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Biological productivity and optimal harvesting of a biological resource — An uncomplicated exercise in comparative dynamics.☆

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ABSTRACT

The paper presents a very simple mathematical proposition that enables easy examination of comparative dynamics when the stock of a renewable biological resource is very low. The proposition is used to prove that in the canonical schooling fisheries model the optimal harvest rate is a decreasing function of the biological productivity of the resource for low stock levels even if the optimal steady state harvest rate is an increasing function of biological productivity. The results presented here carries over to the effect of technological change in the Ramsey model as the fisheries model used here and the Ramsey model are formally equivalent.

1. Introduction

Natural resource economics was given its capital theoretic foundations in Clark (1973), Clark and Munro (1975) and Clark et al. (1979). Natural resource economics makes extensive use of dynamic optimization and as in any other branch of economics it is interesting to study how changes in the parameters of a model affect the optimal solution. In resource economics this is usually done by analyzing changes in a steady state when parameters are perturbed. Clark (1990), pp 130–135, derived the discounted supply curve where steady state harvest rate is a backwards bending function of price implying that when the price is high enough the steady state stock will be below the stock yielding the maximum sustainable yield and steady harvest rates will decrease as a function of price. Nøstbakken and Bjørndal (2003) estimated discounted supply functions for herring based on Clark's model. Caputo (1989) analyzed the effect of changes in parameters such as price and technology and described the optimal paths from one steady state to another. The examination of behavior outside of steady state is often done with numerical methods, see e.g. Kvamsdal et al. (2016) and Nøstbakken and Bjørndal (2006).

Theoretical analysis of optimal paths outside of steady state, termed comparative dynamics, was initiated by Oniki (1973) where a variational calculus is developed. Caputo (1990a) developed dynamic versions of the envelope theorem. Several applications have been published based on these papers. For example, the analysis of an increase in price on the extraction rate of a non-renewable resource examined in Caputo (1990b) is based on Oniki (1973). Nævdal (2022) uses a different approach by examining how parameters affect the stable manifold in a phase diagram. Nævdal (2022) demonstrates that for low stock levels increased productivity in the harvest technology leads to a lower harvest rate even though the steady state harvest rate increases, thus providing an example that underscores that changes in optimal policy outside of steady state may be opposite of changes in steady state. The analysis in Nævdal (2022) hinges on the existence of an interval of low stock levels where it is optimal to set the harvest rate equal to zero. In the present paper it is optimal with a positive harvest rate for all positive stock levels.

Both the variational calculus approach pioneered by Oniki (1973) and the analysis in Nævdal (2022) are algebraically demanding and it is perhaps therefore interesting to derive results where comparative dynamics are easily checked. The present paper provides an example.

2. Comparative dynamics on the shadow price at low stock levels

Consider the following two optimal control problems:

$$V_i(y_0) = \max_{u \in R_+} \int_0^\infty F^i(y, u) e^{-\rho t} dt \quad \text{subject to}$$

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$$\dot{y} = f^i(y,u) \text{ and } y(0) = y_0,\ i = 1,2 \tag{1}$$

Here i is an index whose purpose is to distinguish between two control problems that may be very different. Both problems are assumed to satisfy sufficiency conditions for a unique optimal solution, such as Theorem 9.11.1 in Sydsæter et al. (2008). Pontryagin's maximum principle gives us conditions for optimality:

$$F_u^i(y,u) + \mu_i f_u^i(y,u) \leq 0\ (= 0 \text{ if } u > 0) \tag{2}$$

$$\dot{\mu}_i = \rho\mu_i - F_y^i(y,u) - \mu_i f_y^i(y,u) \tag{3}$$

Here μ_i is the current value shadow price for the problems. If $u > 0$, then Eq. (2) implies that u may be written as a function of y and μ_i:

$$u = \chi_i(y,\mu_i) \tag{4}$$

These equations, together with the differential equation for y and the transversality condition, can be used to solve for the variables' optimal time paths. However, one can also use them to construct a differential equation for μ_i as function of y.

Assume that the solution to the problems in (1) converges towards long run steady state (y_i^*, μ_i^*). Then one can solve the following ordinary differential equation:

$$\frac{d\mu_i}{dy} = \frac{\dot{\mu}_i}{\dot{y}} = \frac{\rho\mu_i - F_y^i(y,\chi_i(y,\mu_i)) - \mu_i f_y^i(y,\chi_i(y,\mu_i))}{f^i(y,\chi_i(y,\mu_i))}, \quad \mu_i(y_i^*) = \mu_i^* \tag{5}$$

See Conrad and Clark (1987), Ch. 1.6.6 or Judd (1998), Ch. 10.7 for details. The solution to (5) is a function $\mu_i(y; y_i^*, \mu_i^*)$. As this function maps from y to μ_i along an optimal path $\mu_i(y; y_i^*, \mu_i^*)$ is in fact the derivative of the value function, i.e. $V_i'(y) = \mu_i(y; y_i^*, \mu_i^*)$ from hereon abbreviated $\mu_i(y)$. Inserting $\mu_i(y)$ into Eq. (4) yields a feedback control

$$u = \chi_i(y,\mu_i(y)) \tag{6}$$

This feedback control is a rule that specifies the value of the control variable for any given value of y. With this setup we can prove the following proposition:

Proposition 1. *Consider the two problems in (1). Assume that we know a priori that $V_i(0) = 0$ for $i = 1,2$, and that $V_1(y) > V_2(y)$ for all $y > 0$. Then there exists an interval $(0, y_c)$ where $\mu_1(y) > \mu_2(y)$.*

Proof. The value functions may be written $V_i(y) = V_i(0) + \lim_{c\downarrow 0}\int_c^y \mu_i(z)\,dz$. $V_1(y) > V_2(y) \Rightarrow \lim_{c\downarrow 0}\int_c^y \mu_1(z)\,dz > \lim_{c\downarrow 0}\int_c^y \mu_2(z)\,dz$. As $\mu_i(\cdot)$ is a continuous function over $(0,y)$ there must be an interval $(0,y_c)$ where $\mu_1(y) > \mu_2(y)$. ■

The problems analyzed in Proposition 1 are deterministic, but the proposition can be extended to stochastic dynamic optimization problems. As long as the derivative of the value function is continuous and piecewise differentiable, Proposition 1 applies. Proposition 1 can also be used to compare stochastic dynamic optimization problems with their corresponding deterministic problems.

As propositions come in dynamic optimization, Proposition 1 is certainly a very simple one.[1] But its simplicity should not detract from its usefulness as it can be used to construct interesting comparative dynamics results. The next section provides an example.

[1] The result is indeed so simple that one may wonder if it has not been discovered before. The only similar result I have found is Brock and Dochert (1983) who use the converse of Proposition 1 by integrating the shadow prices to find properties of the value function.

3. Biological productivity and optimal harvest rates in a fishery

We examine the following standard model of a schooling fishery

$$\max_{h\geq 0}\int_0^\infty B(h)\,e^{-\rho t}dt \quad \text{subject to } \dot{x} = \alpha G(x) - h \text{ and } x(0) = x_0 \tag{7}$$

Here $B(h)$ is the instantaneous benefit of harvesting h. It is assumed that $B(0) = 0$, $B'(0) = \infty$ and $B''(h) < 0$. $\alpha G(x)$ is the biological growth function defined over the interval $[0,K]$. $G(x)$ is an everywhere twice differentiable, strictly concave and nonnegative function satisfying $G(0) = 0$, and α is a parameter used to indicate biological productivity. An increase in α has the same effect as increasing the intrinsic growth rate if $G(x)$ is the logistic growth function. It is assumed that $\rho < \alpha G'(0)$ which ensures a steady state with positive values of $x = x^*$ and $\mu = \mu^*$. The conditions in the Maximum principle then become:

$$B'(h) - \mu \leq 0\ (= 0 \text{ if } h > 0) \tag{8}$$

As we have assumed that $B'(0) = \infty$ we can ignore the nonnegativity constraint for h and write $B'(h) = \mu$. The shadow price is determined by the following differential equation:

$$\dot{\mu} = \rho\mu - \mu\alpha G'(x) \tag{9}$$

Applying the method in (5) we can derive a function $\mu(x;\alpha)$ that is the stable manifold. From Eq. (8) we have that

$$h = B'^{-1}(\mu(x;\alpha)) := h(x;\alpha) \tag{10}$$

Here $B'^{-1}(\cdot)$ is the inverse of $B'(\cdot)$, and $h(x;\alpha)$ is a feedback control giving the optimal control as a function of the state variable.

3.1. Comparative statics in steady state

The effect of α in steady state for this problem is readily found and follows from $B'(h) = \mu$ and Theorem 1 in Caputo (1997) where the effect of parameter changes in the Ramsey model is also demonstrated. The steady state is a triple $(h*, x*, \mu*)$ defined by the equations $\dot{x} = \dot{\mu} = 0$ and Eq. (8). Implicit differentiation of these equations with respect to α and rearranging yields:

$$\begin{aligned}
\frac{dh*}{d\alpha} &= G(x*) - \frac{\rho^2}{\alpha^2 G''(x*)} > 0\\
\frac{dx*}{d\alpha} &= -\frac{G'(x*)}{\alpha G''(x*)} = -\frac{\rho}{\alpha^2 G''(x*)} > 0\\
\frac{d\mu*}{d\alpha} &= \left(G(x*) - \frac{\rho^2}{\alpha^2 G''(x*)}\right)B''(h*) < 0
\end{aligned} \tag{11}$$

The signs of these expressions are what one would expect. In steady state the harvest rate and stock become larger and the shadow price smaller if α increases.

3.2. Comparative dynamics for low stock levels

Let us now compare two fisheries with different biological population dynamics differing by the value of α. The biologically most productive fishery has a biological productivity parameter $\alpha_1 > \alpha_2$. Both fisheries satisfy that the value of the fishery is zero if there is no fish. Clearly the fishery with the highest biological productivity has the largest value function for all strictly positive stock levels. It then follows from Proposition 1 that there exists an interval $(0,x_c)$ where $\mu(x,\alpha_1) > \mu(x,\alpha_2)$ for all x in $(0,x_c)$. From (8) it follows that $B''(h)\,dh = d\mu$ which implies that h is a monotonically decreasing function of the shadow price. This again implies that for all x in $(0, x_c)$ it holds that $h(x, \alpha_1) < h(x, \alpha_2)$. Thus we have that even if an increase in biological productivity leads to larger harvest rates in steady state, for sufficiently low stock levels it is optimal to harvest less if a fishery is biologically more productive.

The explanation for this result is that in a more biologically productive fishery it is on margin more profitable to be close to steady state. It therefore pays to reduce harvest rates at low stock levels in order to get close to steady state sooner. Surprisingly this effect does not depend on the discount rate as it holds for all positive values of ρ.

4. Summary

The present article presents a simple mathematical proposition that can be used to examine the comparative dynamics of optimal control in certain classes of natural resource models. The proposition is used to analyze how higher biological productivity implies lower harvest rates when fish stocks are sufficiently low and it is shown that higher biological productivity implies lower harvest rates for sufficiently low stock levels even if higher biological productivity implies higher harvest rates in steady state. The analysis is performed with general functional forms, but is akin to an increase in the intrinsic growth rate in the logistic growth function.

The methodology obviously applies to other branches of economics that use similar models. For example can Proposition 1 be used to analyze changes in technology in the Ramsey model. It is hoped that the mathematical result can be useful for other researchers examining comparative dynamics as well as for instructors looking for an easily accessible example of comparative dynamics in natural resource economics classes.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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