out, an exterior unit can be light-years away from other units, but still be an active peer. This should however make it eligible for an outlier analysis, i.e. a screening of units that should enter the DEA model. In our article we are concerned with how much we can say about the efficiency of units that are presumed to be observed without error, which could be viewed as a screening of units before any use of the DEA-estimated efficiencies, e.g. in a second-stage regression analysis.

[11] One could extend the concepts of self-evaluators to the FDH setting. In the minimax approach of FDH, each inefficient unit such as F is referenced by one (though not necessarily unique) FDH peer, which in figure 2 would be unit B. The FDH referencing set corresponding to (7) would still be empty for unit C, making it a self-evaluator also in the FDH method, even though it is not efficient by default. Since we in this article are interested in the information content of the efficiency measures under a convexity assumption, it is the DEA classification which is relevant here.

[12] To ensure that the data quality was good enough extensive quality control was performed. We strongly feel that one should not automatically remove outliers, but if possible contact the municipality in question and ask if the data is correct. This is especially important if the methodology is frontier based (such as DEA) because the units defining the frontier are outliers by definition. This led to many changes in the dataset and required quite a lot of work, but as a result we could be much more confident in the quality of the data (see Aas (2000) for details).

[13] In the empirical application the the multipliers in (3) and (4) are unlikely to be non-unique, and if they are this will have only a minor influence on the results presented. We have therefore used these original weights rather than the maximal weights from (5) and (6) in computing the partitions into self-evaluators and active peers.

[14] A detailed analysis is found in Edvardsen et al. (2003).

[15] A simple regression analysis carried out in Edvardsen et al. (2003) shows that removing the 25 exterior self evaluators increases the multiple correlation coefficient and the significance level of some key explanatory variables.