Technology and Time Inconsistency

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Policy makers have time-inconsistent preferences if they fear losing power or are endowed with hyperbolic discount factors. Politicians may thus seek to influence future policy choices, for example, by investing in green technologies that motivate later politicians to act sustainably. I show that optimal investment subsidies are larger for technologies that are strategic complements to future investments, that are further upstream in the supply chain, or that are characterized by longer maturity. Time inconsistency can rationalize subsidies at similar levels as market failures such as externalities can. Furthermore, the two are superadditive: time inconsistency and strategic investments are especially important for long-term policies associated with externalities.

I. Introduction

> The right way is to adopt policies that spur investment in the new technologies needed to reduce greenhouse gas emissions more cost effectively in the longer term without placing unreasonable burdens on American consumers and workers in the short term. (President Bush's speech on climate change, April 16, 2008)

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Cutting emissions today in order to improve the future quality of life is the wrong way of approaching climate change, according to President Bush's 2008 speech. The right way, instead, is to invest in technology that can be used to cut tomorrow's emissions.[1] This paper sheds new light on such policy preferences.

Many projects generate costs and benefits for future years and generations. Reducing emissions today improves the environment in the future, conserving nature now makes it available for future users, extracting resources today reduces the amount available later, investments in public infrastructure generate future utilities, and costly research creates knowledge we can draw on later. When evaluating whether such projects are worthwhile, we are faced with the fundamental question of how to compare costs and benefits that occur at different points in time. This question is a deep and difficult one, and philosophers as well as economists have struggled with it for centuries.

Over the past decades, our profession has settled on employing exponential discounting, partly because preferences are then likely to be time consistent. Apart from the convenience, however, there are few reasons to impose exponential discounting as a reasonable model of decision-making. The lack of empirical and theoretical foundations for exponential discounting will be reviewed in section II, suggesting that individuals often rely on hyperbolic discounting. I also explain why, even if every individual and voter applies constant discount factors, policy makers who rotate being in office will evaluate investment projects using discount factors that increase (i.e., discount rates decrease) in relative time. Intuitively, even if everyone wants a future government to invest for the future, those ending up in office may rather prefer perks. This time inconsistency problem turns out to be particularly severe for investment projects that are associated with externalities, such as climate change.

Whether the foundation is behavioral or political, time inconsistency implies that today's decision maker disagrees with the choice of the future decision maker. Even without the ability to commit, today's decision maker can influence the future choice by investing in capital, capacity, technology, or knowledge, since such investments affect the costs or benefits of future actions. This possibility raises a number of important questions. Can time inconsistency motivate political measures, such as investment subsidies or taxes, normally reserved for traditional market failures? How will the strategic investment and investment policy depend on the type of technology, its position in the production chain, and the discount factors?

[1] In his 2008 speech, President Bush also said, "There is a wrong way and a right way to approach reducing greenhouse gas emissions. . . . The wrong way is to . . . demand sudden and drastic emissions cuts that have no chance of being realized and every chance of hurting our economy. The right way is to set realistic goals for reducing emissions consistent with advances in technology."

What is the interaction between these strategic concerns and traditional market failures, such as spillovers and externalities?

To address these questions, I consider a time-inconsistent but sophisticated decision maker who is able and willing to distort current investments in order to influence the choices made in the future. Although the game can be between the current decision maker and her future self, I find it useful to measure the strategic concern by the investment subsidy level that the decision maker would have liked to introduce if the investments were instead made in a (perhaps hypothetical) perfect market by private investors sharing the same discount factors as the decision maker. In that situation, the best policy would simply be laissez-faire (zero subsidies) if preferences were time consistent. This analysis results in three contributions.

First, I show how investments in technology and capital that are complementary to future investments should be subsidized and how investments in strategic substitutes for future investments should be taxed. An important policy implication is that so-called green technology (which reduces the cost of pollution abatement) should be subsidized, while so-called brown technology (e.g., drilling technology or investments in fossil fuel–dependent industries) should be taxed. This result holds even if we abstract from standard market failures, such as public good problems, externalities, and technological spillovers.

Second, the investment policy also depends on the technology's position in the production hierarchy. If technologies are strategic complements, technologies that are further upstream should be invested in more heavily or subsidized at a higher rate because they will impact all the subsequent steps in the production chain. In other words, the decision maker benefits from subsidizing basic research rather than investments in infrastructure at the highest rate.

The consequence is that the investment expenditures shift toward more basic/upstream technologies relative to the situation with time consistency. As an illustration, figure 1 shows that the expenditures on abatements might be higher under exponential discounting than under hyperbolic discounting, although expenditures on windmill infrastructure can be highest under hyperbolic discounting. The difference in expenditures is even larger when it comes to the investments in technology that is necessary in order to build windmills. (The numbers supporting the diagram are presented in sec. IV.B.)

These results hinge on the discount factors in interesting ways. Under exponential discounting, the equilibrium subsidies are always zero (this will follow from the envelope theorem). Furthermore, the result that upstream technologies should be subsidized more—and how this depends on the investment lags—does not hold under quasi-hyperbolic discounting, which is therefore a poor approximation for hyperbolic discounting.

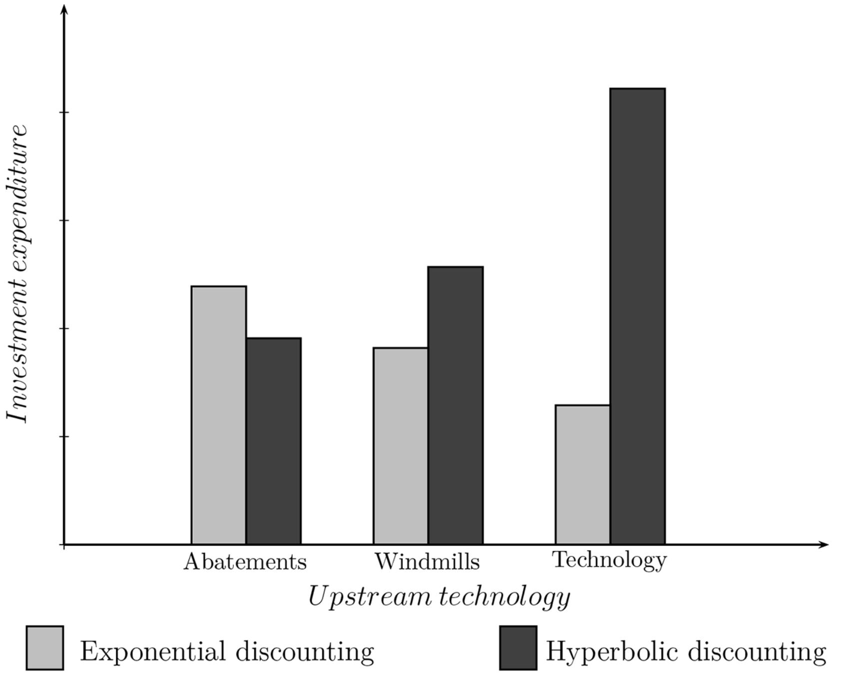

FIG. 1.—The equilibrium investment portfolio allocates more to upstream technologies under hyperbolic discounting than under exponential discounting.

Third, a quantitative assessment suggests that time inconsistency motivates subsidies of similar magnitude as do externalities and spillovers. Furthermore, the two effects are superadditive, in that the effect of time inconsistency is larger when international spillovers are also present, and vice versa. In other words, the time inconsistency problem is especially severe for environmental problems, such as climate change.

Outline.—The next section explains why time inconsistency is realistic, especially in political settings associated with externalities. Section III presents a simple model that describes how the investment policy varies with the type of technology (e.g., green vs. brown technology) and its position in the supply chain. The basic model is then extended in two important directions: section IV allows for multiple technology levels, while section V permits multiple countries, externalities, and spillovers and provides a quantitative assessment. Section VI concludes, and the appendix contains all proofs.

II. Background, Foundation, and Literature

A. *A Brief History on Discounting*

In the nineteenth century, the debate regarding how to evaluate future utility gains and losses included a large number of philosophical and psychological factors (Rae 1834; Senior 1836; Jevons 1871; von Böhm-Bawerk 1889). Ramsey (1928) suggested maximizing a weighted sum of future utilities,

$$v_t = \sum_{\tau=t}^{\infty} D(\tau - t) u_\tau,$$

where $D(0) = 1$ and $D(\tau)$ measures the weight of utility τ periods ahead relative to utility right now. Although the weight $D(\tau)$ was left unspecified, Samuelson (1937) suggested the now familiar formula for exponential discounting:

$$D(t) = \delta^t = \left(\frac{1}{1+\rho}\right)^t \approx e^{-\rho t},$$

where δ is the corresponding constant discount factor between subsequent periods and ρ is the constant discount rate. With Koopmans's (1960) axiomatic foundation, exponential discounting became the standard way in economics of evaluating future gains and losses.

To many, the appeal of exponential discounting is not that its assumptions regarding individual behavior are reasonable but that it simplifies the analysis.[2] In a seminal paper, Strotz (1955–56) explained why preferences are likely to be time inconsistent and that we, as a consequence, have to search for the best plan that will actually be followed. The next few decades saw an explosion of empirical and experimental evidence that "seems overwhelmingly to support hyperbolic discounting," according to Frederick, Loewenstein, and O'Donoghue (2002, 361).[3] With hyperbolic discounting, utility at time t is given the weight

$$D(t) = \frac{1}{1+\alpha t}, \tag{1}$$

[2] Samuelson himself had reservations when suggesting the exponential formulation both as a representation of an individual's preference ("It is completely arbitrary to assume that the individual behaves so as to maximize an integral of [this] form" [Samuelson 1937, 159]) and as advice for a public planner ("any connection between utility as discussed here and any welfare concept is disavowed" [161]). Nevertheless, and "despite Samuelson's manifest reservations, the simplicity and elegance of this [exponential] formulation was irresistible," according to Frederick, Loewenstein, and O'Donoghue (2002, 355–56).

[3] For empirical evidence, see the survey by Angeletos et al. (2001) or more recent research by Shapiro (2005), Laibson, Repetto, and Tobacman (2007), Paserman (2008), or Augenblick, Niederle, and Sprenger (2015). Salois and Moss (2011) argue that observed asset valuations reject exponential discounting, and Giglio, Maggiori, and Stroebel (2015) find discount rates below 2.6% for 100-year claims. In lab experiments, individuals often prefer a smaller benefit today to a larger benefit tomorrow but reverse the ranking if the two consecutive days are further into the future; see, e.g., Thaler (1981), Ainslie (1992), Benhabib, Bisin, and Schotter (2010), or Halevy (2015). To explain why humans have time-inconsistent preferences, Dasgupta and Maskin (2005) show that individuals evolve and survive better if they are endowed with hyperbolic discount factors, while Budish, Roin, and Williams (2015) argue that firms focus on short-term investments because of the patent system.

where $\alpha > 0$ is a constant that can measure either impatience or the scale of time. In general, the discount factor for time t relative to $t - 1$ is

$$\delta_t \equiv \frac{D(t)}{D(t-1)} \Leftrightarrow D(t) = \prod_{\tau=1}^{t} \delta_\tau, \tag{2}$$

so with hyperbolic discounting, $\delta_t = 1 - \alpha/(1 + \alpha t) \in (0, 1)$, which is concave and increasing in t and approaching 1 as t grows.

Laibson (1997) adopted a simpler approximation of equation (1), often referred to as quasi-hyperbolic discounting. He considered $\beta < 1$ and $\delta < 1$ such that for every $t > 0$,

$$D(t) = \beta\delta^t, \text{ so } \delta_1 = \beta\delta < \delta_t = \delta \; \forall \; t > 1. \tag{3}$$

But even if individuals apply discount factors that increase in relative time, does this imply that a policy maker ought to do the same? There are several reasons for an affirmative answer. First, the government consists of individual decision makers who share these preferences regarding the future, so it is inevitable that policy makers will act in a time-inconsistent way. Second, to be reelected, the government might need to be accountable and apply the same discount factors as the voters do.[4]

B. A Foundation: Rotation of Political Power

Even if everyone were endowed with time-consistent preferences, policy makers are still likely to be time inconsistent for political reasons. It is well know, for example, that political turnover leads to time inconsistency (e.g., Persson and Svensson 1989; Alesina and Tabellini 1990; Tabellini 1991).[5] This section draws on Amador (2003) and Chatterjee and Eyigungor (2016), but unlike them, I emphasize the importance of externalities and incumbency advantage.

Suppose that the party or policy maker in office today expects to remain in office with probability q in the next period. In a simple symmetric setting with two parties, the party outside office gains power with probability $1 - q$. If p_t measures the probability that the party is in power in period t, then p_t follows the Markov process

$$p_t = qp_{t-1} + (1 - q)(1 - p_{t-1}),$$

which is a difference equation with the following solution:

[4] However, citizens may prefer that the government apply a lower discount rate than the citizens themselves would (Caplin and Leahy 2004).

[5] Most of this literature assumes that the reelection probability is exogenous. Battaglini and Harstad (2020) show that incumbents invest in technologies (and treaties) in order to influence future elections.

$$p_t = (2q-1)^t\left(p_0 - \frac{1}{2}\right) + \frac{1}{2}, \tag{4}$$

where $p_0 = 1$ for the incumbent at time zero.

One of the benefits of being in power is that one can allocate the budget to some pet projects, giving the policy maker a marginal benefit normalized to 1. Suppose that the policy maker at time $t-1$ can forgo some of this benefit by investing in a project that pays off in period t. For a dollar benefitting the party in office, the benefit to the party not in office is given by z. If we considered only investments in the policy maker's pet project, then $z = 0$, while investments in public goods imply $z = 1$.[6] In either case, the policy maker's expected present discounted value of per unit of total return is $\delta^t[p_t + z(1-p_t)]/(1+z)$. The expected present discounted value of consumption at $t-1$ is $\delta^{t-1}p_{t-1}$. Thus, the policy maker at time zero benefits from the investment at time $t-1$ if and only if the investment cost (per unit of return) is smaller than

$$\delta_t \equiv \frac{\delta^t[p_t + z(1-p_t)]/(1+z)}{\delta^{t-1}p_{t-1}} = \delta\frac{p_t + z(1-p_t)}{p_{t-1}(1+z)}. \tag{5}$$

In other words, for any pair of future periods $(t-1, t)$, δ_t measures how the policy maker at time zero discounts the total return realized at t relative to the cost at $t-1$. If we combine equations (4) and (5), we learn how the discount factor δ_t depends on t.

PROPOSITION 1. If $q \in (1/2, 1)$, the decision maker's discount factor δ_t is a strictly increasing concave function of t, and it increases more rapidly if z is large:

$$\delta_t = \delta\left[1 - \frac{1-q+(2q-1)z/(1+z)}{\left[1+(2q-1)^{1-t}\right]/2}\right] \Rightarrow \frac{\partial\delta_t}{\partial t} > 0 > \frac{\partial\delta_t}{\partial z} \text{ and } \frac{\partial^2\delta_t}{\partial z\partial t} > 0. \tag{6}$$

The appendix proves and generalizes the proposition by permitting arbitrary numbers of political parties and investment maturation periods.

COROLLARIES TO PROPOSITION 1.

1. If $q \uparrow 1$, then the discount factor is constant and equal to $\delta_t = \delta/(1+z)\ \forall\ t$.
2. If $q \downarrow 1/2$, then $\delta_1 = \delta/2$ but $\delta_t = \delta$ for $t > 1$, so discounting is quasi-hyperbolic.
3. If $q \in (1/2, 1)$, δ_t increases strictly from $\delta_1 = \delta[q - (2q-1)z/(1+z)]$ to $\lim_{t\to\infty}\delta_t = \delta$.

[6] One could also assume that the party not in office faces a cost per each unit invested by the party in power. A larger z relative to that cost would then have the same effect as z has in this section.

Corollary 1 shows that the discount factor would be constant if the incumbency advantage were complete, as in a dictatorship. In that case, there is no reason to commit one's future self.

Corollary 2 shows that with no incumbency advantage, time preferences are represented by quasi-hyperbolic discounting.[7] The incumbent may not be in office in the next period and thus applies a small discount factor. Thereafter, the future discount factor is constant, since the probability of being in power at future dates equals $1/2$ regardless of whether one is in power today.

Corollary 3 shows that for $q \in (1/2, 1)$, δ_t increases in t because the probability that it is the time zero policy maker who actually has to pay for an investment at time $t - 1$ is gradually declining with t. In the very long run, p_{t-1} and p_t approach $1/2$, and thus $\lim_{t\to\infty}\delta_t = \delta$. This leads to an interesting time inconsistency problem: for every investment cost in the interval (δ_0, δ_t), the policy maker at time zero would prefer to commit to invest at $t - 1$, but any policy maker actually in office at that time will prefer to reverse that decision.

Importantly, the time inconsistency problem is more severe for investments that are associated with large externalities. Proposition 1 implies that if z increases, δ_t decreases and the slope $\partial\delta_t/\partial t$ increases. Intuitively, although a larger z does not reduce the investment's attractiveness when $p_t \approx 1/2$, it does reduce the fraction of the total return captured by the party actually in office. To appreciate the magnitudes of these effects, table 1 illustrates the applied discount factors as functions of (t, q, z), assuming $\delta = 0.95$ (this corresponds to a 1% annual discount rate if each period lasts 5 years).[8]

C. *Other Foundations: Preference Aggregation and Intergenerational Altruism*

Time inconsistency can arise in politics for other reasons too. Even if the government is ruled by a benevolent planner or the median voter, and each individual has time-consistent preferences, collective decisions will be time inconsistent as long as the discount factors differ among the individuals.[9]

[7] This result is also derived by Amador (2003) and Chatterjee and Eyigungor (2016, app. B).

[8] The numbers for the incumbency advantage are within the range discussed in the literature. Even in US presidential elections, Mayhew (2008, 213) find that "in-office parties had kept the presidency exactly two-third of the time (20 out of 30 instances) when they ran incumbent candidates, and exactly half the time (11 out of 22 instances) when they did not."

[9] See Gollier and Zeckhauser (2005), Jackson and Yariv (2014, 2015), or Feng and Ke (2018) for the theory, or see Adams et al. (2014) for evidence. Note that the fact that the pure time preference rate depends on the time horizon is orthogonal to the arguments by Gollier and Weitzman (2010) and Weitzman (2001), who have shown that if the growth rate of consumption is uncertain, then it is optimal to discount future consumption at a rate that is decreasing in time in order to reflect risk aversion and the accelerating level of risk.

TABLE 1
APPLIED DISCOUNT FACTOR IS SMALLER FOR SMALL t AND LARGE z

	$q = 3/5$		$q = 4/5$	
DISCOUNT FACTOR	$z = 0$	$z = 1$	$z = 0$	$z = 1$
δ_1	.57	.48	.76	.48
δ_2	.82	.79	.81	.59
δ_3	.92	.91	.85	.70
δ_4	.94	.94	.88	.78
δ_5	.95	.95	.91	.84

Also, when we abstract from heterogeneity and rotation of power, one can argue that a government should—from a normative perspective—discount future utility by using a discount factor that increases in relative time. If parents are thoughtful (as in Barro 1974), the welfare of a generation is a weighted sum of its own utility and the next generation's welfare. We can then write welfare recursively as a weighted sum of all future utilities, and the discount factor will be constant over time, leading to exponential discounting. However, if today's parents also care about the welfare of their grandchildren, then stationarity will be violated and the effective discount factor will indeed increase in relative time (Harstad 1999; Saez-Marti and Weibull 2005; Galperti and Strulovici 2017). In fact, the formula for quasi-hyperbolic discounting, $D(t) = \beta\delta^t$, was first suggested by Phelps and Pollak (1968), who argued that it may represent imperfect altruism between generations.

In sum, there are several reasons for why time inconsistency is especially important in politics and for long-term decisions associated with externalities. This motivates the following analysis, which holds regardless of the exact reason for time inconsistency.

D. Policies in the Presence of Time Inconsistency

There is a large literature on policies when individuals have time-inconsistent preferences. For example, hyperbolic discounters may retire too early (Diamond and Kőszegi 2003) or save too little (Harris and Laibson 2001), so the government can help by subsidizing saving (Krusell, Kuruşçu, and Smith 2009, 2010). But individuals may also try to commit their future selves by exerting self-control (Fudenberg and Levine 2006), limiting their future choice set (Gul and Pesendorfer 2001), signing up for saving plans (Thaler and Benartzi 2004), accumulating debt (Bisin, Lizzeri, and Yariv 2015), or paying today the cost of attending the gym tomorrow (DellaVigna and Malmendier 2006). When the effects of climate change are discounted hyperbolically, Karp (2005) shows how the stock of pollutants can influence future decisions, while the choice of carbon taxes is investigated by Gerlagh and Liski (2018). In contrast to all these papers, I allow for a general class of technology and focus on how the type of that technology and its position in the production chain determine the equilibrium investment strategy and policy. By allowing discount factors to depend on time in a general way, the model encompasses exponential discounting, hyperbolic discounting, and quasi-hyperbolic discounting as special cases and shows that results based on the traditional models are nonrobust.

III. The Basic Model

A central result in this section regards how the technology's type and position in the production chain determine the equilibrium investment policy. To emphasize this, it is useful to present the model stepwise: After notation is introduced in section III.A, section III.B discusses the last stage in the production chain as a simple investment before we consider capital in section III.C and technology in section III.D.

A. *Notation and Measures of Strategic Investments*

If $u_t(\mathbf{k}_t)$ measures the momentary utility t periods from now as a function of past actions, $\mathbf{k}_t = (k_0, \ldots, k_t)$, then the decision maker's objective at time t is to maximize $v_t \equiv \Sigma_{\tau=t}^{\infty} D(\tau - t) u_\tau(\mathbf{k}_\tau)$. Unless otherwise stated, I will assume that the discount factor $\delta_t = D(t)/D(t-1) \in (0, 1)$ is strictly increasing in t.

It is obvious that any action that increases every future u_t will be taken. The interesting decisions are those that require the decision maker to trade off future gains against current losses or, equivalently, vice versa. If the cost of $k_t \in \mathbb{R}$ in terms of utility is $c^t(k_t; \mathbf{k}_{t-1})$ at time t, it may nevertheless be worthwhile if it increases future utility. If we assume differentiable utility functions, the necessary first-order condition for an interior solution is

$$c_1^t \equiv \frac{dc^t(k_t; \mathbf{k}_{t-1})}{dk_t} = \frac{d}{dk_t} \sum_{\tau=t+1}^{\infty} D(\tau - t) u_\tau(\mathbf{k}_\tau), \tag{7}$$

where derivatives are denoted as subscripts.

Since other actions might be taken in the future, it is useful to distinguish between the total derivatives and the partial derivatives. The total derivative $d(\cdot)/dk_t$ in equation (7) recognizes that when taking an action, a sophisticated decision maker takes into account the fact that this choice can influence other future choices that may in turn also influence utilities. If, in contrast, the decision maker did not seek to influence future choices, then the choice of k_t would solve

$$c_1^t = \frac{\partial}{\partial k_t} \sum_{\tau=t+1}^{\infty} D(\tau - t) u_\tau(\mathbf{k}_\tau). \tag{8}$$

If the decision maker were time consistent, then equations (7) and (8) would be equivalent, since future choices would be optimal also from today's point of view, and thus there would be no reason to influence them (this would follow from the envelope theorem). But when preferences are time inconsistent, then we can measure the strategic consideration when choosing k^t in the following way:

$$s_*^t \equiv \frac{\sum_{\tau=t+1}^{\infty} D(\tau - t) du_\tau(\mathbf{k}_\tau)/dk_t}{\sum_{\tau=t+1}^{\infty} D(\tau - t)\partial u_\tau(\mathbf{k}_\tau)/\partial k_t} - 1. \tag{9}$$

That is, when $s_*^t > 0$, the investment level that is chosen according to equation (7) is strategically large when the decision maker takes into account the fact that k_t influences future choices. If $s_*^t < 0$, the investments are instead strategically small when the effect on future decisions is taken into account. In either case, s_*^t measures the extent to which the choice of k_t is distorted because of the decision maker's desire to influence future decisions.

1. A Perfect Market

Interestingly, s_*^t can also be interpreted as the equilibrium subsidy if the actual investment is made by private investors in a competitive market. To see this, consider that a competitive or perfect market—defined as a market in which investors obtain full property rights to the direct revenues of their investments—takes as given the future willingness to pay, $\partial u_\tau(\mathbf{k}_\tau)/\partial k_t$, and that future revenues are discounted according to $D(\tau - t)$. The investment in k_t would then be given by equation (8). With exponential discounting, the first welfare theorem implies that the market equilibrium would be first best and there would be no need for any regulation. However, if the investment cost is subsidized by $\underline{s}^t$, the market solution is

$$\begin{aligned}(1 - \underline{s}^t)c_1^t &= \frac{\partial}{\partial k_t} \sum_{\tau=t+1}^{\infty} D(\tau - t) u_\tau(\mathbf{k}_\tau) \Leftrightarrow \\ c_1^t &= (1 + s^t)\frac{\partial}{\partial k_t} \sum_{\tau=t+1}^{\infty} D(\tau - t) u_\tau(\mathbf{k}_\tau), \text{ with} \qquad (10) \\ s^t &\equiv \frac{1}{1 - \underline{s}^t} - 1.\end{aligned}$$

Here, s^t is equivalent to a subsidy on future revenues. Alternatively, we can let an investment cost subsidy $\underline{s}^t$ be measured by $s^t \equiv 1/(1 - \underline{s}^t) - 1$.

2. Policies

The decision maker at time t can implement her preferred k_t by ensuring that equation (10) coincides with equation (7). This requires that $s^t = s^t_*$, as it is given by equation (9). In fact, this choice of s^t_* is preferred by the decision maker if she considers the subsidies to be simply transfers at no net cost within the society, except that they influence the choice of k_t. Whether the decision maker sets k_t directly or by regulating the market, s^t_* measures the equilibrium level of k_t and how it differs from the choice of k_t in the absence of any strategic considerations.

Note that there is no commitment to any future subsidies in the model. The subsidy is set for current investments, and it is impossible to commit to any future subsidies or policies. The only way to partially commit is to take today's decision k_t in such a way as to influence future choices.

B. A Simple Investment

To illustrate the notation and derive a benchmark comparison, consider a simple and single once-and-for-all investment or action $a \in \mathbb{R}$ (thus, I can ignore subscripts measuring time) generating a future benefit $b\,(a)$ at cost $c^a\,(a; k)$ today, where k is some exogeneously given capital. If the benefit is realized Δ^α periods from now, it is discounted by $D\,(\Delta^\alpha)$. Thus, the decision maker at the time when a is decided on maximizes $v^a \equiv -c^a(a; k) + D(\Delta^a)b(a)$. The necessary first-order condition is

$$c_1^a = D(\Delta^a)b_1, \tag{11}$$

where $c_1^a \equiv dc^a(a; k)/da$ and $b_1 \equiv db(a)/da$. For simplicity, I follow the convention to restrict attention to environments in which the solution is interior and the second-order condition is satisfied.[10] As a comparison, private investors can invest today and earn the marginal revenue b_1 tomorrow. With the subsidy s^a, the first-order condition is

$$\frac{c_1^a}{1 + s^a} = D(\Delta^a)b_1. \tag{12}$$

With only one action, a, equations (11) and (12) are equivalent if and only if the subsidy equals

$$s_*^a = 0.$$

The market makes the same decision as the decision maker does, so laissez-faire works fine.

[10] For example, I here assume that $c^a(\cdot)$ is increasing and convex, $b(\cdot)$ is increasing and concave, and $c_1^a(0; k) - D(\Delta^a)b_1(0) < 0 < \lim_{a\uparrow\infty} c_1^a(a; k) - D(\Delta^a)b_1(a)$.

C. *Investments in Capital*

The investment or action $a \in \mathbb{R}$ can have a large number of interpretations. The investment can be in health, education, infrastructure, or pollution abatement, to mention some examples. For such investments, it is reasonable that the cost of investing depends on the level of capital or infrastructure. The importance of capital is represented by $k \in \mathbb{R}$. When a measures pollution abatement, it is natural to think of two interpretations of k:

Green capital is assumed to be complementary to pollution abatement. Such technology can be cleaning technology or alternative energy sources; in either case, a larger stock of green technology is a strategic complement to abatement, and it reduces the marginal cost of abating. That is, c_1^a decreases in k, so $c_{12}^a \equiv \partial^2 c^a(\cdot)/\partial a \partial k < 0$.

Brown capital refers to drilling technologies or investments in industries that pollute. Such capital may be beneficial in the sense that it increases the utility, but a larger level of k also makes it costly to cut back on pollution. Thus, $c_{12}^a > 0$, meaning that a and k are strategic substitutes.

The proof in the appendix also permits k to influence $b(\cdot)$, as when k represents the extent to which a country has adapted to climate change.

The level of k is given when a is decided upon. If we differentiate equation (11), we can see how the decision on a varies with k:

$$\frac{da}{dk} = \frac{-c_{12}^a}{c_{11}^a - D(\Delta^a) b_{11}} \Rightarrow \operatorname{sign}\left(\frac{da}{dk}\right) = \operatorname{sign}(-c_{12}^a). \tag{13}$$

Thus, $da/dk > 0$ for green capital and $da/dk < 0$ for brown capital.[11]

Figure 2 illustrates that Δ^k measures the number of periods between the decision on k and the decision on a. That is, Δ^k is the time it takes for the capital to be built. Further, $c^k(k; r)$ is the cost of k, given the technology, r. When k is decided upon, the decision maker takes into account that the level of k affects future payoffs not only directly but also indirectly through the choice of a. Private investors, however, would invest to ensure that marginal costs equal the present discounted willingness to pay:

$$\frac{c_1^k}{1 + s^k} = D(\Delta^k)(-c_2^a), \tag{14}$$

where s^k represents the subsidy on k. The decision maker can implement her preferred level of k by setting the appropriate s^k. Even when the decision maker decides on k directly, there exists some s^k, referred to as s_*^k, such that the decision maker's preferred level of k satisfies equation (14) with s_*^k.

[11] The denominator of eq. (13) is positive when the second-order condition holds in the maximization problem over a.

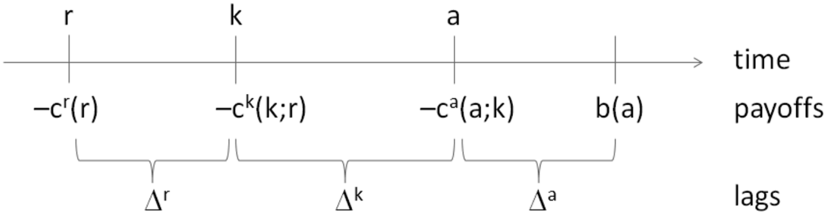

FIG. 2.—Timing of the game.

So, as explained above, s_*^k can measure how much the decision maker strategically distorts investments in k in order to influence the decision on a.

PROPOSITION 2. The decision maker's choice of capital investment level is given by equation (14) if and only if s^k is

$$s_*^k = \left(\prod_{t=1}^{\Delta^a} \frac{\delta_{t+\Delta^k}}{\delta_t} - 1\right) \frac{c_1^a}{-c_2^a} \frac{da}{dk}. \tag{15}$$

The term after the square bracket, $c_1^a/c_2^a < 0$, is simply the slope of the isocost curve.

COROLLARIES TO PROPOSITION 2.

1. With exponential discounting, $s_*^k = 0$.
2. With quasi-hyperbolic discounting, equation (15) simplifies to

$$s_*^k = \left(\frac{1}{\beta} - 1\right) \frac{c_1^a}{-c_2^a} \frac{da}{dk}. \tag{16}$$

3. For a strictly increasing δ_t, $|s_*^k|$ increases in Δ^k.

Corollary 1 verifies that in traditional settings in which decision makers have time-consistent preferences, there is no need to distort the future choices in this model. So if investors capture the full future return of investments, there is no need for regulation. This confirms the earlier finding that laissez-faire is just fine.

Corollary 2 recognizes that a time-inconsistent decision maker is not satisfied with the future choice of a. Today's decision maker would prefer a larger a than the level that will actually be implemented, and the choice of a can be influenced by k. In general, the disagreement between the two decision makers—and thus the equilibrium level of s^k—will depend on every relevant δ_t. With quasi-hyperbolic discounting, however, $\delta_t = \delta$ for $t > 1$, and the formula for s_*^k simplifies.

Corollary 3 shows that when discount factors are strictly increasing in relative time, the disagreement with the future decision maker is larger if the various decisions are made at very different points in time. Thus, the expression in brackets in equation (15) is increasing in the investment lag.[12]

COROLLARIES TO PROPOSITION 2 (Continued).

4. With green capital, the decision maker benefits from a subsidy on investments:

$$c_{12}^a < 0 \Rightarrow s_*^k = \left(\prod_{i=1}^{\Delta^a} \frac{\delta_{t+\Delta^k}}{\delta_t} - 1\right) \frac{c_1^a}{-c_2^a} \frac{da}{dk} > 0.$$

5. With brown capital, the decision maker benefits from a tax on investments:

$$c_{12}^a > 0 \Rightarrow s_*^k = \left(\prod_{t=1}^{\Delta^a} \frac{\delta_{t+\Delta^k}}{\delta_t} - 1\right) \frac{c_1^a}{-c_2^a} \frac{da}{dk} > 0.$$

The intuition for these corollaries is straightforward but important:

Corollary 4 states that, regardless of whether discounting is quasi-hyperbolic or whether δ_t is instead strictly increasing in t, $s_*^k > 0$ for green capital. For this type of capital, a increases in k, and thus the decision maker prefers a strategically large k in order to motivate a larger a in the future.

Corollary 5 recognizes that a decreases in k, if k represents brown capital. To motivate a larger a, which the decision maker would prefer, it is necessary to reduce the investment in brown capital today. Thus, the decision maker benefits from investing strategically little and from taxing this kind of investment.[13]

D. Investments in Technology

Section III.C made a distinction between different types of investments at the same stage in the production chain. This subsection explores how the strategic choice of investment or subsidy also depends on the stage in the production chain. As in figure 2, the technology r is endogenized and invested in at cost $c^r(r)$, Δ^r periods before the k stage. For example, while a larger number of windmills will make it cheaper to reduce pollution, the production cost of each windmill will depend on the amount of technology, knowledge, or basic research.

[12] If either lag is zero, $s_*^k = 0$. Intuitively, if $\Delta^k = 0$, it takes no time to build the capital. It is then the same decision maker selecting k and a, and there is obviously no need to distort either decision. Alternatively, if $\Delta^a = 0$, the decision maker choosing a gets the benefit herself immediately, and the level of a does not influence any future utility, which the two decision makers would evaluate differently.

[13] One can extend the model to permit investments in both capital types at the same time. If the investment cost is a convex function of the sum of green and brown investments, then corollary 2 is strengthened: since time inconsistency motivates larger green investments, the cost of brown capital will increase, and thus investments in brown capital decreases both because of the strategic consideration and also because marginal investments are costlier when green investments are large.

With time-inconsistent preferences, today's decision maker is not satisfied with the future choices of k and a, and in order to influence these choices, it may be beneficial to distort today's investments in r. To see how r influences k, we can simply differentiate the first-order condition for k to show that the cross derivative is, again, crucial:

$$\frac{dk}{dr} = -\frac{c_{12}^k}{-v_{11}^k},$$

where $v_{11}^k < 0$ is the second-order condition when k is chosen (see the appendix). The influence of r on a is given by the product of dk/dr and da/dk.

Just as in section III.C, we can measure the decision maker's decision on r relative to her choice in the absence of the strategic concerns by deriving the level of s^r, which would ensure that equation (17) is in line with the decision maker's preferred level. The competitive market would invest as follows:

$$\frac{c_1^r}{1+s^r} = D(\Delta^r)(-c_2^k). \tag{17}$$

PROPOSITION 3. The decision maker's choice of technology investment level is given by equation (17) if and only if s^r is

$$\begin{aligned} s_*^r = {} & \left(\prod_{t=1}^{\Delta^r}\frac{\delta_{t+\Delta^k}}{\delta_t} - 1\right)\frac{c_1^k}{-c_2^k}\frac{dk}{dr} \\ & + \left(\prod_{t=1}^{\Delta^r}\frac{\delta_{t+\Delta^a+\Delta^k}}{\delta_t} - \prod_{t=1}^{\Delta^r}\frac{\delta_{t+\Delta^k}}{\delta_t}D\right)(\Delta^k + \Delta^a)\frac{b_1}{-c_2^k}\frac{da}{dk}\frac{dk}{dr}. \end{aligned} \tag{18}$$

As before, the contribution of the result is best illustrated by discussing its corollaries.

COROLLARIES TO PROPOSITION 3.

1. With exponential discounting, $s_*^r = 0$.
2. With quasi-hyperbolic discounting, the second term in equation (18) is zero, so s_*^r takes the same form as s_*^k does in equation (16):

$$s_*^r = \left(\frac{1}{\beta} - 1\right)\frac{c_1^k}{-c_2^k}\frac{dk}{dr}. \tag{19}$$

3. For a strictly increasing δ_t, the absolute values of both terms in equation (18) increase in Δ^r, and the second term dominates for sufficiently high long-term discount factors.

Corollary 1 confirms that with exponential discounting, both terms in equation (18) are zero. For the same reasons as before, a time-consistent decision maker would be perfectly satisfied with the future choices of a and k, and she would have no desire to distort r.

Corollary 2 recognizes that with time-inconsistent preferences, the decision maker disagrees with the future choice of k. Thus, r will be chosen in order to influence and increase the investment in k. If the cross derivative c^k_{12} is negative so that r is a strategic complement to the investment in k, then the current decision maker has an incentive to invest strategically more in r in order to motivate a larger investment in k. The equilibrium investment in r is larger if the current decision maker disagrees strongly with her future self. With quasi-hyperbolic discounting, this disagreement is larger if β is small. Note the similarity between s^r_* and s^k_* in this case; we see exactly the same forces at work. If technology r is complementary to k, then r requires a subsidy just as k did when k was complementary to a.

Interestingly, when we derive s^r_* for the case with quasi-hyperbolic discounting, it is important only whether k increases or decreases in r. It is irrelevant whether the capital k is itself green or brown (i.e., whether k increases or decreases a). The explanation for the irrelevance of the capital type is the following. Although the current decision maker disagrees with her future self regarding the appropriate level of investment k, these two selves agree perfectly when trading off utilities between two later dates. With quasi-hyperbolic discounting, the discount factor on utility at time $t + 1$ relative to time t is δ whenever $t > 1$. Thus, the decision maker choosing r agrees with the decision maker choosing k regarding how to influence the decision maker selecting a.

Corollary 3 shows that when δ_t increases strictly in t, then the decision maker investing in r disagrees with the decision maker investing in k on the need to influence the future choice of a. This disagreement explains the second term in s^r_*, which is larger if the long-term discount factors are large. The second term is important because it can overturn the first.

It is natural to define green technology as technology that is complementary to the investment in green capital and brown technology as technology that is complementary to the investment in brown capital.

COROLLARIES TO PROPOSITION 3 (Continued).

4. For green technology, both terms in equation (18) are positive, so $s^r_* > 0$.
5. For brown technology, the first term in equation (18) is positive, the second is negative, and

$$s^r_* < 0 \Leftrightarrow$$

$$\left(\prod_{t=1}^{\Delta^r} \frac{\delta_{t+\Delta^a+\Delta^k}}{\delta_{t+\Delta^k}} - 1\right)\left(-\frac{da}{dk}\right) > \left(1 - 1/\prod_{t=1}^{\Delta^r} \frac{\delta_{t+\Delta^k}}{\delta_t}\right) \frac{c^k_1}{b_1 D(\Delta^k + \Delta^a)}.$$

Corollary 4 recognizes that for green technology, complementary to green capital, the decision maker invests strategically more in r both to induce a larger k and also to induce a larger a. Hence, the expression for s_*^r consists of two positive terms.

Corollary 5 verifies that for brown technology, the second term of s_*^r is negative, while the first term is positive. Thus, $s_*^r < 0$ if the second term dominates the first, positive term. This will be the case when, for example, the degree of substitutability between k and a is particularly large (i.e., when $|da/dk|$ is large) and when the long-term discount factors are large. In this case, the motivation to subsidize investments in technology in order to motivate larger capital investments is outweighed by the concern that the capital stock will subsequently lead to more emissions.

IV. Multiple Technology Levels

A. *The Supply Chain of Technologies*

The analysis above suggests that for investment policies, it is crucial to determine the technology's position in the production hierarchy: while the final investment stage before consumption did not need any regulation, investments in complementary green capital are subsidized. Furthermore, the investment in green technology will be subsidized at a rate that consists of two positive terms rather than just one, where the first corresponds to the equilibrium subsidy on investments in capital. These comparisons suggest that the equilibrium subsidy for complementary investments further upstream might have a tendency to be larger.

To investigate this conjecture, assume now that there are L technology levels, indexed by $l \in \{1, .., L\}$. To recognize the similarity between the stages, refer to a as k^1, with $c^1(k^1; k^2)$ as the investment cost. Capital is referred to as k^2 (instead of simply k), and the capital investment cost is $c^2(k^2; k^3)$, and so on. More generally, the investment cost for technology level l is given by $c^l(k^l; k^{l+1})$, if we just take k_{L+1} as exogenously given when writing $c_L(k_L; k_{L+1})$, measuring the most upstream investment cost. For simplicity, I first assume that the decision maker invests in only one technology type at each point in time. (Section IV.C relaxes this assumption.)

To solve for the decision at stage l, note that with a subsidy s^l, the market will invest according to

$$\frac{c_1^l(k^l; k^{l+1})}{1 + s^l} = D(\Delta^l)\left(-c_2^{l-1}\right). \tag{20}$$

The decision maker, however, will take into account that the choice of k^l influences the next choice of k^{l-1}, and so on. In other words, the decision maker's preferred level of k^l satisfies equation (20) only for some $s^l \neq 0$.

Proposition 4. The decision maker's investment choice satisfies equation (20) with $s_*^1 = 0$, and for every $l \in \{2, \ldots, L\}$,

$$s_*^l = \sum_{i=1}^{l-1}\left(\prod_{t=1}^{\Delta_i}\frac{\delta_{t+\Lambda(l,i)}}{\delta_t} - 1 - s^i\right)\frac{dk^i}{dk^l}\frac{D(\Lambda(l,i))D(\Delta^i)}{D(\Delta^l)}\frac{c_2^{i-1}}{c_2^{l-1}}, \tag{21}$$

if we define $\Lambda(l, i) \equiv \Sigma_{t=i+1}^{l}\Delta^t$ and $c^0(k^0; k^1) \equiv -b(k^1)$.[14]

Equation (21) holds for arbitrary levels of subsequent s^i, $i < l$. In equilibrium, s^{l-1} is also given by equation (21) if just l is replaced by $l - 1$. When the equation for s_*^{l-1} is combined with equation (21), the expression for s_*^{l-1} simplifies if discounting is quasi-hyperbolic (this is proven in the appendix).

Corollaries to Proposition 4.

1. With exponential discounting, $s_*^l = 0$ for every $l \in \{1, \dots, L\}$.
2. With quasi-hyperbolic discounting, s_*^l accounts only for the effect on k^{l-1}:

$$s_*^l = \left(\frac{1}{\beta} - 1\right)\frac{dk^{l-1}}{dk^l}\frac{c_1^{l-1}}{-c_2^{l-1}}.$$

3. With strictly increasing discount factors, s_*^l is the sum of $l - 1$ terms.

Corollaries 2 and 3 confirm that there is a dramatic difference between quasi-hyperbolic discounting and strictly increasing discount factors. With quasi-hyperbolic discounting, the expression for s_*^l consists of only a single term, and that term is written equivalently for every $l > 1$. The explanation is the same as for equation (19): the decision maker deciding on k^l and the decision maker deciding on k^{l-1} agree on how much more the decision maker deciding on k^{l-2} ought to invest, thanks to discount factors (3) that are constant after $t > 1$. With strictly increasing discount factors, however, the equilibrium subsidy consists of a number of terms that equals the number of subsequent decisions.

B. Stepping-Stone Technologies

To investigate the above intuition further, consider now what I will refer to as stepping-stone technologies. For such technologies, each stage is the stepping-stone for the next. The larger one stepping-stone is, k^{l+1}, the larger k^l also is for any given investment cost at stage l. To formalize this idea, it is convenient to assume that the cost of investing in k^l can be written as $c^l(k^l - \phi^{l+1}k^{l+1})$. We can let $\phi^i = 1\ \forall\ i \in \{1, \dots, L\}$ without loss

[14] Note that s_*^l is defined recursively in proposition 4, in contrast to the expressions in propositions 2 and 3. However, with $s^1 = 0$, proposition 4 can be used to write s_*^2 as s_*^k in proposition 2, and s_*^3 as s_*^r in proposition 3.

of generality.[15] With this, technology k^{l+1} becomes a perfect complement to k^l: one more unit of k^{l+1} makes it possible to also raise k^l by one unit while changing neither the cost nor the marginal cost of investing in k^l. For simplicity, assume that $b(a) = a$.

The study of stepping-stone technologies can be motivated in several ways. One motivation is that these technologies capture quite well the way in which environmentally friendly infrastructure enters the production chain: the amount of energy that can be generated by renewable energy sources reduces, one by one, the amount of greenhouse gas that enters the atmosphere for any given level of energy consumption. For this reason, stepping-stone technologies have already been used in other studies of climate change.[16]

PROPOSITION 5. For stepping-stone technologies, where $c^l(k^l; k^{l+1}) = c^l(k^l - k^{l+1})$, the choice of k^l satisfies equation (20) with the following $s^l \geq 0$, increasing in l:

$$s_*^l = \prod_{t=1}^{\Delta^l} \frac{\delta_{t+\Lambda(l-1,0)}}{\delta_t} - 1.$$

Just as before, the subsidy is zero at the last stage ($s_*^1 = 0$). If discounting is exponential, the subsidy is zero at every stage. And, as a confirmation of the intuition discussed above, the subsidy is constant in l under quasi-hyperbolic discounting but increasing in l when δ_t increases in t.

COROLLARIES TO PROPOSITION 5.

1. With exponential discounting or when $l = 1$, then $s_*^l = 0$.
2. With quasi-hyperbolic discounting, $s_*^l = 1/\beta - 1 > 0$ is constant for all $l > 1$.
3. With strictly increasing δ_t, s_*^l increases strictly in l and Δ^i, $\forall\ l > 1$ and $l \geq i \geq 1$.
4. In the simple case in which $\Delta^i = 1$ for every $i \in \{1, \ldots, l\}$, then

$$s_*^l = \frac{\delta_l}{\delta_1} - 1.$$

[15] If the true investment costs were $c^l(k^l - \phi^{l+1}\tilde{k}^{l+1})$, and the technology level $\tilde{k}^{l+1}$ could be invested in at cost $\tilde{c}^{l+1}(\tilde{k}^{l+1} - \phi^{l+2}\tilde{k}^{l+2})$, then we could simply define $k^{l+1} \equiv \phi^{l+1}\tilde{k}^{l+1}$ and let the investment cost for k^{l+1} be $c^{l+1}(k^{l+1} - \phi^{l+1}\phi^{l+2}\tilde{k}^{l+2}) \equiv \tilde{c}^{l+1}(k^{l+1}/\phi^{l+1} - \phi^{l+2}\tilde{k}^{l+2})$. In an analogous way, we can eliminate $\phi^{l+1}\phi^{l+2}$ and write $c^{l+1}(k^{l+1} - k^{l+2})$ by defining $k^{l+2} \equiv \phi^{l+1}\phi^{l+2}\tilde{k}^{l+2}$ and redefining $c^{l+2}(\cdot)$, and so on.

[16] See, e.g., Harstad (2012) or Battaglini and Harstad (2016). The term "stepping-stone technology" is not used in those papers, even though the technology is a perfect substitute for reducing consumption, as assumed here. Another natural interpretation of these cost functions is that each investment is $k^l - k^{l+1}$ and that these accumulate over time, so that the accumulated level is $\Sigma_{l=L}^1 k^l - k^{l+1} = k^1$ if $k^{L+1} = 0$.

Figure 3 illustrates corollary 4 when $\Delta^i = 1$ for every $i \in \{1, \dots, L\}$. The production stage is measured at the horizontal axis. The solid line measures equilibrium marginal investment costs, $c_1^l = D(l) = \prod_{t=1}^{l}\delta_t$, at each stage in the production chain. Since the investment cost function is convex, a higher $c_1^l(k^l - k^{l+1})$ corresponds to a higher $k^l - k^{l+1}$. The lower dashed line similarly measures investments under laissez-faire (i.e., if $s^l = 0$ for every l): then, $c_1^l = \delta_1^l$. The upper dashed line is in a similar way corresponding to investment expenditures at each stage under commitment, if the decision maker deciding on k^L could commit to how much to invest in all future stages. In this case, investments would be larger and given by $c_1^l = D(L)/D(L-l) = \prod_{t=L-l}^{L}\delta_t$. Finally, the dotted line corresponds to the investment expenditures under exponential discounting for some fixed discount factor $\delta \in (\delta_1, \sqrt[L]{\delta_1 \cdot \delta_2 \cdot \dots \delta_L})$. Relative to any of these three benchmarks, the equilibrium investment expenditures are biased toward the investments that are further upstream and away from the downstream investments. In other words, with time-inconsistent preferences, more of the budget is spent on basic research and the development of fundamental technology, whether we compare to a setting with time consistency, commitment, or the investments in a competitive market under laissez-faire.

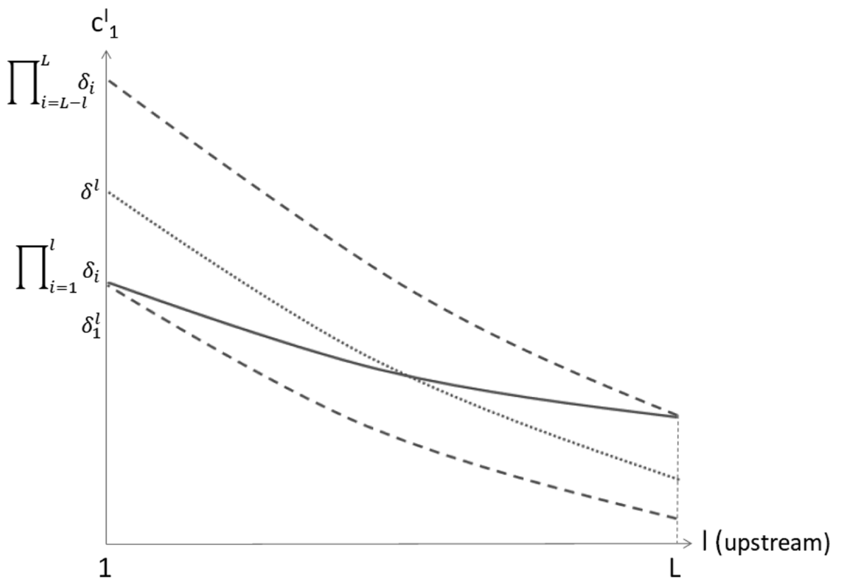

FIG. 3.—Equilibrium upstream investments (solid line) are larger and/or downstream investments are smaller regardless of whether we compare with laissez-faire, exponential discounting, or investments under commitment.

Figure 1 can be derived from figure 3 by choosing specific c^l functions. Suppose that $c^l(k^l - k^{l+1}) \equiv (\varphi^l/2)(k^l - k^{l+1})^2$, where φ^l denotes the constant φ in the power of l. Then, the investment under exponential discounting is $(k^l - k^{l+1}) = (\delta/\varphi)^l$, but $(k^l - k^{l+1}) = 1/(1 + \alpha l)\varphi^l$ under hyperbolic discounting; the former is decreasing in l, but the latter is increasing in l if $\varphi \in (\delta, e^{-\alpha(1+\alpha)})$. Figure 1 is drawn for $(\delta, \varphi, \alpha) = (0.6, 0.63, 0.73)$.

C. *Investments in Multiple Technologies in Multiple Periods*

In the analysis above, the decision on k^l was, for simplicity, taken before the decision on k^{l-1}. The beneficial abatement decision ($a \equiv k^1$) was made only at the end of the sequence. In the climate change application, however, decision makers decide on abatements as well as all kinds of investments in every period. The cost of each investment decision may depend on the upstream level of capital inherited from the previous period. Fortunately, it is straightforward to reformulate the model to capture such a setting.

Suppose now that at every time t a decision maker decides on an investment vector $\mathbf{k}_t = (k_t^1, k_t^2, \dots, k_t^L)$, receives the momentary utility $u_t = b(k_{t-1}^1) - \sum_{l=1}^{L} c^l(k_t^l; k_{t-1}^{l+1})$, and seeks to maximize $v_t(\mathbf{k}_{t-1}) = \max_{k_t} \sum_{\tau=t}^{\infty} D(\tau - t)u_\tau$. (Thus, each lag is $\Delta^l = 1$.) As in section IV.B, $b(\cdot)$ might be a linear function, as when the social cost of carbon stays more or less unchanged when we vary the abatement level ($a_t \equiv k_t^1$).

By inserting the expression for u_τ into $v_t(\mathbf{k}_{t-1})$, we obtain

$$v_t(\mathbf{k}_{t-1}) = \max_{\mathbf{k}_t} \sum_{\tau=t}^{\infty} D(\tau - t)\left[b(k_{\tau-1}^1) - \sum_{l=1}^{L} c^l(k_\tau^l; k_{\tau-1}^{l+1})\right].$$

In this expression, each bracket sums the terms in one column of the payoff matrix illustrated in figure 4. By rearranging the terms, we can instead write

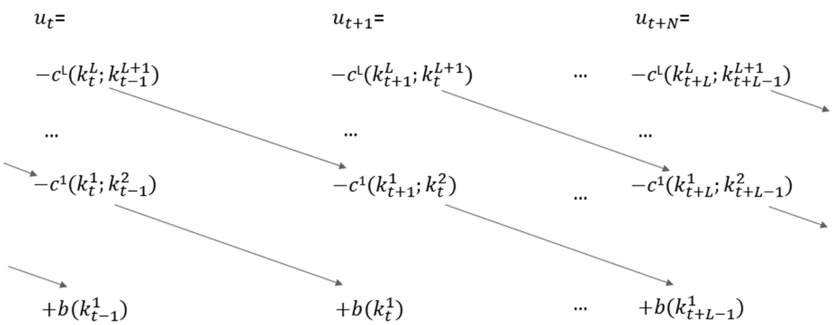

FIG. 4.—Maximizing the vector $(k_t, \dots, k_t^L)$ can be separated into L independent maximization problems.

$$v_t(\mathbf{k}_{t-1}) = \sum_{l=1}^{L} v_t^l\left(k_{t-1}^{l+1}\right) + b\left(k_{t-1}^1\right) \tag{22}$$

$$+ \sum_{\tau=1}^{\infty}\left[D(L+\tau)b\left(k_{t+L+\tau-1}^1\right) - \sum_{j=1}^{L} D(L-j+\tau)c^j\left(k_{t+L-j+\tau}^j; k_{t+L-j+\tau-1}^{j+1}\right)\right], \tag{23}$$

where

$$v_t^l\left(k_{t-1}^{l+1}\right) \equiv \max_{k_t^l} D(l)b\left(k_{t+l-1}^1\right) - \sum_{j=1}^{l} D(l-j)c^j\left(k_{t+l-j}^j; k_{t+l-j-1}^{j+1}\right).$$

Here, each term $v_t^l(k_{t-1}^{l+1})$ summarizes the payoffs along one diagonal arrow in the payoff matrix. The final sum of brackets in equation (23) captures future payoffs that are not influenced by $\mathbf{k}_t$ (i.e., the payoffs along the arrows to the right in fig. 4). Therefore, equation (22) shows that the decision maker's problem of maximizing v_t with respect to $\mathbf{k}_t$ consists of L independent maximization problems. The decision maker decides on k_t^l, $l \in \{1, \ldots, L\}$, taking into account that k_t^l will influence the choice of k_{t+1}^{l-1}, which will influence k_{t+2}^{l-2}, and so on. This problem is independent from the decision maker's problem when choosing $k_t^{l'}$, $l' \neq l$ in this model. Consequently, each maximization problem, as described by $v_t^l(k_{t-1}^{l+1})$, coincides with the problem analyzed in the previous sections where attention was indeed limited to the actions along a single diagonal in figure 4.

V. Multiple Countries

A. *Externalities and Technological Spillovers*

Playing a game with future governments is not so different from playing a game with other contemporary governments. In the previous sections, the future government's investment generates externalities on the present, just as the actions of governments in other countries generate externalities that cross the border. In the latter situation, it may be beneficial to invest strategically much in renewable energy technology, if technological spillovers induce foreign countries to abate more as a result. This requires a subsidy if private investors do not internalize the spillovers because of weak intellectual property rights.[17]

To show the similarity and the interaction between spillovers and time inconsistency, I now permit $n + 1$ identical countries. As in section IV.B, consider stepping-stone technologies; as in section III, limit attention to

[17] This point is made by, e.g., Golombek and Hoel (2004). Harstad, Lanica, and Russo (2019) argue that even without spillovers, countries may need to invest strategically much in green technologies and little in brown technologies to make future cooperation credible in a repeated game between multiple countries. When spillovers are added, e.g., from the north to the south, then the north may want to invest strategically more to motivate the south to continue cooperation. Discounting is exponential in these papers.

three levels, $l \in \{a, k, r\}$. The equilibrium decision in a foreign country has superscript F, and z^l is the externality from each unit of $l \in \{a, k, r\}$ on each of the other n countries. Thus, our decision maker's benefit is $b = a + nz^a a^F$, the cost of a is $c^a(a; K)$, where $K = k + nz^k k^F$, and the cost of k is $c^k(k; R)$, where $R = r + nz^r r^F$. When a is a public good, for example, emission abatement, $z^a = 1$. The technological spillovers z^k and z^r may be larger when intellectual property rights are weak.[18]

When deciding on k, it is understood that a larger k influences K^F. A larger K^F, in turn, leads to a larger a^F, and this is beneficial for everyone. Thus, the home country has an incentive to invest strategically much in k if both z^k and z^a are positive. Just as before, this strategic concern can be measured by the subsidy that the decision maker prefers to impose on private investors. After all, private investors do not take into account the externality on foreign countries, and they invest in k, as explained in section III.

The spillovers are motivating strategic investments in r as well. In general, the larger the spillovers are, the larger the equilibrium subsidies and investment levels are.[19] Yet more importantly, the spillovers are interacting with the discount factors so that the two effects are strategic complements.

Proposition 6. The effects of time inconsistency and spillovers are superadditive.

1. The equilibrium k satisfies equation (14), with

$$s_*^k = \prod_{t=1}^{\Delta^a} \frac{\delta_{t+\Delta^k}}{\delta_t}(1 + nz^a z^k) - 1.$$

2. The equilibrium r satisfies equation (17), with

$$\begin{aligned} s_*^r &= nz^r[z^k + z^a(1 + nz^k)(1 - z^k)] \\ &+ \left(\prod_{t=1}^{\Delta^r} \frac{\delta_{t+\Delta^k}}{\delta_t} - 1\right)\{1 + nz^r[z^k + z^a(1 + nz^k)(1 - z^k)]\} \\ &+ \left(\prod_{t=1}^{\Delta^r} \frac{\delta_{t+\Delta^a+\Delta^k}}{\delta_t} - \prod_{t=1}^{\Delta^r} \frac{\delta_{t+\Delta^k}}{\delta_t}\right)[1 + nz^k z^r + nz^a z^k + nz^a z^r + n(n-1)z^a z^k z^r]. \end{aligned}$$

[18] Since $b_1(a + nz^a) = 1$ with stepping-stone technologies, a^F will not influence the optimal choice of a when the government decides on a by maximizing $v^a(a; K) \equiv -c^a(a; K) + D(\Delta^a)b(a + nz^a a^F, K)$.

[19] An interesting exception is that when z^k is very large, then s_*^r might decrease in z^k. This can be seen from the term $1 - z^k$ in the formula in proposition 6. A very large z^k means that the home country prefers that the foreigners invest more in k^F, and they will do this if they expect the home country to invest less in k (k and k^F are substitutes); thus, the home country might prefer to invest strategically little in r as a commitment device to invest less in k. This possibility can be ruled out by assuming that $z^k < 1/2$.

Interestingly, the effects of spillovers and time inconsistency are superadditive: the effect of one is larger because of the other. Spillovers have a larger effect on the subsidy if the decision maker is time inconsistent, just as time inconsistency has a larger impact on the subsidy if spillovers are important. Since spillovers are likely to be large for both new and green technologies as well as for the international benefits from emission abatements, this complementarity suggests that time inconsistency will have an especially large influence on strategic investments in climate change technologies and policies.

It is also worthwhile to emphasize the following implications of the proposition.

COROLLARIES TO PROPOSITION 6.

1. Even with exponential discounting, the equilibrium subsidies are positive when spillovers are positive:

$$s_*^k = nz^a z^k,$$

$$s_*^r = nz^r[z^k + z^a(1 + nz^k)(1 - z^k)].$$

2. With quasi-hyperbolic discounting, the third term of s_*^r is zero and s_*^r is independent of investment lags.
3. If all investment lags are equal to 1, then

$$s_*^k = \frac{\delta_2}{\delta_1}(1 + nz^a z^k) - 1,$$

$$\begin{aligned} s_*^r = {} & nz^r[z^k + z^a(1 + nz^k)(1 - z^k)] \\ & + \left(\frac{\delta_2 - \delta_1}{\delta_1}\right)\left\{1 + nz^r[z^k + z^a(1 + nz^k)(1 - z^k)]\right\} \\ & + \left(\frac{\delta_3 - \delta_2}{\delta_1}\right)[1 + nz^k z^r + nz^a z^k + nz^a z^r + n(n-1)z^a z^k z^r]. \end{aligned}$$

As before, there are interesting differences between quasi-hyperbolic and strictly increasing discount factors. With quasi-hyperbolic discounting, (1) the lengths of the investment lags are unimportant, (2) there is no superadditivity between time inconsistency and the spillovers in s_*^r when $z^r = 0$ (as when the fundamental research is hard to observe for others), and (3) a larger δ_2 increases s_*^r. These three results are nonrobust and reversed when δ_t is strictly increasing: larger lags then lead to larger s_*^r and s_*^k (and the effect of larger lags and the effect of spillovers are superadditive), the combination of any two effects (from time inconsistency, the lag lengths, the spillovers) is superadditive also when $z^r = 0$, and when all lags are equal to 1, then a larger δ_2 in isolation reduces s_*^r. The last effect arises because a larger δ_2, for a fixed $\delta_3 > \delta_2$, makes the two more similar and reduces the long-term time inconsistency problem. Interestingly, the effect of δ_3 depends on all the spillovers in a symmetric way.

B. A Quantitative Assessment

The international externalities and spillovers described above represent well-known reasons for why governments may want to subsidize investments, even in a situation with time-consistent decision makers. Perhaps surprisingly, the effect of time inconsistency on the equilibrium level of subsidies can be of a similar order, even if we limit ourselves to rather realistic numbers for the discount factors. Fortunately, corollary 3 provides formulas for the subsidies that depend on nothing else than the spillovers, the discount factors, and the number of countries.

Table 2 shows how the equilibrium subsidies (in percentages) vary with discount factors $(\delta_1, \delta_2, \delta_3)$ and international spillovers (z^a, z^k, z^r), assuming three large players (e.g., United States, Europe, and China). The first row assumes exponential discounting and shows how the subsidies increase in the spillovers. If there are no spillovers, then $s_*^k = s_*^r = 0$. If abatement is a public good ($z^a = 1$) while the spillovers z^k and z^r are both 10%, then s_*^k is 20%, while s_*^r is 24% (row A, col. 3). This pair is comparable to the situation without spillovers but with the time-inconsistent discount factors $(\delta_1, \delta_2, \delta_3) = (0.7, 0.8, 0.9)$ (as in row D, col. 1), although the difference s_*^r/s_*^k is larger in the time inconsistency case.

More importantly, when these two effects are combined (as in row D, col. 3), the equilibrium s_*^k (i.e., 0.37) is 9% higher and s_*^r (i.e., 62) is 17% higher than if we simply sum the two individual effects.

Larger discount factor differences lead to larger subsidies. When discount factors are as in table 1 (when $q = 3/5$ and $z = 1/2$), copied in the last row, then s_*^k is as large as 65% and s_*^r is 90%, even in the absence of spillovers. Similarly, larger spillovers lead to larger subsidies. When $(z^a, z^k, z^r) = (1, 2/10, 2/10)$, as in column 5, then s_*^k is 40% and s_*^r is 53%, even with constant discount factors. When these two effects are combined, the equilibrium s_*^k (i.e., 1.30) is 24% higher and the equilibrium s_*^r (i.e., 2.0) is 40% higher than the numbers we find by simply summing the two individual effects.

This quantitative assessment confirms that the two effects are superadditive. In other words, the effect of time inconsistency is larger in a

TABLE 2
Equilibrium Subsidies Are Larger with Time Inconsistency and Spillovers

		(z^a, z^k, z^r)				
		(0, 0, 0)	$(\frac{1}{10}, \frac{1}{10}, \frac{1}{10})$	$(1, \frac{1}{10}, \frac{1}{10})$	$(1, \frac{2}{10}, \frac{1}{10})$	$(1, \frac{2}{10}, \frac{2}{10})$
	$(\delta_1, \delta_2, \delta_3)$	(1)	(2)	(3)	(4)	(5)
A	(.9, .9, .9)	(0, 0)	(2, 4)	(20, 24)	(40, 26)	(40, 53)
B	(.8, .9, .9)	(13, 13)	(15, 17)	(35, 39)	(58, 42)	(58, 72)
C	(.7, .9, .9)	(29, 29)	(31, 34)	(54, 59)	(80, 63)	(80, 96)
D	(.7, .8, .9)	(14, 29)	(17, 34)	(37, 62)	(60, 68)	(60, 103)
E	(.48, .79, .91)	(65, 90)	(68, 98)	(96, 139)	(130, 150)	(130, 200)

Note.—The table reports on the subsidies in percentages $(100s_*^k, 100s_*^r)$ when $n = 2$.

situation in which the players are also affected by externalities and spillovers, such as climate change.

VI. Conclusions

There is a growing body of evidence indicating that individuals are more patient regarding long-term decisions than regarding short-term decisions. Furthermore, policy makers who fear losing elections will find it optimal to apply discount factors that increase in relative time, even if no individual is endowed with time-inconsistent preferences for exogenous reasons. Time inconsistency, possibly due to such political failures, can thus motivate regulatory policies even in the absence of traditional market failures.

The analysis above offers several predictions when today's decision maker seeks to influence the future decision makers: (1) it is beneficial to raise or subsidize investments in green technologies that are complementary to future investments but to tax investments in brown technologies that substitute for future investments, (2) the subsidies should be larger for technologies that are more fundamental and higher upstream in the technology chain, and (3) subsidies should be larger for technologies that have long maturity.

The results can be interpreted as normative recommendations for investment policies in the presence of time inconsistency. If all investments are strategic complements, as in sections IV.B and V, then each decision maker always benefits from the strategic subsidies downstream. After all, one decision maker subsidizes current investments in order to motivate larger investments downstream, and these downstream investments are also larger when the future decision maker is strategic. However, if investments are strategic substitutes, as with brown capital, then one decision maker might invest strategically little in order to raise future investments, but this strategy can be harmful for the earlier decision makers. (A related effect is emphasized by Krusell, Kurusçu, and Smith [2002] and Hiraguchi [2014]). In this case, the strategies that benefit one decision maker do not necessarily benefit others.

The results can also be interpreted as empirically testable predictions. Recent lab experiments are in fact supporting the basic predictions of the model (Dengler et al. 2018). Specifically, quasi-hyperbolic discounting, which is the standard way of modeling time-inconsistent preferences, does not permit predictions 1 and 2. Quasi-hyperbolic discounting is thus distinguishable from and a poor approximation for more general time-inconsistent discounting.

The quantitative assessment suggests that time inconsistency can rationalize subsidies at similar levels as traditional market failures, such as externalities and technological spillovers. Interestingly, the effects of time inconsistency and externalities are superadditive: the effect of one is larger in the presence of the other. A testable prediction is thus that policy measures on investments in research and development may be more substantial than we find rationalizable by externalities alone. Economists and policy commentators often criticize symbolic policies and the reliance on extensive subsidies, especially in environmental policy. The analysis above motivates second thoughts before such policies are repealed.

Appendix

Proofs of Propositions 1–6

A1. Proof of Proposition 1

To generalize proposition 1, suppose that there are $m + 1$ political parties, the return of the investment arrives after $\Delta \geq 1$ periods, and ξ/m is the fraction of the total social return enjoyed by each opposition party (in sec. II.B, where $m = 1, \xi = z/(1 + z)$). The current decision maker stays in power with probability q, and each party outside office gains power with probability $(1 - q)/m$. The probability of staying in power at time t is then

$$p_t = \left(q - \frac{1-q}{m}\right)^t \left(p_0 - \frac{1}{1+m}\right) + \frac{1}{1+m},$$

replacing equation (4), which continues to hold when $m = 1$. When we consider the current incumbent, then $p_0 = 1$ and p_t can be written as

$$p_t = \frac{1}{1+m}\left[1 + m\left[q\left(1 + \frac{1}{m}\right) - \frac{1}{m}\right]^t\right].$$

With this, we obtain the decision maker's present discounted value from one unit of social return at time $t + \Delta$, relative to letting the decision maker at time t consume that unit, evaluated at time zero; that is,

$$\begin{aligned}
\frac{D(t+\Delta)}{D(t)} &= \delta^{t+\Delta}\frac{p_{t+\Delta}(1-\xi) + (1-p_{t+\Delta})\xi/m}{\delta^t p_t} \\
&= \delta^\Delta \frac{\frac{\xi}{m} + \left[1 - \xi\left(1+\frac{1}{m}\right)\right]\frac{1}{1+m}\left\{1 + m\left[q\left(1+\frac{1}{m}\right) - \frac{1}{m}\right]^{t+\Delta}\right\}}{\frac{1}{1+m}\left\{1 + m\left[q\left(1+\frac{1}{m}\right) - \frac{1}{m}\right]^t\right\}} \\
&= \delta^\Delta\left[1 - \xi\left(1+\frac{1}{m}\right)\right]\left[q\left(1+\frac{1}{m}\right) - \frac{1}{m}\right]^\Delta \\
&\quad + \delta^\Delta(1+m)\frac{\frac{\xi}{m} + \left(\frac{1}{1+m} - \frac{\xi}{m}\right)\left\{1 - \left[q\left(1+\frac{1}{m}\right) - \frac{1}{m}\right]^\Delta\right\}}{1 + m\left[q\left(1+\frac{1}{m}\right) - \frac{1}{m}\right]^t} \\
&= \delta^\Delta\left[1 - \xi\left(1+\frac{1}{m}\right)\right]\left[q\left(1+\frac{1}{m}\right) - \frac{1}{m}\right]^\Delta \\
&\quad + \frac{\delta^\Delta}{m}\frac{\xi(1+m) + (1 - \xi[1+m])\left\{1 - \left[q\left(1+\frac{1}{m}\right) - \frac{1}{m}\right]^\Delta\right\}}{1 + m\left[q\left(1+\frac{1}{m}\right) - \frac{1}{m}\right]^t},
\end{aligned}$$

which increases strictly in t. With this, it is easy to check that the derivative with respect to t decreases in t and increases in ξ (and thus in $z = \xi/(1-\xi)$), proving the statement in proposition 1 for this somewhat more general model. This discount factor simplifies to equation (6) if we rewrite after imposing $m = \Delta = 1$ and using the definition $\delta_t \equiv D(t)/D(t-1)$. QED

A2. *Proofs of Propositions 2 and 6 (Part 1)*

The proof permits $n+1$ countries and spillovers z^a and z^k (see sec. V). Each of the other foreign countries contributes a^F and invests k^F in equilibrium.

A2.1. The Abatement Stage

At stage a, the decision maker maximizes

$$v^a(a;K) \equiv -c^a(a;K) + D(\Delta^a)b(a + nz^a a^F, K),$$

so the first-order condition for a is given by

$$c_1^a = D(\Delta^a)b_1, \tag{A1}$$

while the second-order condition is

$$v_{11}^a = -c_{11}^a + D(\Delta^a)b_{11} \le 0.$$

To see how a depends on K, we can differentiate equation (A1) to obtain

$$\begin{aligned} c_{11}^a da + c_{12} dK = D(\Delta^a)b_{11} da + D(\Delta^a)b_{12} dK \Leftrightarrow \frac{da}{dK} &= \frac{D(\Delta^a)b_{12} - c_{12}^a}{c_{11}^a - D(\Delta^a)b_{11}} \\ &= \frac{D(\Delta^a)b_{12} - c_{12}^a}{-v_{11}^a}. \end{aligned}$$

Note that I here could ignore how the modified a influences a^F (which in turn could influence a) because (1) proposition 2 assumes that $n=1$, while (2) proposition 6 assumes that b_1 is independent of a^F (and then the optimal a^F does not change with a). Furthermore, when $b_2 = 0$, we obtain

$$\frac{da}{dK} = \frac{-c_{12}^a}{-v_{11}^a},$$

and when, in addition, b is linear in a,

$$\frac{da}{dK} = \frac{-c_{12}^a}{c_{11}^a}.$$

A2.2. The Investment Stage

At stage k, the decision maker's objective is to maximize the continuation value:

$$\begin{aligned} v^k(k;R) \equiv &-c^k(k;R) - D(\Delta^k)c^a(a; k + nz^k k^F) \\ &+ D(\Delta^k + \Delta^a)b(a + nz^a a^F, k + nz^k k^F). \end{aligned}$$

By taking the total derivative of $v^k(k;R)$ with respect to k, we obtain the first-order condition for k:

$$v_1^k \equiv -c_1^k - D(\Delta^k)\left(c_1^a \frac{da}{dK} + c_2^a\right) + D(\Delta^k + \Delta^a)\left[\left(\frac{da}{dK} + nz^a \frac{da^F}{dK^F}\frac{dK^F}{dk}\right)b_1 + b_2\right] = 0.$$

The second-order condition is $v_{11}^k \leq 0$ and holds when we assume that c_{11}^k is sufficiently large.

When we substitute with equation (A1) and take into account that in equilibrium, $da^F/dK^F = da/dK$, then we can write the first-order condition k as

$$c_1^k = -D(\Delta^k)c_2^a + D(\Delta^k + \Delta^a)b_2 - D(\Delta^k)D(\Delta^a)b_1 \frac{da}{dK} + D(\Delta^k + \Delta^a)(1 + nz^a z^k)b_1 \frac{da}{dk}. \quad \text{(A2)}$$

The market solution is

$$\frac{c_1^k}{1 + s^k} = -D(\Delta^k)c_2^a + D(\Delta^k + \Delta^a)b_2. \quad \text{(A3)}$$

A2.3. The Investment Policy

Equation (A3) coincides with equation (A2) if and only if s^k is

$$\begin{aligned} s_*^k &= \frac{D(\Delta^k + \Delta^a)(1 + nz^a z^k) - D(\Delta^k)D(\Delta^a)}{-D(\Delta^k)c_2^a + D(\Delta^k + \Delta^a)b_2} b_1 \frac{da}{dK} \\ &= \frac{[D(\Delta^k + \Delta^a)/D(\Delta^k)D(\Delta^a)](1 + nz^a z^k) - 1}{-c_2^a + D(\Delta^k + \Delta^a)b_2/D(\Delta^k)} c_1^a \frac{da}{dk}. \end{aligned}$$

When $b_2 = 0$, then

$$s_*^k = \left[\frac{D(\Delta^k + \Delta^a)}{D(\Delta^k)D(\Delta^a)}(1 + nz^a z^k) - 1\right] \frac{c_1^a}{-c_2^a} \frac{da}{dk}, \text{ where } \frac{D(\Delta^k + \Delta^a)}{D(\Delta^k)D(\Delta^a)} = \prod_{t=1}^{\Delta^a} \frac{\delta_{t+\Delta^k}}{\delta_t},$$

when we use equation (2). When, in addition, the spillovers are zero, as in proposition 2, then s_*^k simplifies to equation (15). If, instead, $c^a(a; k) = \tilde{c}^a(a - k)$, as with stepping-stone technology, then $[c_1^a/(-c_2^a)](da/dk) = 1$, and s_*^k can be rewritten as in proposition 6 (point 1). QED

A3. Remark on Adaptation Technology

Adaptation technology or capital refers to investments that enhance the economy's ability to deal with pollution. For example, one can invest in agricultural products that can cope with pollution or climate change, or one can build infrastructure that is robust to pollution, climate change, or sea-level rises. Such capital not only increases the future benefit, $b(a, k)$, but also reduces the marginal environmental harm; in other words, a larger level of k reduces the value of a so that $b_{12} = \partial^2 b(a, k)/\partial a \partial k < 0$. Such adaptation capital does not affect the cost of abating, so $\partial c(a; k)/\partial k = 0$. When we combine the formula for s_*^k and the formula for da/dk, we obtain, when there are no spillovers,

$$s_*^k = \left[1 - \frac{D(\Delta^k)D(\Delta^a)}{D(\Delta^k + \Delta^a)}\right] \frac{b_1}{b_2} \frac{D(\Delta^a)b_{12}}{-v_{11}^a} < 0.$$

Adaptation can be a good thing in that $\partial b(a,k)/\partial k > 0$. However, even private investors will account for the value $\partial b(a,k)/\partial k$ in a perfect market, so this creates no reason to strategically distort k. On the contrary, more investments in adaptation will reduce the cost of polluting, and the level of abatement will thus be reduced as well. The decision maker of today prefers a larger a in the future, and this can be achieved by strategically reducing and taxing investments in adaptation capital. For simplicity, I henceforth assume $\partial b(\cdot)/\partial k = 0$.

A4. Proof of Propositions 3 and 6 (Part 2)

At the r stage, the decision maker prefers to maximize the continuation value

$$\begin{aligned} v^r(r) &= -c^r(r) - D(\Delta^r)c^k(k; r + nz^r r^F) - D(\Delta^r + \Delta^k)c^a(a; k + nz^k k^F) \\ &\quad + D(\Delta^r + \Delta^k + \Delta^a)b(a + nz^a a^F). \end{aligned}$$

Thus, the first-order condition for r can be written as

$$\begin{aligned} 0 = v_1^r &= -c_1^r - D(\Delta^r)c_1^k \frac{dk}{dR} - D(\Delta^r)c_2^k - D(\Delta^r + \Delta^k)c_1^a \frac{da}{dK}\cdot\left(\frac{dk}{\mathrm{dR}} + nz^k z^r \frac{dk^F}{dR^F}\right) \\ &\quad - D(\Delta^r + \Delta^k)c_2^a\left(\frac{dk}{dR} + nz^k z^r \frac{dk^F}{dR^F}\right) \\ &\quad + D(\Delta^r + \Delta^k + \Delta^a)b_1\left\{\frac{da}{dK}\left(\frac{dk}{dR} + nz^k z^r \frac{dk^F}{dR^F}\right) + nz^a \frac{da^F}{dK^F}\left[z^r \frac{dk^F}{dR^F} + (n-1)z^k z^r \frac{dk^F}{dR^F} + z^k \frac{dk}{dR}\right]\right\}, \end{aligned} \tag{A4}$$

where I have taken into account that

$$\begin{aligned} \frac{dR}{dr} &= \frac{d(r + nz^r r^F)}{dr} = 1, \\ \frac{dR^F}{dr} &= z^r, \\ \frac{dK}{dr} &= \frac{d(k + nz^k k^F)}{dr} = \frac{dk}{dR} + nz^k z^r \frac{dk^F}{dR^F}, \\ \frac{dK^F}{dr} &= \frac{d[k^F + (n-1)z^k k^F + z^k k]}{dr} = z^r \frac{dk^F}{dR^F} + (n-1)z^k z^r \frac{dk^F}{dR^F} + z^k \frac{dk}{dR}. \end{aligned}$$

Note that as in the proof of propositions 2 and 6 (point 1), I could ignore that the modified a could influence a^F (which in turn could influence a) because (1) $n = 1$ in proposition 3 and (2) $b_{11} = 0$ in proposition 6. Similarly, I can here ignore that when r changes, the modified choice of k could influence the choice of k^F (which in turn could influence k) because (1) $n = 1$ in proposition 3 and (2) in proposition 6, I consider stepping-stone technologies where b_1 is a constant.

If we differentiate the first-order condition for k, we can derive $dk/dR = v_{12}^k/(-v_{11}^k)$, where $-v_{11}^k \geq 0$ is the second-order condition associated with the first-order condition for k. The second-order condition associated with the first-order condition for r (eq. [A4]) is $v_{11}^r \leq 0$ and holds if c_{11}^r is sufficiently large, which I henceforth assume.

Since in equilibrium $dk^F/dR^F = dk/dR$ and $da^F/dK^F = da/dK$, we can write the first-order condition for r (eq. [A4]) as

$$
\begin{aligned}
c_1^r = & -D(\Delta^r)c_2^k - D(\Delta^r)c_1^k\frac{dk}{dR} - D(\Delta^r+\Delta^k)(1+nz^kz^r)c_2^a\frac{dk}{dR} \\
& - D(\Delta^r+\Delta^k)(1+nz^kz^r)c_1^a\frac{da}{dK}\frac{dk}{dR} \\
& + D(\Delta^r+\Delta^k+\Delta^a)[1+nz^kz^r+nz^a[z^r+(n-1)z^kz^r+z^k]]b_1\frac{da}{dK}\frac{dk}{dR}.
\end{aligned}
$$

Then, by using the first-order condition for k to substitute for c_2^a, the first-order condition for r (eq. [A4]) can be written as

$$
\begin{aligned}
c_1^r = & -D(\Delta^r)c_2^k - D(\Delta^r)c_1^k\frac{dk}{dR} \\
& -(1+nz^kz^r)\frac{D(\Delta^r+\Delta^k)}{D(\Delta^k)}\left[-c_1^k+[D(\Delta^k+\Delta^a)(1+nz^az^k)-D(\Delta^k)D(\Delta^a)]b_1\frac{da}{dK}\right]\frac{dk}{dR} \\
& - D(\Delta^r+\Delta^k)(1+nz^kz^r)c_1^a\frac{da}{dK}\frac{dk}{dR} \\
& + D(\Delta^r+\Delta^k+\Delta^a)[1+nz^kz^r+nz^a[z^r+(n-1)z^kz^r+z^k]]b_1\frac{da}{dK}\frac{dk}{dR}.
\end{aligned}
$$

Next, by using the first-order condition for a to substitute in for c_1^a and rewriting the equation, we obtain

$$
\begin{aligned}
c_1^r = & -D(\Delta^r)c_2^k + \left[\frac{D(\Delta^r+\Delta^k)}{D(\Delta^k)}(1+nz^kz^r) - D(\Delta^r)\right]c_1^k\frac{dk}{dR} \\
& + \left[D(\Delta^r+\Delta^k+\Delta^a) - \frac{D(\Delta^r+\Delta^k)D(\Delta^k+\Delta^a)}{D(\Delta^k)}\right](1+nz^kz^r)(1+nz^az^k)b_1\frac{da}{dK}\frac{dk}{dR} \\
& + D(\Delta^r+\Delta^k+\Delta^a)nz^az^r(1-z^k)(1+nz^k)b_1\frac{da}{dK}\frac{dk}{dR}.
\end{aligned}
$$

Since the market equilibrium is $c_1^r/(1+s^r) = -D(\Delta^r)c_2^k$, the two are equal if and only if s^r equals

$$
\begin{aligned}
s_*^r = & \left[\frac{D(\Delta^r+\Delta^k)}{D(\Delta^k)D(\Delta^r)}(1+nz^kz^r) - 1\right]\frac{c_1^k}{-c_2^k}\frac{dk}{dR} \\
& + \left[\frac{D(\Delta^r+\Delta^k+\Delta^a)}{D(\Delta^r)} - \frac{D(\Delta^r+\Delta^k)D(\Delta^k+\Delta^a)}{D(\Delta^k)D(\Delta^r)}\right](1+nz^kz^r)(1+nz^az^k)\frac{b_1}{-c_2^k}\frac{da}{dK}\frac{dk}{dR} \\
& + \frac{D(\Delta^r+\Delta^k+\Delta^a)}{D(\Delta^r)}nz^az^r(1-z^k)(1+nz^k)\frac{b_1}{-c_2^k}\frac{da}{dK}\frac{dk}{dR}.
\end{aligned}
$$

When the spillovers are zero, as in proposition 3, then

$$
\begin{aligned}
s_*^r = & \left[\frac{D(\Delta^r+\Delta^k)}{D(\Delta^k)D(\Delta^r)} - 1\right]\frac{c_1^k}{-c_2^k}\frac{dk}{dR} \\
& + \left[\frac{D(\Delta^r+\Delta^k+\Delta^a)}{D(\Delta^r)D(\Delta^k+\Delta^a)} - \frac{D(\Delta^r+\Delta^k)}{D(\Delta^k)D(\Delta^r)}\right]D(\Delta^k+\Delta^a)\frac{b_1}{-c_2^k}\frac{da}{dK}\frac{dk}{dR},
\end{aligned}
$$

which can be written as in proposition 3 when we use point 2. If, instead, $c^a(a;k) = \tilde{c}^a(a-k)$, $c^k(k;r) = \tilde{c}^k(k-r)$, and $b_1 = 1$, as with the stepping-stone technologies in proposition 6, then (since $-c_2^k = c_1^k = D(\Delta^k + \Delta^a)b_1$) s_*^r can be written as in proposition 6 (point 2). QED

A4.1. Proof of Proposition 4

Given that $c^0(k^0;k^1) \equiv -b(k^1)$, the l-stage decision maker investing k^l maximizes

$$v^l = \sum_{j=0}^{l} -D(l-j)c^j\left(k^j;k^{j+1}\right).$$

Maximizing v^l with respect to k^l is directly giving the first-order condition:

$$\begin{aligned}
v_1^l\left(k^l;k^{l+1}\right) &\equiv \frac{d}{dk^l}\sum_{j=0}^{l} -D(l-j)c^j\left(k^j;k^{j+1}\right) = 0 \Leftrightarrow \\
c_1^l\left(k^l;k^{l+1}\right) &= D(\Delta^l)\left(-c_2^{l-1}\right) - \sum_{j=1}^{l-1} D(\Lambda(l,j))\frac{dk^j}{dk^l}\left(c_1^j + \frac{D(\Lambda(l,j-1))}{D(\Lambda(l,j))}c_2^{j-1}\right) \\
&= D(\Delta^l)\left(-c_2^{l-1}\right) - \sum_{j=1}^{l-1} D(\Lambda(l,j))\frac{dk^j}{dk^l}\left[(1+s^j)\left(-c_2^{j-1}\right)D(\Delta^j) + \frac{D(\Lambda(l,j))}{D(\Lambda(l,j))}c_2^{j-1}\right] \\
&= D(\Delta^l)\left(-c_2^{l-1}\right) + \sum_{j=1}^{l-1} D(\Lambda(l,j))\frac{dk^j}{dk^l}\left[\prod_{t=1}^{\Delta^j}\frac{\delta_{t+\Lambda(l,j)}}{\delta_t} - (1+s^j)\right]D(\Delta^j)\left(-c_2^{j-1}\right).
\end{aligned}$$

The second-order condition is $v_{11}^l \equiv dv_1^l(k^l;k^{l+1})/dk^l < 0$, which I require to hold, and it does hold if c^l is sufficiently convex in k^l. By differentiating the first-order condition $v_1^l(k^l;k^{l+1}) = 0$ with respect to k^{l+1}, we obtain

$$\frac{dk^l}{dk^{l+1}} = -\frac{\partial^2 c^l\left(k^l;k^{l+1}\right)}{\partial k^l \partial k^{l+1}}\left(\frac{1}{-v_{11}^l}\right), \tag{A5}$$

and $dk^{l-i}/dk^l = \prod_{j=l-i+1}^{l}(dk^{j-1}/dk^j)$. Since private investments ensure that $c_1^l = (1+s^l)D(\Delta^l)(-c_2^{l-1})$, the two are equal if equation (21) holds. QED

A4.2. Proof of Corollary 2 to Proposition 4

With quasi-hyperbolic discounting, equation (21) can be written as

$$\begin{aligned}
s_*^l &= \sum_{j=1}^{l-1}\left[\frac{1}{\beta} - (1+s^j)\right]\frac{dk^j}{dk^l}D(\Lambda(l,j))\frac{D(\Delta^j)}{D(\Delta^l)}\frac{c_2^{j-1}}{c_2^{l-1}} \Rightarrow \\
s_*^l &= \sum_{j=1}^{l-1}\left[\frac{1}{\beta} - (1+s^j)\right]\frac{dk^j}{dk^l}\beta\delta^{\Lambda(l-1,j-1)}\frac{c_2^{j-1}}{c_2^{l-1}} \Rightarrow \\
s_*^{l-1} &= \sum_{j=1}^{l-2}\left[\frac{1}{\beta} - (1+s^j)\right]\frac{dk^j}{dk^{l-1}}\beta\delta^{\Lambda(l-2,j-1)}\frac{c_2^{j-1}}{c_2^{l-2}}.
\end{aligned}$$

When the last term is separated from the sum, we can write

$$
\begin{aligned}
s_*^l = {} & \left[\frac{1}{\beta} - \left(1 + s^{l-1}\right)\right] \frac{dk^{l-1}}{dk^l} D\left(\Delta^{l-1}\right) \frac{c_2^{l-2}}{c_2^{l-1}} \\
& + \left[\sum_{j=1}^{l-2}\left[\frac{1}{\beta} - (1 + s^j)\right] \frac{dk^j}{dk^{l-1}} D(\Lambda(l,j)) \frac{D(\Delta^j)}{D(\Delta^l)} \frac{c_2^{j-1}}{c_2^{l-2}}\right] \frac{c_2^{l-2}}{c_2^{l-1}} \frac{dk^{l-1}}{dk^l} \\
= {} & \left(\frac{1}{\beta} - 1\right) \frac{dk^{l-1}}{dk^l} \frac{c_1^{l-1}}{\left(-c_2^{l-1}\right)}.
\end{aligned}
$$

QED

A4.3. Proof of Proposition 5

With stepping-stone technologies, $c_1^l/(-c_2^l) = 1$. Thus, the first-order conditions when deciding on k^1 is $c_1^1(k_l - k_{l+1}) = D(\Delta^1) \Rightarrow k^1 = (c_1^1)^{-1}(D(\Delta^1)) + k^2$, so $dk^1/dk^2 = 1$. When deciding on k^2, it is $c_1^2(k^2 - k^3) = D(\Delta^1 + \Delta^2) \Rightarrow k^2 = (c_1^2)^{-1}[D(\Delta^1 + \Delta^2)] + k^3$, so $dk^2/dk^3 = 1$, and so on. Recursively, when $dk^{l'-1}/dk^{l'} = 1$ for every $l' \leq 1$, the first-order condition for k^l can be written as $k^l = (c_1^l)^{-1}(D(\Lambda(l-1,0))) + k^{l+1}$, so $dk^l/dk^{l+1} = 1$, and the decision maker's first-order condition simplifies to $c_1^l = D(\Lambda(l,0))$. Similarly, $c_1^{l-1} = D(\Lambda(l-1,0))$, which measures the decision maker's willingness to pay when deciding on $l-1$. The market thus ensures that

$$
c_1^l(k_l; k_{l+1}) = (1 + s_l)D\left(\Lambda^{l-1,0}\right),
$$

which coincides with the decision maker's preferred choice, $c_1^l = D(\Lambda(l,0))$, if and only if

$$
s^l = \frac{D(\Lambda(l,0))}{D(\Lambda(l-1,0))} - 1 = \prod_{t=1}^{\Delta_l} \frac{\delta_{t+\Lambda(l-1,0)}}{\delta_t} - 1,
$$

when we use equation (2). The second-order conditions hold as long as every c^l is convex. QED

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