European Economic Review 33 (1989) 825–838. North-Holland

ECONOMIC PERFORMANCE OF FLEXIBLE FUNCTIONAL FORMS

A Correction and Comment

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Received April 1987

In a recent article in the *European Economic Review* Kostas A. Despotakis (1986) examines the properties of flexible functional forms, applying the concept of the inner domain. In the numerical experiments he unfortunately uses an incorrect form of the transcendental logarithmic function. When the correct functional form is used his experimental results are always weakened and sometimes invalidated. In particular, the differences between the considered flexible functional forms are smaller than his experiments would suggest, while the importance of differences in the elasticity of substitution is strengthened.

1. Introduction

In a recent article in the *European Economic Review*, Despotakis (1986) examines the local properties of flexible functional forms (FFF) used as cost functions to represent real-world production technology in economic models. He defines the 'economic effects at a point' (EEP) as the total cost, the factor value shares and the Allen–Uzawa elasticities of substitution at a point given by a set of factor prices and an output level. By examining the EEP generated by each FFF under consideration he seeks to discover the extent of the 'inner domain', defined as 'the sub-region of the outer domain over which the function is a "good approximation" of "true" technology' (p. 1109).

After a general introduction and theoretical discussion of the relationship between EEPs and the cost function in sections 1 and 2, Despotakis proceeds in section 3.1 to analyze the local properties of the general cost function and three specific functional forms: (a) constant elasticity of substitution (CES) [Uzawa (1962)], (b) generalized Leontief (GL) [Diewert (1971)] and (c) transcendental logarithmic or translog (TL) [Christensen et al. (1971, 1973)]. Confining himself to constant returns to scale (CRS) functions, in section 3.2 he designs experiments which are carried through for two-input CES, GL and TL functions in section 3.3, and for three-input GL and TL functions in

*I am grateful to Tor Jakob Klette and Petter Frenger for useful comments.

0014–2921/89/$3.50

section 3.4. After the concluding section 4 Despotakis includes 4 appendices: The first three (A–C) give the detailed formulae for the TL, GL and CES cost functions and their relationships to EEPs, while Appendix D contains additional tables for the three-input experiments of section 3.4.

I wish to emphasize that I have no major reservations about the theoretical sections 1 to 3.2 and the mathematical formulae of Appendices A to C. On the contrary, I find these parts of the article both interesting and illuminating, and totally agree with Despotakis in his emphasis on the need to study the properties of relevant functional forms, both generally and in the building and use of specific economic models. Unfortunately, Despotakis in the numerical experiments uses an incorrect form of the TL function which severely affects his results.

2. The Error

In Appendix A.2 Despotakis correctly states that the translog cost function has the following general form:[1]

$$\ln C(P,Y) = a_0 + a_Y \ln Y + b_{YY}(\ln Y)^2 + \sum_i a_i \ln P_i$$

$$+ 1/2 \sum_i \sum_j b_{ij} \ln P_i \ln P_j + \sum_i b_{Yi} \ln Y \ln P_i \tag{1}$$

where

$$\sum_i a_i = 1;\ \sum_i b_{Yi} = 0;\ \sum_i b_{ij} = \sum_j b_{ij} = 0;\ b_{ij} = b_{ji},\ i \neq j.$$

and $\ln X$ is the natural logarithm of X. The independent EEP are in the CRS case given by total cost and

$$s_i = a_i + \sum_j b_{ij} \ln P_j + b_{Yi} \ln Y, \qquad \sum_i s_i = 1, \tag{2}$$

$$\sigma_{ij} = (b_{ij} + s_i s_j)/s_i s_j, \qquad i \neq j. \tag{3}$$

The problem is that the results that Despotakis gives for the TL function in his tables 2 to 4 and Appendix D are not reproducible using the formulation (1)–(3). Instead his tables can be exactly reproduced if one in (1) and (2) replaces the natural logarithm $\ln X \equiv \log_e X$ (to the base e), with the common or Briggs logarithm $\lg X \equiv \log_{10} X$ (to the base 10):

[1]The notation follows Despotakis (1986): P is a vector of input prices, Y is total output, $C(P, Y)$ is the cost function, C_i and C_{ij} are its first and second derivatives with respect to P_i and P_j, $s_i \equiv (P_i C_i)/C$ is the value share of input i and $\sigma_{ij} \equiv (C_{ij}C)/(C_i C_j)$ is the Allen–Uzawa elasticity of substitution (AUES) between input i and j. a and b are parameters in the cost function.

$$\lg C(P,Y)=a_0+a_Y \lg Y+b_{YY}(\lg Y)^2+\sum_i a_i \lg P_i$$

$$+1/2\sum_i\sum_j b_{ij}\lg P_i \lg P_j+\sum_i b_{Yi}\lg Y \lg P_i, \tag{1a}$$

where

$$\sum_i a_i=1;\ \sum_i b_{Yi}=0;\ \sum_i b_{ij}=\sum_j b_{ij}=0;\ b_{ij}=b_{ji}, \qquad i\neq j.$$

$$s_i=a_i+\sum_j b_{ij}\lg P_j+b_{Yi}\lg Y. \tag{2a}$$

The function (1a) is not the translog function suggested by Christensen, Jorgenson and Lau (1971). It is still a perfectly valid logarithmic approximation to the true production technology, and therefore a valid FFF even if it is not the original translog. The corresponding value shares are indeed given by (2a), but the AUES are not given by (3), but rather by

$$\sigma_{ij}=(b_{ij}+s_i s_j \ln 10)/s_i s_j \ln 10 \tag{3a}$$

The difference between (3) and (3a) stems from the fact that while the derivative of $\ln X$ is $1/X$, the derivative of $\lg X$ is $1/(X\ln 10)$.[2] Naming the system (1a), (2a) and (3a) the Briggs translog (BTL), one can show that the TL and BTL are different parametrisations of identical cost functions.[3]

Despotakis, on the other hand, incorrectly uses (3) for calculating the AUES in conjunction with the BTL function (1a) and its value shares given by (2a). This mixture of equations from TL and BTL does not give a valid representation of a cost function, and I will nickname it Despotakis' translog (DTL). Put another way, Despotakis in his experiments reports a wrong AUES for a valid BTL cost function calibrated for a different base-point AUES than reported. The need for recomputation of the TL results in his experiments is obvious.

3. The corrected results

The experiments documented here are identical to the ones Despotakis probably intended to carry out. Like him, I will only consider constant returns to scale (CRS) functions. The differences between TL and DTL are such that the experimental section of Despotakis' paper is severely mislead-

[2]See for example Chiang (1974, p. 309). While the constant ln 10 cancels out in the derivation of the expression for the value shares (2a), it does not disappear from (3a).

[3]Reformulating the equations for the TL function in Appendix A.2 of Despotakis to fit the BTL function, one can express the parameters of one function in terms of the parameters of the other (they are not generally equal), and thereby show that (1a) is in fact just (1) divided on both sides by (ln 10).

ing. While a detailed discussion of results lies outside the scope of this comment, I will for both the two-input and three-input experiments comment briefly on what I perceive to be Despotakis' main observations, and add a point or two of my own.

3.1. Experiments with two-input functions

For the two-input case the GL, TL and CES cost functions are calibrated for a common base-point set of EEPs (c^*, s^* and σ^*), with cross-AUES ranging from 0.1 to 2.0. The resulting EEPs are for each function evaluated at relative prices ranging from 0.2 to 5.0 times the base-point value. The results are documented in table 2 for the case with equal base-point value shares ($s_1^*=s_2^*=0.5$) and in table 3 for unequal value shares ($s_1^*=0.2$, $s_2^*=0.8$).[4]

3.1.1. Despotakis' results

Comparing the EEPs of the correct TL function with the DTL results tabulated in Despotakis, the main observation is that the input demands and value shares of the TL function generally lie closer to the CES function than the DTL does, while the AUES diverges more for the TL than the DTL. Two of Despotakis' conclusions need further commenting:

(a) Despotakis states that 'Reallocation of resources generated by TL are noticeably less sensitive to values of elasticity of substitution at the starting point' (p. 1115). His conclusion is based on the responsiveness of input demand c_i as reported in the table on page 1120, which I have reproduced below with the correct results for TL added:[5]

Function responsiveness: Sensitivity to values, of σ_{12}^*.

	$s_1^*=0.5$		$s_1^*=0.2$			
$P_1/P_2=0.5$	$c_1/c_1^*(\%)$		$c_1/c_1^*(\%)$		$c_2/c_2^*(\%)$	
σ_{12}^*	0.1	2.0	0.1	2.0	0.1	2.0
GL	104.1	182.7	106.7	232.4	98.8	76.6
DTL	125.2	158.5	138.4	212.5	93.2	80.5
TL	102.7	179.3	90.3	260.5	101.4	72.2
CES	104.1	178.0	105.8	277.4	98.9	69.5

While the DTL 'ranges are considerably narrower and responsiveness accordingly more insensitive to values of σ_{12}^*' (p. 1120), the correct TL

[4]To ease comparison with Despotakis (1986) the tables are given the same numbering and appearance as in his article. His Table 1 is correct and therefore not reproduced here.
One of the weaknesses of retaining the table layout is the misleading use of relative price in the headings of Tables 2 and 3; while the AUES, value shares and input demands are functions only of relative prices, unit cost also depends on the price level. The results reported are those that appear when P_1 is varied while P_2 is kept constant.

[5]Computer rounding errors may cause differences in the last digit.

Table 2

Economic performance of two-input CRS cost functions; base-point value shares $s_1^* = s_2^* = 0.5$[a].
Correct TL function

	Unit cost: c					Cross AUES: σ_{12}					Value share: s_1					Input demand response (%)									
																c_1/c_1^*					c_2/c_2^*				
P_1/P_2	0.2	0.5	1.0	2.0	5.0	0.2	0.5	1.0	2.0	5.0	0.2	0.5	1.0	2.0	5.0	0.2	0.5	1.0	2.0	5.0	0.2	0.5	1.0	2.0	5.0
GL	58.5	74.6	100.0	149.1	292.4	0.123	0.104	0.100	0.104	0.123	0.192	0.349	0.500	0.651	0.808	112.4	104.1	100.0	97.1	94.5	94.5	97.1	100.0	104.1	112.4
TL	59.9	74.6	100.0	149.3	299.3	−0.893	0.003	0.100	0.003	−0.893	0.138	0.344	0.500	0.656	0.862	82.5	102.7	100.0	97.9	103.2	103.2	97.9	100.0	102.7	82.5
CES	58.5	74.6	100.0	149.1	292.6			0.100			0.190	0.349	0.500	0.651	0.810	111.3	104.1	100.0	97.1	94.8	94.8	97.1	100.0	104.1	111.3
GL	53.9	73.3	100.0	146.6	269.4	0.414	0.403	0.400	0.403	0.414	0.277	0.398	0.500	0.602	0.723	149.4	116.6	100.0	88.3	77.9	77.9	88.3	100.0	116.6	149.4
TL	54.3	73.3	100.0	146.6	271.6	0.218	0.373	0.400	0.373	0.218	0.259	0.396	0.500	0.604	0.741	140.4	116.1	100.0	88.5	80.5	80.5	88.5	100.0	116.1	140.4
CES	53.9	73.3	100.0	146.6	269.6			0.400			0.276	0.398	0.500	0.602	0.724	148.7	116.5	100.0	88.3	78.1	78.1	88.3	100.0	116.5	148.7
GL	52.4	72.9	100.0	145.7	261.8	0.500	0.500	0.500	0.500	0.500	0.309	0.414	0.500	0.586	0.691	161.8	120.7	100.0	85.4	72.4	72.4	85.4	100.0	120.7	161.8
TL	52.6	72.9	100.0	145.7	262.9	0.403	0.485	0.500	0.485	0.403	0.299	0.413	0.500	0.587	0.701	157.1	120.5	100.0	85.5	73.7	73.7	85.5	100.0	120.5	157.1
CES	52.4	72.9	100.0	145.7	261.8			0.500			0.309	0.414	0.500	0.586	0.691	161.8	120.7	100.0	85.4	72.4	72.4	85.4	100.0	120.7	161.8
GL	50.8	72.4	100.0	144.9	254.2	0.586	0.597	0.600	0.597	0.586	0.343	0.431	0.500	0.569	0.657	174.2	124.9	100.0	82.4	66.8	66.8	82.4	100.0	124.9	174.2
TL	50.9	72.4	100.0	144.9	254.5	0.554	0.592	0.600	0.592	0.554	0.339	0.431	0.500	0.569	0.661	172.6	124.8	100.0	82.5	67.3	67.3	82.5	100.0	124.8	172.6
CES	50.8	72.4	100.0	144.8	254.0			0.600			0.344	0.431	0.500	0.569	0.656	174.9	124.9	100.0	82.4	66.6	66.6	82.4	100.0	124.9	174.9
GL	48.5	71.8	100.0	143.6	242.7	0.722	0.744	0.750	0.744	0.722	0.397	0.456	0.500	0.544	0.603	192.7	131.1	100.0	78.0	58.5	58.5	78.0	100.0	131.1	192.7
TL	48.5	71.8	100.0	143.6	242.5	0.739	0.748	0.750	0.748	0.739	0.399	0.457	0.500	0.543	0.601	193.7	131.1	100.0	78.0	58.2	58.2	78.0	100.0	131.1	193.7
CES	48.5	71.8	100.0	143.6	242.3			0.750			0.401	0.457	0.500	0.543	0.599	194.2	131.2	100.0	78.0	58.1	58.1	78.0	100.0	131.2	194.2
GL	46.2	71.1	100.0	142.3	231.2	0.877	0.896	0.900	0.896	0.877	0.457	0.482	0.500	0.518	0.543	211.2	137.3	100.0	73.6	50.2	50.2	73.6	100.0	137.3	211.2
TL	46.2	71.1	100.0	142.3	231.0	0.899	0.900	0.900	0.900	0.899	0.460	0.483	0.500	0.517	0.540	212.4	137.3	100.0	73.6	49.9	49.9	73.6	100.0	137.3	212.4
CES	46.2	71.1	100.0	142.3	231.0			0.900			0.460	0.483	0.500	0.517	0.540	212.4	137.3	100.0	73.6	49.9	49.9	73.6	100.0	137.3	212.4
GL	44.7	70.7	100.0	141.4	223.6	1.000	1.000	1.000	1.000	1.000	0.500	0.500	0.500	0.500	0.500	223.6	141.4	100.0	70.7	44.7	44.7	70.7	100.0	141.4	223.6
TL	44.7	70.7	100.0	141.4	223.6	1.000	1.000	1.000	1.000	1.000	0.500	0.500	0.500	0.500	0.500	223.6	141.4	100.0	70.7	44.7	44.7	70.7	100.0	141.4	223.6
CES	44.7	70.7	100.0	141.4	223.6			1.000			0.500	0.500	0.500	0.500	0.500	223.6	141.4	100.0	70.7	44.7	44.7	70.7	100.0	141.4	223.6
GL	43.2	70.3	100.0	140.6	216.0	1.149	1.108	1.100	1.108	1.149	0.546	0.518	0.500	0.482	0.454	236.0	145.6	100.0	67.8	39.2	39.2	67.8	100.0	145.6	236.0
TL	43.3	70.3	100.0	140.6	216.5	1.101	1.100	1.100	1.100	1.101	0.540	0.517	0.500	0.483	0.460	233.9	145.4	100.0	67.9	39.8	39.8	67.9	100.0	145.4	233.9
CES	43.3	70.3	100.0	140.6	216.5			1.100			0.540	0.517	0.500	0.483	0.460	233.9	145.4	100.0	67.9	39.8	39.8	67.9	100.0	145.4	233.9
GL	37.1	68.6	100.0	137.1	185.4	2.551	1.600	1.500	1.600	2.551	0.770	0.591	0.500	0.409	0.230	285.4	162.1	100.0	56.1	17.1	17.1	56.1	100.0	162.1	285.4
TL	38.0	68.6	100.0	137.2	190.2	1.597	1.515	1.500	1.515	1.597	0.701	0.587	0.500	0.413	0.299	266.7	161.0	100.0	56.7	22.7	22.7	56.7	100.0	161.0	266.7
CES	38.2	68.6	100.0	137.3	191.0			1.500			0.691	0.586	0.500	0.414	0.309	263.9	160.8	100.0	56.9	23.6	23.6	56.9	100.0	160.8	263.9
GL	29.4	66.4	100.0	132.8	147.2	−3.592	2.481	2.000	2.481	−3.592	1.179	0.688	0.500	0.312	−0.179	347.2	182.8	100.0	41.4	−10.6	−10.6	41.4	100.0	182.8	347.2
TL	32.4	66.6	100.0	133.2	161.8	3.837	2.137	2.000	2.137	3.837	0.902	0.673	0.500	0.327	0.098	291.9	179.3	100.0	43.5	6.3	6.3	43.5	100.0	179.3	291.9
CES	33.3	66.7	100.0	133.3	166.7			2.000			0.833	0.667	0.500	0.333	0.167	277.8	177.8	100.0	44.4	11.1	11.1	44.4	100.0	177.8	277.8

[a] – corresponds to negative input demand: the function is not well behaved (out of its outer domain). c_i is the first price derivative of the unit cost function, c, equal to the demand for input i per unit of output: $i = 1, 2$. c_i^* is the base-point value of c_i. P_1/P_2 is the relative price, ranging from 0.2 to 5.0 times its base-point value, P_1^*/P_2^*. CES values of σ_{12} don't change with P_1/P_2 and are therefore reported only for the base point. AUES stands for Allen–Uzawa elasticities of substitution.

Table 3

Economic performance of two-input CRS cost functions; base-point value shares $s_1^*=0.2$, $s_2^*=0.8$[a].
Correct TL function

	Unit cost: c					Cross AUES: σ_{12}					Value share: s_1					Input demand response (%) c_1/c_1^*					c_2/c_2^*				
P_1/P_2	0.2	0.5	1.0	2.0	5.0	0.2	0.5	1.0	2.0	5.0	0.2	0.5	1.0	2.0	5.0	0.2	0.5	1.0	2.0	5.0	0.2	0.5	1.0	2.0	5.0
GL	83.0	89.7	100.0	119.5	175.1	0.158	1.120	0.100	0.087	0.082	0.058	0.119	0.200	0.319	0.521	119.8	106.6	100.0	95.3	91.2	97.8	98.8	100.0	101.7	104.9
TL	87.3	90.1	100.0	118.9	166.3	5.395	−0.597	0.100	0.314	0.413	−0.032	0.100	0.200	0.300	0.432	−69.3	90.3	100.0	89.1	71.8	112.6	101.4	100.0	104.1	118.1
CES	83.1	89.7	100.0	119.4	174.6			0.100			0.055	0.118	0.200	0.318	0.516	115.3	106.0	100.0	95.0	90.0	98.2	98.9	100.0	101.8	105.7
GL	80.1	88.9	100.0	117.8	160.4	0.439	0.417	0.400	0.385	0.371	0.089	0.142	0.200	0.276	0.403	179.1	126.5	100.0	81.3	64.6	91.2	95.3	100.0	106.6	119.8
TL	82.1	89.1	100.0	117.5	156.2	−1.211	0.170	0.400	0.509	0.580	0.045	0.133	0.200	0.267	0.355	93.3	118.9	100.0	78.3	55.4	97.9	96.5	100.0	107.8	126.1
CES	80.2	88.9	100.0	117.8	159.9			0.400			0.087	0.142	0.200	0.275	0.396	174.3	125.9	100.0	80.9	63.4	91.6	95.4	100.0	106.8	120.7
GL	79.1	88.6	100.0	117.3	155.6	0.500	0.500	0.500	0.500	0.500	0.101	0.150	0.200	0.261	0.359	198.9	133.1	100.0	76.6	55.8	88.9	94.1	100.0	108.3	124.7
TL	80.4	88.7	100.0	117.1	153.0	−0.209	0.353	0.500	0.579	0.637	0.071	0.145	0.200	0.255	0.329	143.2	128.3	100.0	74.8	50.3	93.3	94.9	100.0	109.0	128.4
CES	79.1	88.6	100.0	117.3	155.6			0.500			0.101	0.150	0.200	0.261	0.359	198.9	133.1	100.0	76.6	55.8	88.9	94.1	100.0	108.3	124.7
GL	78.1	88.4	100.0	116.7	150.7	0.553	0.577	0.600	0.627	0.664	0.112	0.158	0.200	0.246	0.312	218.7	139.8	100.0	71.9	46.9	86.7	93.0	100.0	109.9	129.7
TL	78.7	88.4	100.0	116.6	149.9	0.269	0.513	0.600	0.653	0.697	0.097	0.156	0.200	0.244	0.303	190.9	137.6	100.0	71.3	45.4	88.9	93.3	100.0	110.2	130.6
CES	77.9	88.3	100.0	116.7	151.5			0.600			0.116	0.159	0.200	0.248	0.322	226.2	140.7	100.0	72.4	48.8	86.1	92.8	100.0	109.7	128.3
GL	76.7	87.9	100.0	115.9	143.3	0.621	0.683	0.750	0.843	0.042	0.130	0.170	0.200	0.224	0.235	248.3	149.7	100.0	64.9	33.7	83.4	91.2	100.0	112.4	137.1
TL	76.3	87.9	100.0	116.0	145.3	0.659	0.719	0.750	0.773	0.794	0.136	0.172	0.200	0.228	0.264	258.8	151.4	100.0	66.0	38.4	82.5	90.9	100.0	112.0	133.6
CES	76.0	87.9	100.0	116.0	145.9			0.750			0.143	0.174	0.200	0.229	0.272	272.2	152.6	100.0	66.5	39.7	81.4	90.8	100.0	111.8	132.8
GL	75.2	87.5	100.0	115.1	136.0	0.680	0.780	0.900	1.102	1.875	0.148	0.182	0.200	0.201	0.150	278.0	159.6	100.0	57.8	20.4	80.1	89.5	100.0	114.9	144.5
TL	74.0	87.4	100.0	115.3	140.9	0.889	0.896	0.900	0.904	0.908	0.174	0.189	0.200	0.211	0.226	322.3	165.1	100.0	60.9	31.8	76.4	88.6	100.0	113.7	136.3
CES	73.9	87.4	100.0	115.3	141.0			0.900			0.175	0.189	0.200	0.211	0.227	324.4	165.3	100.0	60.9	32.0	76.2	88.6	100.0	113.7	136.2
GL	74.2	87.3	100.0	114.5	131.1	0.716	0.841	1.000	1.307	3.396	0.160	0.191	0.200	0.186	0.088	297.8	166.3	100.0	53.1	11.6	77.9	88.3	100.0	116.6	149.4
TL	72.5	87.1	100.0	114.9	138.0	1.000	1.000	1.000	1.000	1.000	0.200	0.200	0.200	0.200	0.200	362.4	174.1	100.0	57.4	27.6	72.5	87.1	100.0	114.9	138.0
CES	72.5	87.1	100.0	114.9	138.0			1.000			0.200	0.200	0.200	0.200	0.200	362.4	174.1	100.0	57.4	27.6	72.5	87.1	100.0	114.9	138.0
GL	73.2	87.0	100.0	114.0	126.2	0.750	0.898	1.100	1.547	14.843	0.173	0.199	0.200	0.170	0.021	317.5	172.9	100.0	48.5	2.7	75.7	87.1	100.0	118.2	154.4
TL	71.0	86.7	100.0	114.4	135.1	1.092	1.096	1.100	1.104	1.111	0.226	0.211	0.200	0.189	0.174	400.7	183.1	100.0	54.0	23.5	68.7	85.5	100.0	116.0	139.5
CES	70.9	86.7	100.0	114.4	135.2			1.100			0.227	0.211	0.200	0.189	0.175	402.6	183.3	100.0	54.1	23.7	68.6	85.5	100.0	116.0	139.4
GL	69.3	85.9	100.0	111.8	106.7	0.877	0.108	1.500	3.196	−1.258	0.229	0.232	0.200	0.106	−0.306	396.7	199.4	100.0	29.7	−32.7	66.8	82.4	100.0	124.9	174.2
TL	65.3	85.4	100.0	112.7	124.4	1.363	1.421	1.500	1.647	2.209	0.329	0.255	0.200	0.145	0.071	537.1	218.2	100.0	40.7	8.9	54.8	79.5	100.0	120.5	144.4
CES	64.3	85.3	100.0	112.8	126.4			1.500			0.359	0.261	0.200	0.150	0.101	576.3	222.8	100.0	42.4	12.7	51.5	78.8	100.0	119.9	142.1
GL	64.4	84.5	100.0	109.0	82.2	1.043	0.342	2.000	18.457	−0.481	0.308	0.275	0.200	0.023	−0.935	495.5	232.5	100.0	6.3	−76.9	55.8	76.6	100.0	133.1	198.9
TL	58.9	83.8	100.0	110.5	112.1	1.645	1.747	2.000	2.971	−1.631	0.458	0.311	0.200	0.089	−0.058	673.8	260.5	100.0	24.6	−6.4	39.9	72.2	100.0	125.9	148.2
CES	55.6	83.3	100.0	111.1	119.0			2.000			0.556	0.333	0.200	0.111	0.048	771.6	277.8	100.0	30.9	5.7	30.9	69.4	100.0	123.5	141.7

[a]See Table 2.

exhibits ranges that are essentially of the same magnitude as for GL and CES. As the input demand c_i is a first-order property of the cost function, this closeness is as one should expect from three second-order approximations of the same underlying cost function.

(b) Despotakis observes that 'the significance of elasticity of substitution underlying a FFF at a point, for subsequent resource allocation following relative price changes, is function-specific' (p. 1115). While this is true in a strict sense, his emphasis on it stems from the experimental behaviour of the DTL function. He presents a typical example to support this view on page 1121:

Function-specific significance of σ^*_{12} for resource allocation ($\sigma^*_{12}=2.0$, $s^*_1=0.5$, $P_1/P_2=2.0$).

	σ_{12}	s_1	c_1/c^*_1 (%)	c_2/c^*_2 (%)
GL	2.48	0.312	41.4	182.7
DTL	2.02	0.425	58.5	158.5
TL	2.14	0.327	43.5	179.3
CES	2.00	0.333	44.4	178.0

While DTL is close to CES in terms of elasticity of substitution, GL is considerably closer in the other respects. Using the correct TL function this eccentric result disappears; the TL lies nicely between the GL and CES functions in all respects.

3.1.2. Further comments

The closeness of the first-order properties of the CES, GL and TL as illustrated above is not just accidental. Despotakis in section 3.1 stresses how the first- and second-order derivatives are common to the three functions, and only the derivatives of the AUES depend on function-specific third-order properties. If the functions are 'well-behaved' in the region under consideration, one would therefore expect unit cost, input demand and value shares always to differ less across FFFs than the AUES does as prices change. As (a) and (b) above show, the DTL does not conform to this.

To highlight the importance of the second-order properties of the cost function, one can in the two-input case reformulate Despotakis' expression (C.2) for the derivative of the value share with respect to own price in terms of the (single) AUES:

$$\frac{\partial s_i}{\partial P_i}=\frac{s_i(1-s_i)}{P_i}(1-\sigma). \tag{4}$$

Together with eq. (2) in Despotakis' paper, eq. (4) implies that the development of the unit cost, input demand and value shares as prices change, depend only on the function-specific development of the elasticities of substitution. This dependence itself is *not* function-specific.[6]

For the two-input case one can more specifically make an observation which is also apparent from tables 2 and 3:

(i) Assume two FFFs are calibrated to common base-point EEPs, and as prices change away from the base-point in a specified direction the AUES of one FFF is always higher than the AUES of the other. Then the function that at a point has the higher AUES will have the smaller unit demand and value share for the input whose relative price increases, and the higher unit demand and value share for the other input.

Using the correct TL function therefore simplifies the observation of the relative economic performance of the FFFs in the two-input case. As a consequence of (i) above one can focus on the development of the elasticity of substitution at prices away from the base-point. The development of the AUES for the two-input CES, GL and TL functions is illustrated graphically in fig. 1. In each of the three panels the horizontal axis shows the common base-point AUES σ^*, while the vertical axis measures the AUES at relative prices of 0.2 and 5.0 the base-point relative prices, respectively.[7]

The AUES values where the TL or GL curves coincide with the CES curve (which always lies on the diagonal) are those that Despotakis identified as 'stable'; the strongly stable value of 0.0 and the weakly stable value of 0.5 for the GL, and the strongly stable value of 1.0 for the TL. In view of this it is not surprising that the figures (and the tables) suggest another important observation:

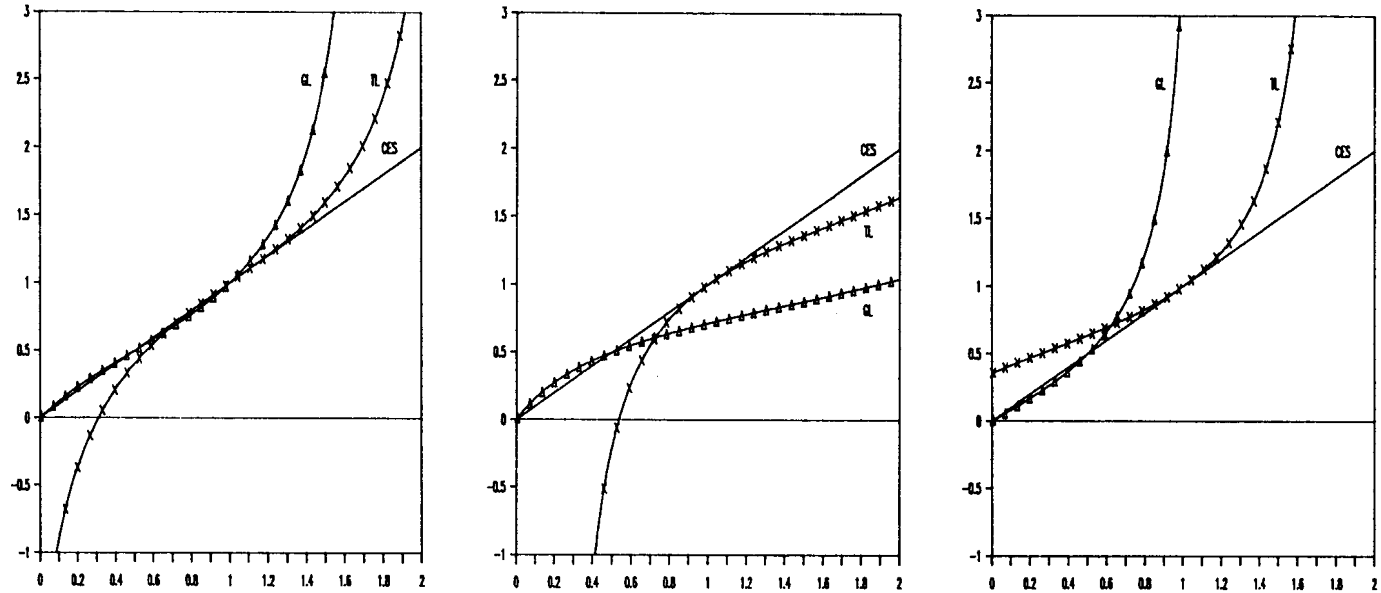
(ii) When two-input GL and TL cost functions are calibrated to common base-point EEPs, the GL AUES will be more stable as prices change for base-point AUES less than a point between 0.5 and 1.0,[8] and the TL AUES will be at least as stable for base-point AUES larger than this.

It is worth noting that the equal value shares of panel (a) increases stability (especially for the GL), and that the GL function acquires an extra stable point at 1.0. While the stability of the elasticity of substitution is not necessarily a preferred property of a cost function, it does extend the outer (mathematical) domain, and increases the transparency of function behaviour.

[6]This is just another way of saying that knowledge of the derivative of a function over a range, combined with a boundary condition, is sufficient to use integration to find the value of the function over the same range. There is no need for reference to higher-order derivatives.

[7]This is equivalent to comparing the figures for the cross-AUES down the columns marked 0.2 and 5.0 in tables 2 and 3.

[8]In these experiments the cross-over value of AUES is approximately 0.7.



(a) Equal base-point value shares $s_1^* = s_2^* = 0.5$. Relative prices equal to 0.2 and 0.5 their base-point value. Y-axis: AUES at $P_1/P_2 = 0.2$ and $P_1/P_2 = 5.0$, X-axis: Base point AUES at $P_1/P_2 = 1.0$.

(b) Unequal base-point value shares $s_1^* = 0.2$, $s_2^* = 0.8$. Relative prices equal to 0.2 their base point value. Y-axis: AUES at $P_1/P_2 = 0.2$, X-axis: Base point AUES at $P_1/P_2 = 1.0$.

(c) Unequal base-point value shares $s_1^* = 0.2$, $s_2^* = 0.5$. Relative prices equal to 5.0 their base-point value. Y-axis: AUES at $P_1/P_2 = 5.0$, X-axis: Base point AUES at $P_1/P_2 = 1.0$.

Fig. 1. Differing Allen–Uzawa elasticities of substitution (AUES) at prices away from the base-point for two-input CES cost functions calibrated to the same base-point AUES and value shares. Y-axis: σ_{12}, X-axis: σ_{12}^*.

3.2. Experiments with three-input functions

As for the two-input case I have carried out the same experiments as Despotakis, but with the correct TL function. The three inputs are identified as capital, labour and energy with base-point value shares of 0.36, 0.60 and 0.04 respectively. In experiment group (A) the cross-AUES between capital and energy σ^*_{EK} is varied from -3.0 to $+1.5$, with the other two cross-AUES having 'literature average' values of $\sigma^*_{KL}=0.80$ and $\sigma^*_{LE}=1.2$. In group (B) the last two AUES values are interchanged, and the capital-energy elasticity given the values -1.0 and $+1.0$.

3.2.1. Despotakis' results

While the results of these experiments could have been documented in numerous tables and figures, in the interest of brevity only one is included here. Table 4 is the correct version of his table 4 and gives the input demands as prices vary in percentage of the base-point input demands. Comparing the DTL and TL results the general conclusion is that the TL input demands are closer to the GL than the DTL demands are, though there are exceptions. Remembering that the cross-price elasticity of input demand has the same sign as the corresponding AUES, the incorrectness of the DTL results are again revealed by the fact that in Despotakis' table 4 for values of the capital-energy elasticity of -1.0 and -0.3 the demand for energy rises as the price of capital rises and vice versa.

Since the three-input case is more complicated to analyze, there are fewer general results. Some of Despotakis' observations still hold with the correct TL function, while some do not:

(c) As in the two-input case, Despotakis states that 'reallocation of capital and energy inputs following changes in the price of capital or energy are noticeably less sensitive to values of σ^*_{EK} for the TL function...' (p. 1123). As table 4 shows, this does not hold generally; the relative sensitivity of the two functions varies with the direction of the price changes in each case.

(d) The specific observation that 'if energy and capital are substitutes...', as energy prices change, '...GL constitutes a considerably more energy conserving representation of technology' (p. 1123) is weakened but still holds when the correct TL function is used.

(e) One lesser point: The observation that neither the GL nor the TL function are well-behaved when the base-point capital-energy elasticity is -3.0, leads Despotakis to suggest that 'complementarity between energy and capital is an economic relationship that can be empirically measured with accuracy only to a limited extent, because it tends to drive the functions that can capture it beyond the limits of their outer domain' (p.

1122). It is worth noting that if the cost function is concave, complementarity defined as a property of this function can only *exist* 'to a limited extent'. Concavity places restrictions on the allowable matrix of second derivatives and therefore on the AUES matrix. Having chosen the value shares and the capital–labour and labour–energy AUES in the conducted experiments, the requirement that the own-AUES σ_{11} be non-positive combined with the Cournot aggregation condition imposes a lower limit on the allowable value of the capital–energy AUES. In case (A) of table 4 this lower limit is -1.333 and in case (B) the limit is -2.0.

3.2.2. Further comments

The point stressed in the two-input case that the second-order properties of the cost function determines the first-order properties is valid generally. Although the implications are intuitively and mathematically more complicated, it is still true that the dependence of the input demands and value shares on the elasticities of substitution is not function-specific.[9]

The question that poses itself is whether there in the many-input case is an analogy for the observation (ii) above that the elasticity of substitution is more stable for GL when its base-point value is low, and more stable for the TL when its base-point value is high. As a hypothesis for further research I suggest that the degree of concavity of the cost function determines the relative performance of the different functional forms.[10]

In these three-input experiments the cross-AUES between capital and labour, which represent 96% of total input value, probably dominates the concavity of the functions. In case (A) the AUES is 0.8 which is close to the 'cross-over' value identified in the section on two-input FFFs where the two functions coincided. In case (B) the base-point capital-labour elasticity is 1.2, which in the two-input analysis would place it in the region where AUES of the TL is the more stable and the two functions behave markedly different. Close scrutiny of table 4 reveals that in most experiments in group (A) the two FFFs are closer to each other than in group (B), which would seem to support the suggested hypothesis.

4. Concluding remarks

Despotakis in his experimental section uses a variant of the translog cost function with logarithms to the base 10 instead of the natural logarithm, while retaining the ordinary translog formula for the elasticity of substitution. This leads to severely misleading conclusions. Among the more serious is that the translog is 'noticeably less sensitive to values of elasticities of

[9]In formal analysis the best analogy for eq. (4) is probably in terms of the Hicks (1963)–Samuelson (1968) own elasticity of substitution (HSES).

[10]For a measure of the concavity of the cost function [see Frenger (1985)].

Table 4

Resource allocation implications. Economic performance of three-input CRS cost functions. A case-study with capital, labour and energy inputs to production. Base-point value shares: $s_K^*=0.36$, $s_L^*=0.60$, $s_E^*=0.04$.[b]
Correct TL function

		c_K/c_K^* (%)					c_L/c_L^* (%)					c_E/c_E^* (%)				
DP_i	σ_{KE}^*	0.50	0.75	1.00	1.50	2.00	0.50	0.75	1.00	1.50	2.00	0.50	0.75	1.00	1.50	2.00
(A) $\sigma_{KL}^*=0.80$; $\sigma_{LE}^*=1.20$																
TL; $i=K$	−3.00[a]	128.7	111.0	100.0	86.6	78.5	83.8	92.4	100.0	113.3	124.9	159.5	128.0	100.0	48.6	0.2
	−1.00	136.6	113.6	100.0	83.9	74.3	83.2	92.3	100.0	113.0	124.1	118.8	109.2	100.0	82.4	65.4
	−0.30	139.3	114.6	100.0	82.9	72.9	83.0	92.3	100.0	112.9	123.8	104.7	102.6	100.0	94.2	88.0
	0.30	141.6	115.4	100.0	82.1	71.7	82.9	92.2	100.0	112.8	123.5	92.7	96.9	100.0	104.3	107.3
	0.80	143.6	116.0	100.0	81.5	70.6	82.7	92.2	100.0	112.8	123.2	82.7	92.2	100.0	112.8	123.3
	1.00	144.4	116.3	100.0	81.2	70.2	82.7	92.2	100.0	112.7	123.2	78.7	90.3	100.0	116.1	129.7
	1.25	145.3	116.6	100.0	80.8	69.7	82.6	92.2	100.0	112.7	123.1	73.8	88.0	100.0	120.3	137.7
	1.50	146.3	116.9	100.0	80.5	69.2	82.5	92.2	100.0	112.7	123.0	68.8	85.6	100.0	124.5	145.6
GL; $i=K$	−3.00[a]	129.8	111.1	100.0	86.8	78.9	83.1	92.3	100.0	112.9	123.9	163.3	128.9	100.0	51.5	10.5
	−1.00	136.5	113.6	100.0	83.9	74.2	83.1	92.3	100.0	112.9	123.9	121.1	109.6	100.0	83.8	70.2
	−0.30	138.8	114.5	100.0	82.8	72.6	83.1	92.3	100.0	112.9	123.9	106.3	102.9	100.0	95.1	91.1
	0.30	140.8	115.2	100.0	81.9	71.2	83.1	92.3	100.0	112.9	123.9	93.7	97.1	100.0	104.9	108.9
	0.80	142.4	115.8	100.0	81.2	70.0	83.1	92.3	100.0	112.9	123.9	83.1	92.3	100.0	112.9	123.9
	1.00	143.1	116.1	100.0	80.9	69.5	83.1	92.3	100.0	112.9	123.9	78.9	90.4	100.0	116.2	129.8
	1.25	143.9	116.4	100.0	80.5	69.0	83.1	92.3	100.0	112.9	123.9	73.6	87.9	100.0	120.2	137.3
	1.50	144.7	116.7	100.0	80.2	68.4	83.1	92.3	100.0	112.9	123.9	68.4	85.5	100.0	124.3	144.7
TL; $i=E$	−3.00[a]	109.4	103.6	100.0	95.5	92.6	98.0	98.8	100.0	102.4	104.7	16.8	81.9	100.0	104.5	99.7
	−1.00	103.3	101.2	100.0	98.5	97.7	97.3	98.7	100.0	102.2	104.0	114.3	109.2	100.0	84.4	73.2
	−0.30	101.1	100.4	100.0	99.6	99.4	97.1	98.7	100.0	102.1	103.7	148.1	118.7	100.0	77.4	64.0
	0.30	99.3	99.7	100.0	100.5	100.9	96.8	98.6	100.0	102.0	103.5	177.0	126.8	100.0	71.4	56.2
	0.80	97.8	99.1	100.0	101.3	102.2	96.7	98.6	100.0	101.9	103.3	200.9	133.6	100.0	66.4	49.7
	1.00	97.2	98.8	100.0	101.6	102.7	96.6	98.6	100.0	101.9	103.3	210.5	136.3	100.0	64.4	47.1
	1.25	96.4	98.5	100.0	102.0	103.3	96.5	98.6	100.0	101.9	103.2	222.4	139.7	100.0	61.9	43.8
	1.50	95.6	98.2	100.0	102.4	103.9	96.4	98.6	100.0	101.9	103.1	234.3	143.1	100.0	59.5	40.6

GL; $i=E$	−3.00[a]	107.0	103.2	100.0	94.6	90.1	97.2	98.7	100.0	102.2	104.0	70.2	88.9	100.0	113.2	121.1
	−1.00	102.3	101.1	100.0	98.2	96.7	97.2	98.7	100.0	102.2	104.0	129.8	111.1	100.0	86.8	78.9
	−0.30	100.7	100.3	100.0	99.5	99.0	97.2	98.7	100.0	102.2	104.0	150.7	118.9	100.0	77.5	64.1
	0.30	99.3	99.7	100.0	100.5	101.0	97.2	98.7	100.0	102.2	104.0	168.6	125.6	100.0	69.6	51.5
	0.80	98.0	99.1	100.0	101.4	102.7	97.2	98.7	100.0	102.2	104.0	183.5	131.2	100.0	63.0	41.0
	1.00	97.7	98.9	100.0	101.8	103.3	97.2	98.7	100.0	102.2	104.0	189.5	133.4	100.0	60.4	36.7
	1.25	97.1	98.7	100.0	102.2	104.1	97.2	98.7	100.0	102.2	104.0	196.9	136.2	100.0	57.1	31.5
	1.50	96.5	98.4	100.0	102.7	105.0	97.2	98.7	100.0	102.2	104.0	204.4	139.0	100.0	53.8	26.2
TL; $i=L$	.+.	72.1	87.2	100.0	121.7	140.3	127.3	110.3	100.0	87.5	79.9	61.0	81.4	100.0	134.2	165.7
GL; $i=L$	.+.	71.9	87.1	100.0	121.6	139.8	127.8	110.4	100.0	87.7	80.3	57.8	80.7	100.0	132.4	159.6
(B) $\sigma^*_{KL}=1.20$; $\sigma^*_{LE}=0.80$																
T; $i=K$	−1.00	159.6	121.5	100.0	75.8	62.2	73.8	88.2	100.0	119.0	134.3	116.4	108.8	100.0	81.8	64.1
	1.00	167.1	124.1	100.0	73.1	58.2	73.3	88.1	100.0	118.7	133.4	77.1	90.0	100.0	115.3	127.0
GL; $i=K$	−1.00	156.3	121.0	100.0	75.0	60.2	74.7	88.4	100.0	119.4	135.8	121.1	109.6	100.0	83.8	70.2
	1.00	163.0	123.5	100.0	72.1	55.5	74.7	88.4	100.0	119.4	135.8	78.9	90.4	100.0	116.2	129.8
TL; $i=E$	−1.00	103.5	101.3	100.0	98.6	97.9	98.6	99.2	100.0	101.6	103.1	81.9	100.1	100.0	91.1	82.0
	1.00	97.4	98.9	100.0	101.7	102.9	97.9	99.1	100.0	101.3	102.4	178.6	127.3	100.0	71.1	55.7
GL; $i=E$	−1.00	102.3	101.1	100.0	98.2	96.7	98.1	99.1	100.0	101.4	102.7	109.9	103.7	100.0	95.6	93.0
	1.00	97.7	98.9	100.0	101.8	103.3	98.1	99.1	100.0	101.4	102.7	169.6	126.0	100.0	69.2	50.8
TL; $i=L$	.+.	59.9	81.1	100.0	133.3	162.7	136.5	114.1	100.0	82.6	71.8	70.8	86.9	100.0	121.0	137.7
GL; $i=L$	.+.	57.8	80.7	100.0	132.4	159.6	138.4	114.4	100.0	83.0	72.8	71.9	87.1	100.0	121.6	139.9

[a]Functions were not well-behaved within the considered price ranges.

[b]c_X; $X=K, L, E$: demand for input X per unit of output. c^*_X: base-point value of c_X. σ^*_{XY}; $X, Y=K, L, E$: base-point Allen–Uzawa elasticities of substitution between inputs X and Y. $i=Z$; $Z=K, L, E$: indicates that only the price of Z changes relative to its base-point value. DP_i: the ratio of price P_i relative to its base-point value P^*_i. .+. Results are identical regardless of the value of c^*_{EK}.

substitution' and that 'the significance of elasticity of substitution... is function-specific'. In this comment I have included the correct tables and stressed how the input demands and value shares depend on the development of the second-order properties of a function. As a consequence of this I have concentrated on the Allen–Uzawa elasticity of substitution and found that in the two-input case the stability of this elasticity depends on its initial value. Using the correct translog function reveals that the elasticity of substitution is more stable for the translog function than the generalized Leontief when the initial value is above approximately 0.7, and conversely for lower initial values.

Further research in this area is of obvious practical importance for model builders and users. Despotakis has taken the choice of functional forms seriously and despite having found less difference between functional forms than he did, I still agree with his general conclusion that

> 'differences in economic performance of FFF, and accordingly in results of economic models that employ FFF in partial or general equilibrium, can well be substantial. Careful examination and understanding of case-specific differences should therefore be one of the first steps in model building.'

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